Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
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2	
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TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
January 2012

Mathematics MFP4

Unit Further Pure 4

Friday 27 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 The vectors **a** and **b** are such that $\mathbf{a} \cdot \mathbf{b} = 21$, $|\mathbf{a}| = 5\sqrt{2}$ and $|\mathbf{b}| = 3$.

Determine the exact value of $|\mathbf{a} \times \mathbf{b}|$. *(5 marks)*

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2 Describe the single transformation represented by each of the matrices:

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$; *(2 marks)*

(b) $\begin{bmatrix} 0.6 & 0 & -0.8 \\ 0 & 1 & 0 \\ 0.8 & 0 & 0.6 \end{bmatrix}$. *(3 marks)*

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3 (a) Find the eigenvalues and corresponding eigenvectors of the matrix $\mathbf{M} = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$.

(6 marks)

(b) The plane transformation T is given by the matrix **M**. Write down the coordinates of the invariant point of T. *(1 mark)*

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4 Let $\mathbf{X} = \begin{bmatrix} 3 & x \\ -1 & 7 \end{bmatrix}$.

(a) Determine $\mathbf{X}\mathbf{X}^{\mathrm{T}}$. *(2 marks)*

(b) Show that $\mathrm{Det}(\mathbf{X}\mathbf{X}^{\mathrm{T}} - \mathbf{X}^{\mathrm{T}}\mathbf{X}) \leqslant 0$ for all real values of x. *(4 marks)*

(c) Find the value of x for which the matrix $(\mathbf{X}\mathbf{X}^{\mathrm{T}} - \mathbf{X}^{\mathrm{T}}\mathbf{X})$ is singular. *(1 mark)*

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5 (a) Determine the two values of the integer n for which the system of equations

$$\begin{aligned} 2x + ny + \quad z &= 5 \\ 3x - \quad y + nz &= 1 \\ -x + 7y + \quad z &= n \end{aligned}$$

does not have a unique solution. *(4 marks)*

(b) For the positive value of n found in part **(a)**, determine whether the system is consistent or inconsistent, and interpret this result geometrically. *(6 marks)*

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6 The planes Π_1 and Π_2 have equations

$$\mathbf{r}\,.\begin{bmatrix}2\\1\\7\end{bmatrix} = 10 \quad \text{and} \quad \mathbf{r}\,.\begin{bmatrix}3\\1\\-4\end{bmatrix} = 7$$

respectively.

(a) Determine, to the nearest degree, the acute angle between Π_1 and Π_2. *(4 marks)*

(b) By setting $z = t$, find cartesian equations for the line of intersection of Π_1 and Π_2 in the form

$$\frac{x-a}{l} = \frac{y-b}{m} = z = t$$ *(6 marks)*

(c) The line L, with equation $\mathbf{r} = \begin{bmatrix}20\\-1\\7\end{bmatrix} + \lambda\begin{bmatrix}1\\9\\4\end{bmatrix}$, intersects Π_1 at the point P and Π_2 at the point Q.

Show that $PQ = k\sqrt{2}$, where k is an integer. *(6 marks)*

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7 The plane transformation T is a rotation through θ radians anticlockwise about O, and maps points (x, y) onto image points (X, Y) such that

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

where $c = \cos\theta$ and $s = \sin\theta$.

(a) Write down the inverse of the matrix $\begin{bmatrix} c & -s \\ s & c \end{bmatrix}$ and hence show that

$$x = cX + sY \quad \text{and} \quad y = -sX + cY$$ *(3 marks)*

(b) The curve C has equation $x^2 - 6xy - 7y^2 = 8$.

The image of C under T is the curve C' with equation $pX^2 + qXY + rY^2 = 8$.

(i) Use the results of part **(a)** to show that

$$q = 6s^2 + 16sc - 6c^2$$

and express p and r similarly in terms of c and s. *(4 marks)*

(ii) Given that θ is an acute angle, find the values of c and s for which $q = 0$ and hence in this case express the equation of C' in the form

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$$ *(8 marks)*

(iii) Hence explain why C is a hyperbola. *(1 mark)*

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8 For $n \neq 1$, the vectors **a**, **b** and **c** are such that

$$\mathbf{a} = \begin{bmatrix} 1 \\ n \\ n^2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2n \\ 2n^2 + n \\ -1 \end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix} n-1 \\ n^2 - 1 \\ 1 - n^2 \end{bmatrix}$$

Determine the value of n for which **a**, **b** and **c** are linearly dependent. *(9 marks)*

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END OF QUESTIONS