Exploration of questions from GCSE Maths A: identifying the types of question that candidates find the most and least demanding, within a topic.

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Introduction

Following each live examination session, senior examiners for GCSE and A-level produce reports on candidate performance in that examination. This paper explores how such reporting might be further enhanced, through the inclusion of the outcomes from detailed statistical analysis of live test data.

The qualification selected for this initial piece of work was the AQA GCSE Mathematics A specification, at Higher Tier. As a Higher Tier qualification, grades A* - D are available. The data analysed came from the 2008-2011 examinations. This qualification is linear, with candidates completing their examination at the end of two years of study.

The topic areas selected for this study were:

- Percentages
- Quadratic equations
- Fractions
- Probabilities

It is important to note that there are two potential sources of difficulty in an examination. The first is the difficulty of the topics; the second is the difficulty of the specific questions asked. It would, for example, be possible to ask an easy question about a difficult topic, or a difficult question about an easy topic. For that reason, the topics selected for this report were selected because the format of the questions had been consistent over time.

Methodology

Candidates' raw marks were first converted into mean percentages, to give the cohort average score per question. For example, a score of 60% signifies that, on average, 60% of the total marks available for a question were obtained by candidates. As questions were taken from different years, this definition of 'difficulty' assumes that candidates' abilities remained unchanged in this period. It also assumes that the question order did not affect the difficulty of the questions.

To relate these percentages to the grading standard, a concept of mastery learning was adopted. Candidates from each grade band were assumed to have mastered a topic when, on average, they gained 80% of the available marks for a question.

Interpreting the outcomes of the analysis

Figures 1 – 4 represent the difficulty of questions within each of the four topics selected, and the level of student ability, represented by grade, at which mastery of each question was achieved.

The height of each question box relates to the average percentage score of the candidates in that cohort. Thus, the lower the height of the box, the more difficult the question. The colour of the question box represents the grade at which the question appeared to have been mastered.

So, for example, in Figure 2 on quadratic equations, the question 'factorise x2 + 3x' was less demanding for candidates, with a mean percentage score of 72%, thus it has a greater height. It was mastered at grade B. The question 'solve 2x2 + 3x - 7 = 0' was more difficult for candidates, with a mean percentage score of 25%, so the height is much lower. It was mastered at grade A*.

Further work

Following this initial exploratory study, it would be useful to consider issues such as:

- To what extent do the teachers' perceptions of the specific difficulties faced by their candidates align with the outcomes from this analysis?
- To what extent are the assumptions of key stakeholders about what students at different grades are likely to know, understand and be able to do, supported by the outcomes from this analysis?

A more comprehensive review, incorporating additional data and seeking to understand in more detail the specific underlying difficulties experienced by candidates would clearly be of interest here.

Figure 1: Percentages

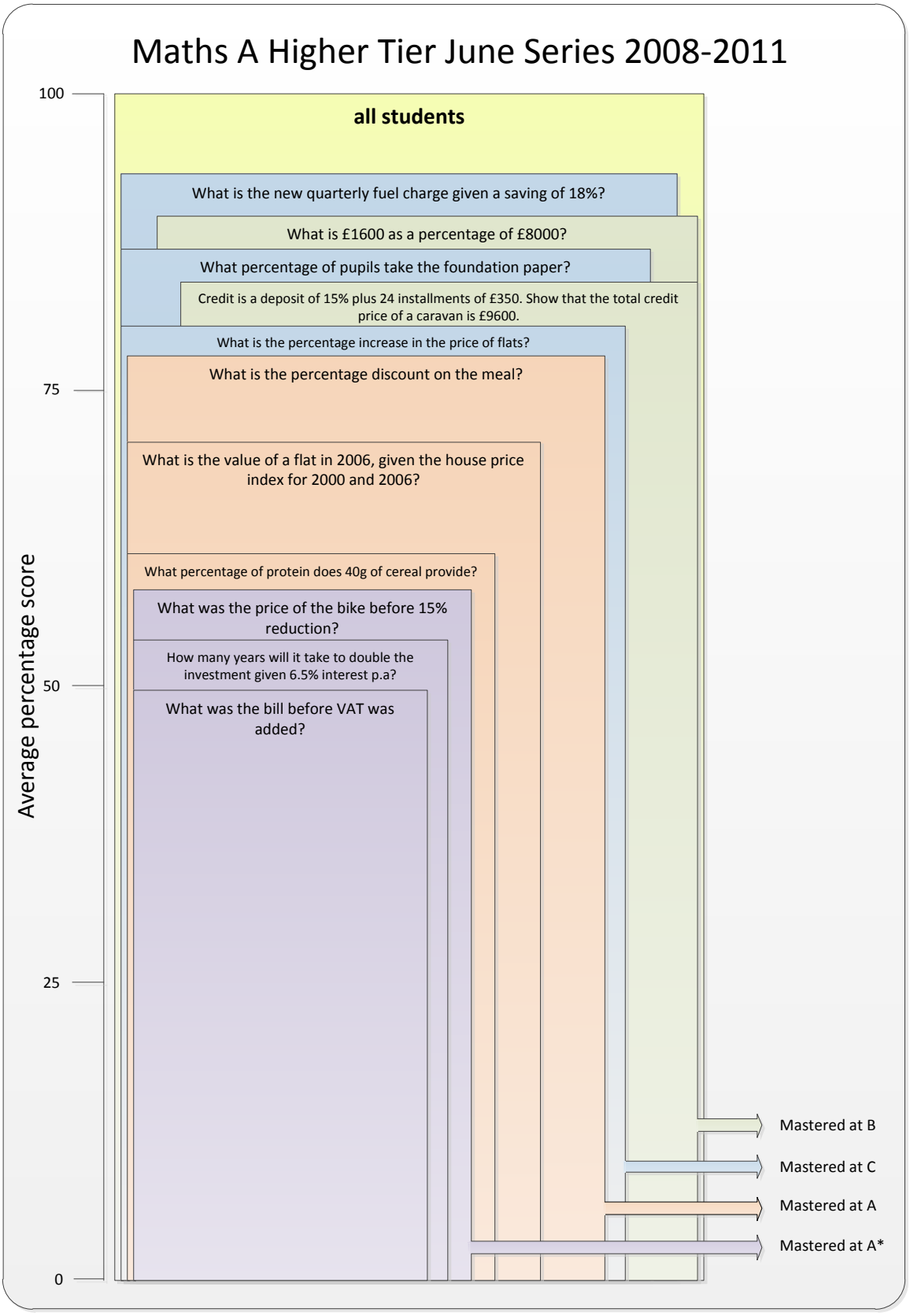


Figure 2: Quadratic equations

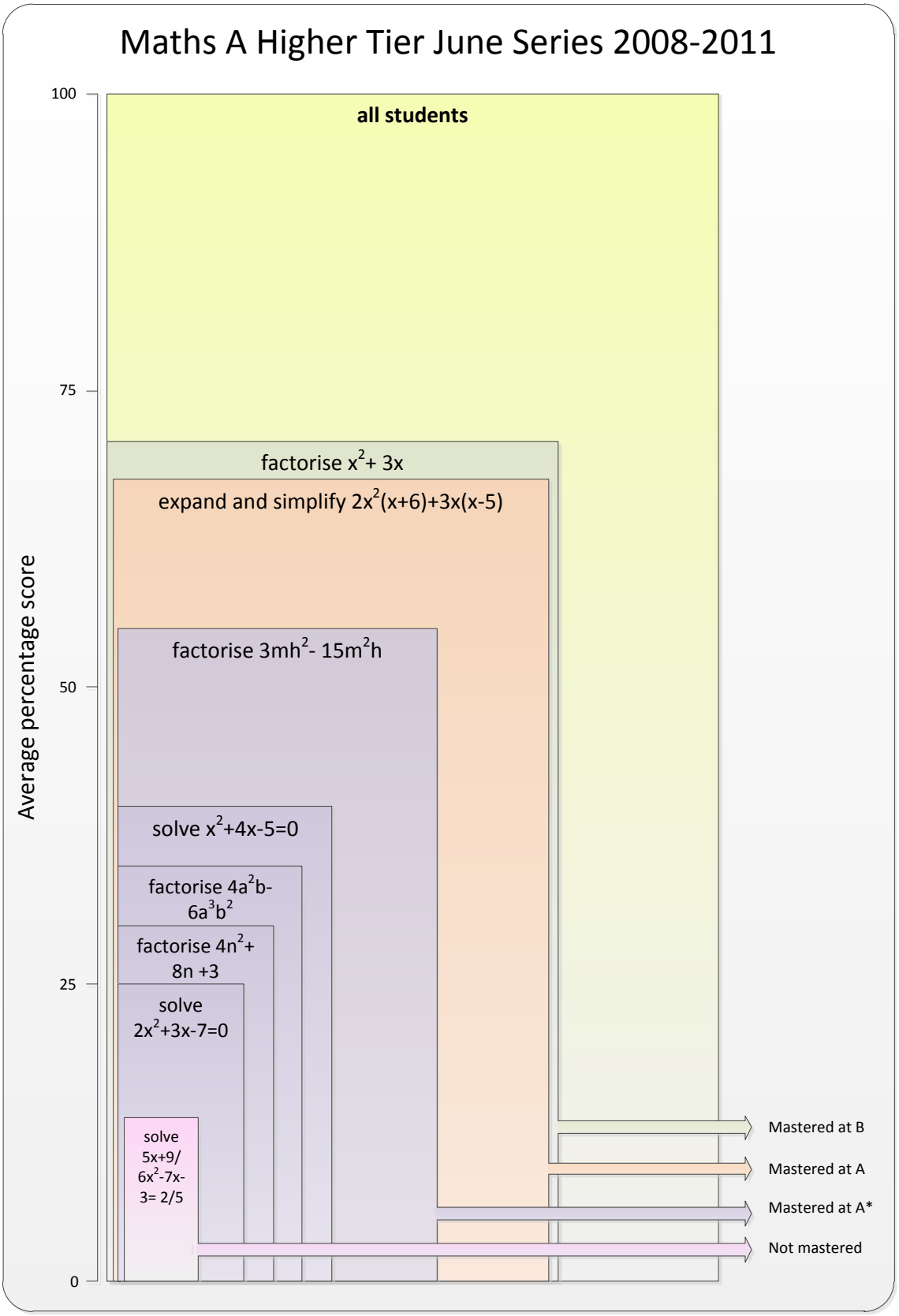


Figure 3: Fractions

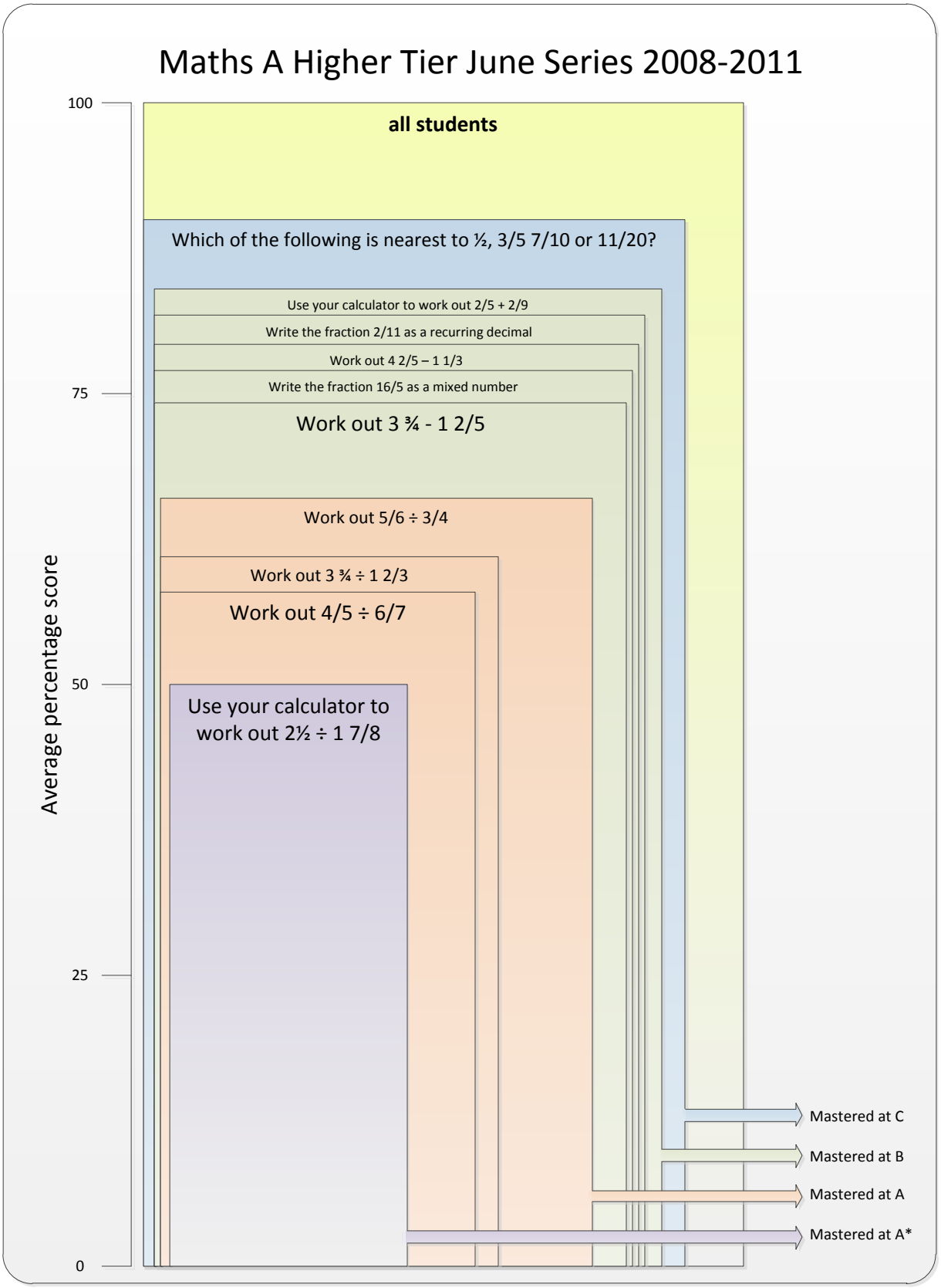
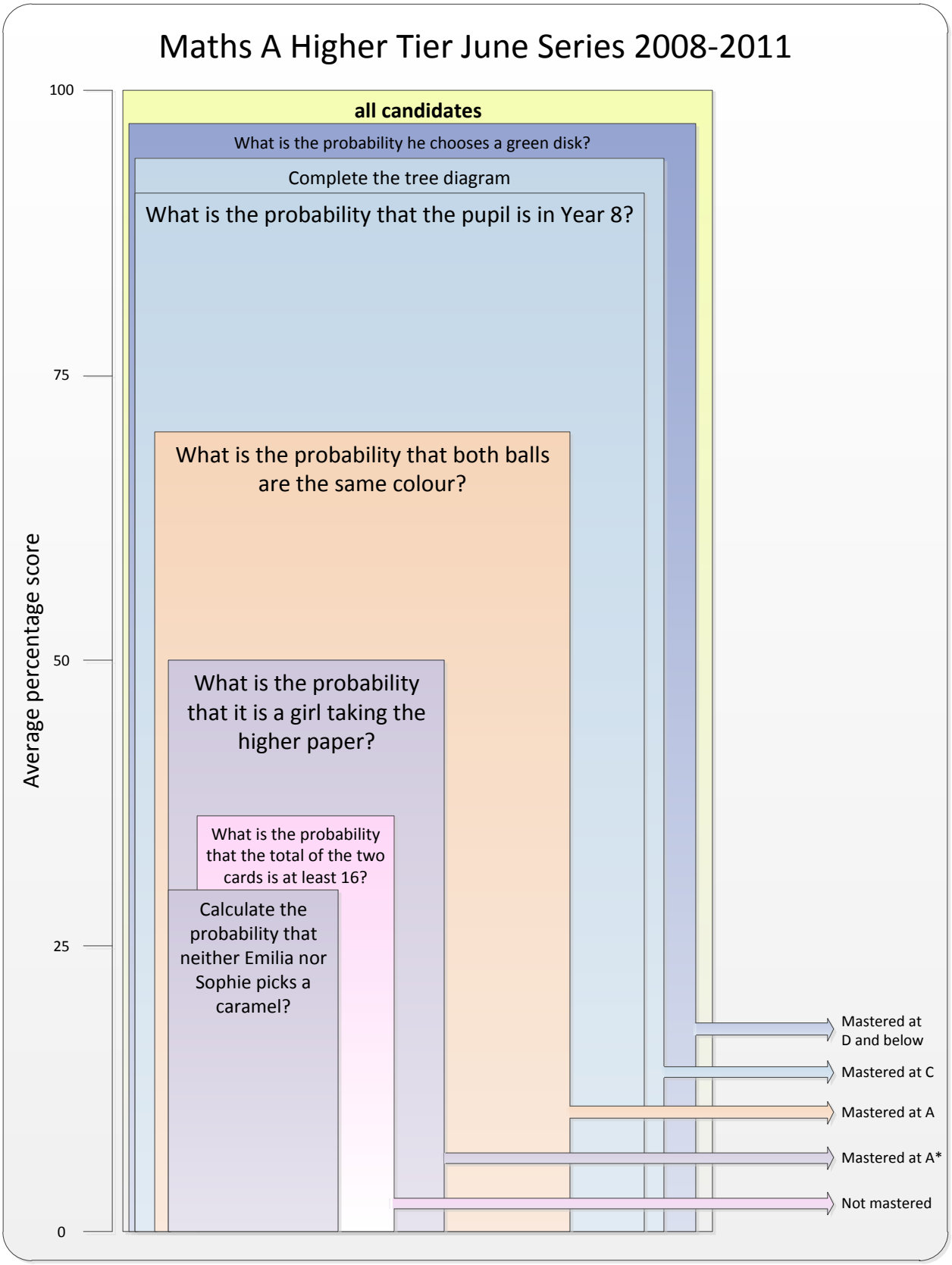


Figure 4: Probabilities



Appendix A Fractions Questions

June 2008 Paper 1H

8 (a) Work out $\frac{4}{5} \div \frac{6}{7}$

Give your answer in its simplest form.

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Answer ..... *(3 marks)*

8 (b) Work out $3\frac{3}{4} - 1\frac{2}{5}$

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Answer ..... *(3 marks)*

June 2009 Paper 1H

1 Which of the following fractions is nearest to $\frac{1}{2}$?
You **must** show your working.

$$\frac{3}{5} \qquad \frac{7}{10} \qquad \frac{11}{20}$$

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Answer ..... *(2 marks)*

June 2009 Paper 1H

18 Work out $3\frac{3}{4} \div 1\frac{2}{3}$

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Answer ..... *(3 marks)*

June 2010 Paper 1H

8 (a) Work out $\frac{5}{6} \div \frac{3}{4}$

Give your answer in its simplest form.

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Answer ..... *(3 marks)*

8 (b) Work out $4\frac{2}{5} - 1\frac{1}{3}$

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Answer ..... *(3 marks)*

June 2011 Paper 2H

8 (a) Write the fraction $\frac{16}{5}$ as a mixed number.

Answer ..... *(1 mark)*

8 (b) Use your calculator to work out $\frac{2}{5} + \frac{2}{9}$

Give your answer as a fraction.

Answer ..... *(1 mark)*

8 (c) Use your calculator to work out $2\frac{1}{2} \div 1\frac{7}{8}$

Give your answer as a mixed number.

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Answer ..... *(1 mark)*

8 (d) Write the fraction $\frac{2}{11}$ as a recurring decimal.

Answer ..... *(1 mark)*

Appendix B Quadratic Equations

June 2008 Paper 1H

7 (a) Factorise $x^2 + 3x$

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Answer ..... *(1 mark)*

7 (b) Factorise fully $4a^2b - 6a^3b^2$

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Answer ..... *(2 marks)*

20 Solve the equation $2x^2 + 3x - 7 = 0$

Give your answers correct to 2 decimal places.
You **must** show your working.

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Answer ..... *(3 marks)*

June 2009 Paper 1H

14 (a) Expand and simplify $2x^2(x + 6) + 3x(x - 5)$

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Answer ..... *(3 marks)*

14 (b) Factorise fully $3mh^2 - 15m^2h$

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Answer ..... *(2 marks)*

June 2011 Paper 1H

20 (a) Factorise $4n^2 + 8n + 3$

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Answer ..... *(2 marks)*

June 2011 Paper 2H

18 Solve the equation $x^2 + 4x - 5 = 0$

Do **not** use trial and improvement.

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Answer ..... *(3 marks)*

June 2011 Paper 2H

24 (c) Hence, or otherwise, find a complete solution to the equation

$$\frac{5x + 9}{6x^2 - 7x - 3} = \frac{2}{5}$$

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Answer ..... *(2 marks)*

Appendix C Percentage Questions

June 2008 Paper 1H

12 The house price index for a flat in Leeds was 190 in August 2006, compared with a base of 100 in April 2000.

12 (a) Write down the percentage increase in the price of flats in Leeds in that period.

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Answer ..... % *(1 mark)*

12 (b) A flat cost £80 000 in April 2000.

What was its likely value in August 2006?

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Answer £ ..... *(2 marks)*

June 2008 Paper 2H

6 (b) There are also 200 pupils in year 11.
The table shows the mathematics GCSE paper they are taking.

	Foundation	**Higher**
Boys	32	76
Girls	28	64

6 (b) (i) What percentage of the pupils are taking the Foundation paper?

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Answer ..... % *(1 mark)*

June 2009 Paper 2H

1 A householder pays £350 for her quarterly fuel charge.
She estimates that if she insulates her house she will reduce her quarterly fuel charge by 18%.

What will her expected quarterly fuel charge be after an 18% reduction?

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Answer £ ..... *(3 marks)*

June 2009 Paper 2H

17 The VAT rate in Spain is 16%
A hotel bill, including VAT, was €324.80

What was the bill before VAT was added?

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Answer € ..... *(3 marks)*

June 2010 Paper 1H

11 A restaurant offers a family discount.
The Taylor family have a meal at this restaurant.
Before the discount the meal costs £140
After the discount the cost is £112

Calculate the percentage discount.

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Answer ..... % *(3 marks)*

June 2010 Paper 2H

9 In a sale the price of a bike is reduced by 15%.
The sale price is £178.50

What was the price of the bike before the reduction?

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Answer £ ..... *(3 marks)*

June 2011 Paper 1H

6 Lewis is buying a caravan.
The cash price is £8000.
He buys it on credit in the following way.

A deposit of 15% of the cash price plus 24 monthly instalments of £350

6 (a) Show that the total credit price of the caravan is £9600.

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(3 marks)

6 (b) Lewis pays £1600 extra buying the caravan on credit.

What is £1600 as a percentage of £8000?

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Answer ..... % *(2 marks)*

June 2011 Paper 2H

10 A packet of breakfast cereal has the following information on it.

Protein: 10.3 g per 100 g of cereal

The recommended daily amount of protein for an adult male is 50 g.

What percentage of the recommended daily amount of protein does 40 g of cereal provide?

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Answer ..... % *(3 marks)*

June 2011 Paper 2H

17 Frank invests £3000 in a savings account that pays 6.5% interest per annum.

How many years will it take to double the investment?

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Answer ..... *(4 marks)*

Appendix D Probability Questions

June 2008 Paper 2H

6 (a) There are 200 pupils in year 10.
All pupils study at least one language.
No pupil studies all three languages.
The table shows how many pupils study each language.

	French	Spanish	German
Number of pupils	97	116	45

How many pupils study two languages?

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Answer ..... *(1 mark)*

6 (b) There are also 200 pupils in year 11.
The table shows the mathematics GCSE paper they are taking.

	Foundation	Higher
Boys	32	76
Girls	28	64

6 (b) (i) What percentage of the pupils are taking the Foundation paper?

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Answer ..... % *(1 mark)*

6 (b) (ii) One of the pupils is absent for the examination.

What is the probability that it is a girl taking the Higher paper?

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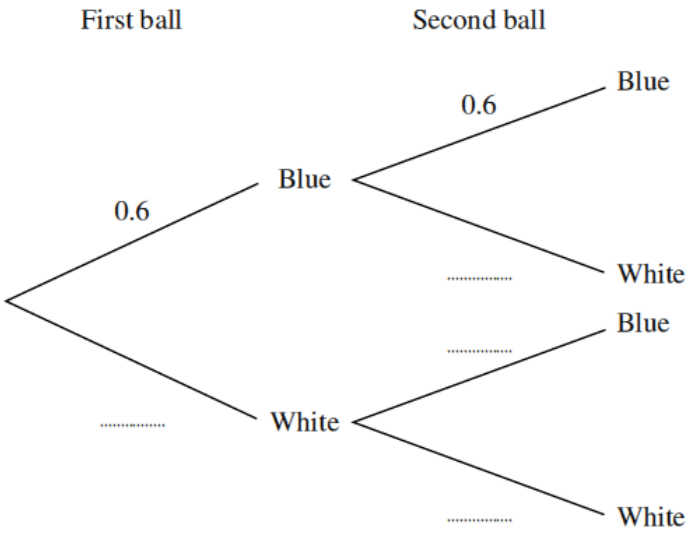
Answer ..... *(1 mark)*

N.B Only question 6b(ii) was used in the analysis; the other parts of the question have been included here to provide context

June 2009 Paper 2H

19 A bag contains 6 blue and 4 white balls.
A ball is taken from the bag at random and replaced.
Another ball is then taken from the bag at random.

19 (a) Complete the tree diagram.



(1 mark)

19 (b) What is the probability that both balls are the same colour?

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Answer ..... *(3 marks)*

June 2010 Paper 1H

2 Jack has a box of 100 coloured discs.
The discs are Red, Blue, Green and Yellow.
The table shows some of the probabilities of choosing a colour.

Colour	Red	Blue	Green	Yellow
Probability	0.6	0.1		0.1

2 (a) Jack chooses a disc at random from the box.
Work out the probability that he chooses a Green disc.

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Answer ..... *(2 marks)*

June 2010 Paper 2H

4 The table shows the year group and gender of a sample of 50 pupils.

Gender	Year Group					Total
	Yr7	Yr8	Yr9	Yr10	Yr11	
Number of boys	3	5	6	2	6	22
Number of girls	4	5	7	6	6	28

4 (b) A pupil from the sample is picked at random.

What is the probability that the pupil is in Year 8?
Give your answer as a fraction in its lowest terms.

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Answer ..... *(2 marks)*

June 2011 Paper 2H

21 Nine cards with the digits from 1 to 9 on them are placed in a bag.

1	2	3	4	5	6	7	8	9

Two cards are taken from the bag at random.

What is the probability that the total of the two cards is at least 16?

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Answer ..... *(4 marks)*