3.3.2   Converting between number bases

Lesson plan and printable activities

Materials needed

1. 3.3.2 Lesson PowerPoint.

2. Numerical conversion tests (Quiz 1, Quiz 2, Quiz 3, Quiz 4).

3. Data Representation Assistant computer software (v1.0) running on Microsoft Windows.

4. Files for Hex colours! (Extension task) on students' computers.

Lesson aims

1. To develop students' theoretical and hands-on skills in converting human-readable numbers into binary representation.

Lesson objectives

1. Understand how binary can be used to represent whole numbers.

2. Understand how hexadecimal can be used to represent whole numbers.

3. Be able to convert in both directions between: binary and decimal, binary and hexadecimal, decimal and hexadecimal.

Starter activity (5 minutes)

Slide 3: Refer back to the previous lesson on the history of data representation and get students to explain why a knowledge of binary is useful in this context.

The importance of binary in the world of computing

Area	Explanation
Programming	Students will probably have seen simple programming examples using languages e.g. Python. This belongs to a group of so-called 'high-level' languages. At CPU level, only binary is understood.
Computer architecture	Knowledge of binary is needed to understand how data are transferred. Explain that there is mathematical reason for the fact that the 32-bit computers can only use up to 4GB of RAM. (See Teacher notes.)
Computer networks	Network engineers need to make binary calculations rapidly when devising ranges of network addresses. The internet uses IP addresses, such as 192.168.1.1. In fact, these are manipulated by routers as purely binary information when sending (routing) IP packets to the correct destinations.

Main activities (40 - 45 minutes)

Use Lesson PowerPoint as a background to the lesson. There are three key areas to discuss. Knowledge is demonstrated through hands-on tests at regular intervals. Explain each key concept then test as listed below. It is suggested that students swap papers when marking.

1. **Slide 4:** Explain how one byte can be used to represent the decimal number range of 0-255.

2. **Slide 5:** Show how decimal numbers within this range can be converted into binary.

3. **Slide 6:** Spend 10 minutes testing learning using the resource Quiz 1

4. **Slide 7:** Show how binary numbers within the range can be converted into decimal.

5. **Slide 8:** Spend 10 minutes testing learning using the resource Quiz 2

6. **Slides 9 and 10:** Show students how to convert from decimal to hexadecimal via binary. *Note: Students will also need to be able to convert hexadecimal to decimal – see* Quiz 4

7. **Slide 11:** Explain why it is better to use hexadecimal to represent binary numbers. Using knowledge gained from the last point, teacher could, for example, display the Unicode value 450F rendered in the Data Representation Assistant and ask students to convert to the binary value – which is easier to remember?

Explain that hexadecimal values:

- can usually be represented in fewer digits than the same value represented in binary
- offer less chance of making mistakes if using them in programming
- are easily converted to binary.

8. **Slide 12:** Spend ten minutes testing learning using the resource Quiz 3

Plenary (10 minutes)

1. **Slide 13:** Round off the session by getting students to compete in an interactive session, answering questions related to binary conversions – play Binary Bingo!

   Using the **Data Representation Assistant** software to help (leave running on board ready to use in Binary calculator mode):

   - Instruct the students to shout something appropriate when finished.
   - Distribute the accompanying handout sheet and then put up a list of five decimal numbers between 0-255 on the board.
   - Collect the 'winning' paper and get another student to enter the decimal numbers one by one displaying the result of the conversion – check of the winning entries.
   - Exercise can be repeated several times as the activity sheet is reusable.

Extension activity (to end or as homework)

1. **Slide 14:** Using the Hex Colours! (Extension task), get students to colour in the web page template provided by using appropriate RGB colour values set in Hexadecimal.

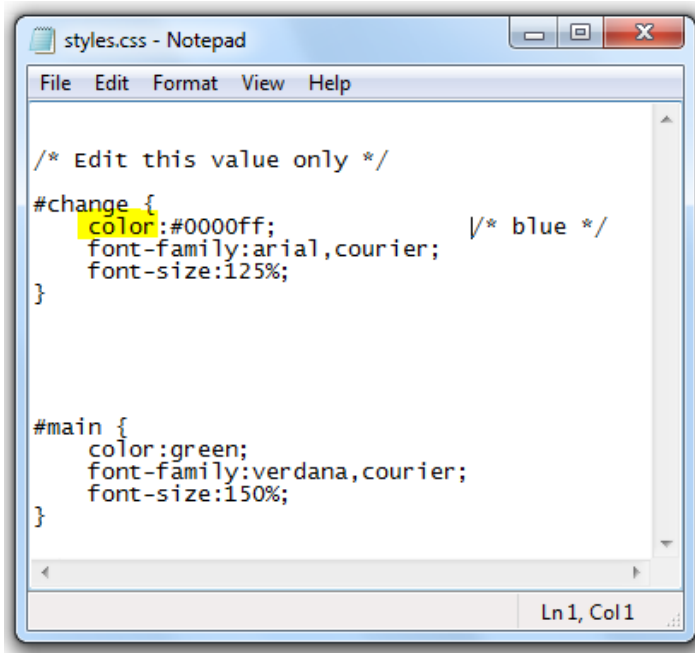   Student instructions are included in the Extension task.

   Navigate to the page **index.html** provided, and open the page using a web browser.

   Open the **style.css** page provided using **Notepad**.

   Edit the RGB colour of the '**#change**' attribute in the CSS file – use any of the values given above or locate your own.

   Note: Quiz 4 is included for extra practice at conversions. It could be set as homework.

This will influence the colour of the font in the second line displayed in the page.

Lesson

3.3 Fundamentals of data representation

3.3.2 Converting between number bases
Lesson

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Objectives

Understand how binary can be used to represent whole numbers.

Understand how hexadecimal can be used to represent whole numbers.

Be able to convert in both directions between: binary and decimal, binary and hexadecimal, decimal and hexadecimal.

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Starter activity

In pairs…
Give three reasons why a knowledge of binary is useful to working with computers and understanding how computers work.

Did you think of any of these?

- Programming
- Computer architecture
- Computer networks

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Converting between number bases

A byte is a collection of eight bits and may be represented as follows:

Value as power of 2	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal value	128	64	32	16	8	4	2	1
Sample 8-bit pattern	0	1	0	1	1	0	0	1
Individual values	0	64	0	16	8	0	0	1
Total decimal value		64 + 16 + 8 + 1 = 89						

Key point
When one byte of storage is used, it does not matter how much you rearrange the ones and zeros, the maximum number in base 10 that can be stored is 255 (and 0 is the minimum).

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Converting decimal to binary

Example: Convert the number 115_{10} into 8-bit binary.
Reading from left to right, look at each column in turn.

	128	64	32	16	8	4	2	1
Step 1	0	115 – 128 = negative, write 0 in that column.						
Step 2	0	1	115 – 64 = 51, write 1 in that column.					
Step 3	0	1	1	51 – 32 = 19, write 1 in that column.				
Step 4	0	1	1	1	19 – 16 = 3, write 1 in that column.			
Step 5	0	1	1	1	0	3 – 8 = negative, write 0		
Step 6	0	1	1	1	0	0	3 – 4 = negative, write 0	
Step 7	0	1	1	1	0	0	1	3 - 2 = 1, write 1
Final answer	0	1	1	1	0	0	1	1*

*Remainder 1 goes in final column.

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Now its your turn…

Activity: Quiz 1

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Converting binary to decimal

Look at the binary pattern and work from right to left, identifying whether each successive bit has a value of 0 or 1. Then add up the totals where the bit is set.

Binary pattern   00101100

	128	64	32	16	8	4	2	1	
	0	0	1	0	1	1	0	0	Decimal value
Totals			32		8	4			44

Now its your turn…

Activity: Quiz 2

Making binary easier to represent

Using the hexadecimal number system

Decimal	Hexadecimal
15	F
14	E
13	D
12	C
11	B
10	A
9	9
8	8
7	7
6	6
5	5
4	4
3	3
2	2
1	1
0	0

Binary is not very easy to read and there are many applications where it is beneficial to convert from binary to hexadecimal.

Key point
Hexadecimal in itself means nothing to a CPU. However, it is useful to abbreviate binary notation.

Converting from binary to hexadecimal

Conversion involves looking at four bits at a time and grouping them as a single hexadecimal ('hex') character.

In this case, the binary value has been split into two groups of four. In each group the highest power of 2 is 2^3.

The first group of four converts to D in hex and the second group converts to 1.

Combining these two values gives D1.

Example 2	Value							
Binary	1	1	0	1	0	0	0	1
Decimal	8	4	2	1	8	4	2	1
Sum (base 10)	13				1			
Hex	D				1			

Benefits of hexadecimal over binary

Hexadecimal:

- is shorter than binary

- offers less chance of making mistakes if using it in programming

- is easily converted to binary.

Now its your turn…

Activity: Quiz 3

Plenary

Binary bingo!

Extension activity

Using the **Hex colours! (Extension task)** sheet, colour in the web page template with appropriate RGB colour values set in hexadecimal.

Quiz 1 Converting decimal to binary – questions

Student name:... **Total score:**......................

Convert each of the decimal values to binary patterns. The first one has been done for you.

1.

Decimal	Write the binary pattern in the boxes below.							
87	128	64	32	16	8	4	2	1
	0	1	0	1	0	1	1	1

2.

Decimal	Write the binary pattern in the boxes below.							
143	128	64	32	16	8	4	2	1

3.

Decimal	Write the binary pattern in the boxes below.							
140	128	64	32	16	8	4	2	1

4.

Decimal	Write the binary pattern in the boxes below.							
56	128	64	32	16	8	4	2	1

5.

Decimal	Write the binary pattern in the boxes below.							
86	128	64	32	16	8	4	2	1

6.

Decimal	Write the binary pattern in the boxes below.							
17	128	64	32	16	8	4	2	1

7.

Decimal	Write the binary pattern in the boxes below.							
196	128	64	32	16	8	4	2	1

8.

Decimal	Write the binary pattern in the boxes below.							
125	128	64	32	16	8	4	2	1

9.

Decimal	Write the binary pattern in the boxes below.							
201	128	64	32	16	8	4	2	1

10.

Decimal	Write the binary pattern in the boxes below.							
15	128	64	32	16	8	4	2	1

11.

Decimal	Write the binary pattern in the boxes below.							
224	128	64	32	16	8	4	2	1

12.

Decimal	Write the binary pattern in the boxes below.							
253	128	64	32	16	8	4	2	1

Quiz 1 Converting decimal to binary – answers

1.

Decimal	Write the binary pattern in the boxes below.							
87	**128**	**64**	**32**	**16**	**8**	**4**	**2**	**1**
	0	1	0	1	0	1	1	1

2.

Decimal	Write the binary pattern in the boxes below.							
143	**128**	**64**	**32**	**16**	**8**	**4**	**2**	**1**
	1	0	0	0	1	1	1	1

3.

Decimal	Write the binary pattern in the boxes below.							
140	**128**	**64**	**32**	**16**	**8**	**4**	**2**	**1**
	1	0	0	0	1	1	0	0

4.

Decimal	Write the binary pattern in the boxes below.							
56	**128**	**64**	**32**	**16**	**8**	**4**	**2**	**1**
	0	0	1	1	1	0	0	0

5.

Decimal	Write the binary pattern in the boxes below.							
86	128	64	32	16	8	4	2	1
	0	1	0	1	0	1	1	0

6.

Decimal	Write the binary pattern in the boxes below.							
17	128	64	32	16	8	4	2	1
	0	0	0	1	0	0	0	1

7.

Decimal	Write the binary pattern in the boxes below.							
196	128	64	32	16	8	4	2	1
	1	1	0	0	0	1	0	0

8.

Decimal	Write the binary pattern in the boxes below.							
125	128	64	32	16	8	4	2	1
	0	1	1	1	1	1	0	1

9.

Decimal	Write the binary pattern in the boxes below.							
201	128	64	32	16	8	4	2	1
	1	1	0	0	1	0	0	1

10.

Decimal	Write the binary pattern in the boxes below.							
15	128	64	32	16	8	4	2	1
	0	0	0	0	1	1	1	1

11.

Decimal	Write the binary pattern in the boxes below.							
224	128	64	32	16	8	4	2	1
	1	1	1	0	0	0	0	0

12.

Decimal	Write the binary pattern in the boxes below.							
253	128	64	32	16	8	4	2	1
	1	1	1	1	1	1	0	1

Quiz 2 Converting binary to decimal – questions

Student name:... **Total score:**.....................

Convert each binary pattern to a decimal value. The first one has been done for you.

1. **Binary pattern** **00101100**

128	64	32	16	8	4	2	1	
0	0	1	0	1	1	0	0	**Decimal value**
Totals		32		8	4			**44**

2. **Binary pattern** **10100111**

128	64	32	16	8	4	2	1	
								Decimal value
Totals								

3. **Binary pattern** **10010001**

128	64	32	16	8	4	2	1	
								Decimal value
Totals								

4. **Binary pattern** **00110010**

128	64	32	16	8	4	2	1	
								Decimal value
Totals								

5. **Binary pattern** **11110010**

	128	64	32	16	8	4	2	1	
									Decimal value
Totals									

6. **Binary pattern** **01111111**

	128	64	32	16	8	4	2	1	
									Decimal value
Totals									

7. **Binary pattern** **10010010**

	128	64	32	16	8	4	2	1	
									Decimal value
Totals									

8. **Binary pattern** **10000011**

	128	64	32	16	8	4	2	1	
									Decimal value
Totals									

9. **Binary pattern** **11000010**

	128	64	32	16	8	4	2	1	
									Decimal value
Totals									

Quiz 2 Converting binary to decimal – answers

1. Binary pattern   00101100

128	64	32	16	8	4	2	1	
0	0	1	0	1	1	0	0	Decimal value
Totals		32		8	4			**44**

2. Binary pattern   10100111

128	64	32	16	8	4	2	1	
1	0	1	0	0	1	1	1	Decimal value
Totals	128		32			4	2	1	**167**

3. Binary pattern   10010001

128	64	32	16	8	4	2	1	
1	0	0	1	0	0	0	1	Decimal value
Totals	128			16				1	**145**

4. Binary pattern   00110010

128	64	32	16	8	4	2	1	
0	0	1	1	0	0	1	0	Decimal value
Totals			32	16			2		**50**

5. **Binary pattern** **11110010**

128	64	32	16	8	4	2	1	
1	1	1	1	0	0	1	0	Decimal value
Totals	128	64	32	16			2		**242**

6. **Binary pattern** **01111111**

128	64	32	16	8	4	2	1	
0	1	1	1	1	1	1	1	Decimal value
Totals		64	32	16	8	4	2	1	**127**

7. **Binary pattern** **10010010**

128	64	32	16	8	4	2	1	
1	0	0	1	0	0	1	0	Decimal value
Totals	128			16			2		**146**

8. **Binary pattern** **10000011**

128	64	32	16	8	4	2	1	
1	0	0	0	0	0	1	1	Decimal value
Totals	128						2	1	**131**

9. **Binary pattern** **11000010**

128	64	32	16	8	4	2	1	
1	1	0	0	0	0	1	0	Decimal value
Totals	128	64					2		**194**

Quiz 3 Converting decimal to binary to hexadecimal – questions

Student name: .... **Total score:**......................

Worked example

Write the hexadecimal pattern required in the boxes below.									Correct?
Decimal value	127								
Binary pattern									
128	64	32	16	8	4	2	1		**Remainder**
0									127
	1								63
		1							31
			1						15
				1					7
					1				3
						1			1
							1		0
0	1	1	1	1	1	1	1		**Binary**
7				F					**Hex**
7F									

Write the hexadecimal pattern required in the boxes below.									Correct?
Q 1	**Decimal value**	71							
	Binary pattern								
	128	64	32	16	8	4	2	1	**Remainder**
									Binary
									Hex

Write the hexadecimal pattern required in the boxes below.									Correct?
Q 2	**Decimal value**	150							
	Binary pattern								
	128	64	32	16	8	4	2	1	**Remainder**
									Binary
									Hex

Quiz 3 Converting decimal to binary to hexadecimal – answers

Write the hexadecimal pattern required in the boxes below.									Correct?
Q 1	**Decimal value**	71							
	Binary pattern								
	128	64	32	16	8	4	2	1	**Remainder**
	0								**71**
		1							**7**
			0						**7**
				0					**7**
					0				**7**
						1			**1**
							0		**1**
								1	**0**
	0	**1**	**0**	**0**	**0**	**1**	**0**	**1**	**Binary**
	4				**5**				**Hex**
	45								

Write the hexadecimal pattern required in the boxes below.									Correct?
Q 2	**Decimal value**	150							
	Binary pattern								
	128	64	32	16	8	4	2	1	**Remainder**
	1								**22**
		0							**22**
			0						**22**
				1					**6**
					0				**6**
						1			**2**
							1		**0**
								0	**0**
	1	**0**	**0**	**1**	**0**	**1**	**1**	**0**	**Binary**
	9				**6**				**Hex**
	96								

Quiz 4
Extra questions on converting between number bases

Convert the number shown below to decimal.			Correct?
Question 1	1000 0111$_2$		
Question 2	1100 0111$_2$		
Convert the number shown below to binary.			Correct?
Question 3	45$_{10}$		
Question 4	201$_{10}$		
Convert the number shown below to hexadecimal.			Correct?
Question 5	1010 1110$_2$		
Question 6	0011 1001$_2$		
Question 7	143$_{10}$		
Question 8	77$_{10}$		

Quiz 4
Extension questions – Converting from hexadecimal to binary

Use what you have learnt about converting numbers to hexadecimal, to convert numbers from hexadecimal. There is a conversion table if you need it.

Convert the number shown below to decimal			Correct?
Question 1	25_{16}		
Question 2	250_{16}		
Question 3	AB_{16}		
Question 4	$1CC_{16}$		
Question 5	$2F8_{16}$		
Convert the number shown below to binary			**Correct?**
Question 6	A_{16}		
Question 7	22_{16}		
Question 8	$1F_{16}$		
Question 9	$1A1_{16}$		
Question 10	$4D_{16}$		

How to convert hexadecimal to decimal

1. Get the decimal equivalent of each hex digit from the table.

Hexadecimal	Decimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
A	10
B	11
C	12
D	13
E	14
F	15

2. Multiply each digit with a power of 16 according to its original location.

	16^3 (or 4096)	16^2 (or 256)	16^1 (or 16)	16^0 (or 1)

3. Add the resulting totals from step 2.

eg $80E1_{16} = 32993_{10}$

How to convert hexadecimal to binary

Use this table:

Hexadecimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

e.g. $0A2B_{16} = 0000\ 1010\ 0010\ 1011_2$

Quiz 4 – answers
Extra questions on converting between number bases

Convert the number shown below to decimal			Correct?
Question 1	$1000\ 0111_2$	135_{10}	
Question 2	$1100\ 0111_2$	199_{10}	
Convert the number shown below to binary			Correct?
Question 3	45_{10}	$0010\ 1101_2$	
Question 4	201_{10}	$1100\ 1001_2$	
Convert the number shown below to hexadecimal			Correct?
Question 5	$1010\ 1110_2$	AE_{16}	
Question 6	$0011\ 1001_2$	39_{16}	
Question 7	143_{10}	$8F_{16}$	
Question 8	77_{10}	$4D_{16}$	

Quiz 4 – answers
Extension questions – Converting from hexadecimal to binary

Convert the number shown below to decimal			Correct?
Question 1	25_{16}	37_{10}	
Question 2	250_{16}	592_{10}	
Question 3	AB_{16}	171_{10}	
Question 4	$1CC_{16}$	460_{10}	
Question 5	$2F8_{16}$	760_{10}	
Convert the number shown below to binary			Correct?
Question 6	A_{16}	1010_2	
Question 7	22_{16}	$0011\ 0010_2$	
Question 8	$1F_{16}$	$0001\ 1111_2$	
Question 9	$1A1_{16}$	$0001\ 1010\ 0001_2$	
Question 10	$4D_{16}$	$0100\ 1101_2$	

Data Representation Assistant instructions

Using the **Data Representation Assistant** program (v1.0)

Launch the program

The executable has been built to work on computers running Microsoft Windows with an installed **.Net** framework of 3.5 or greater. Contact your IT services if you are experiencing any issues with running the program itself, it needs no special installation.

The basic interface looks like this:

Menu   File → *Exit*

Use this option to cleanly shut down the program.

Menu   Help→ *About*

Choosing the menu option **Help→ *About*** will give you information on the version of the program itself.
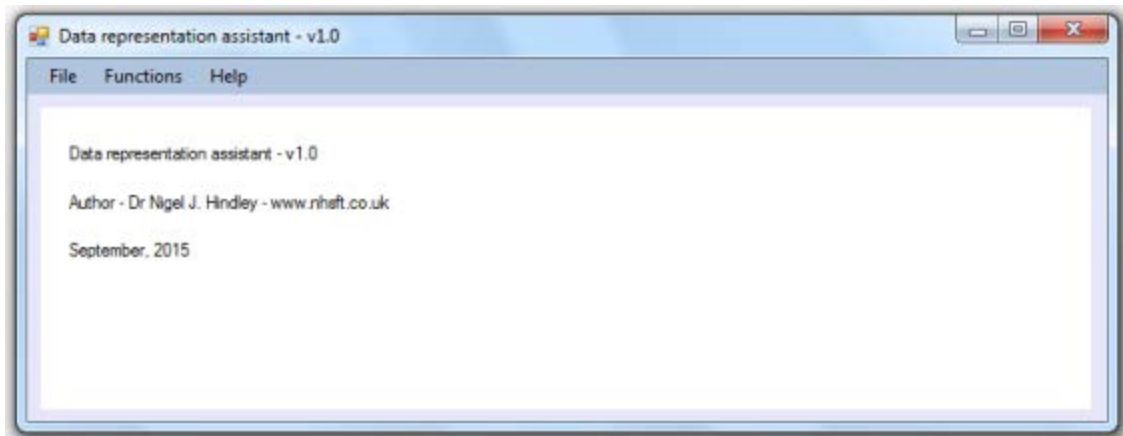
Menu   **Functions →** *Show ASCII*

Select this option to display the dialogue below.
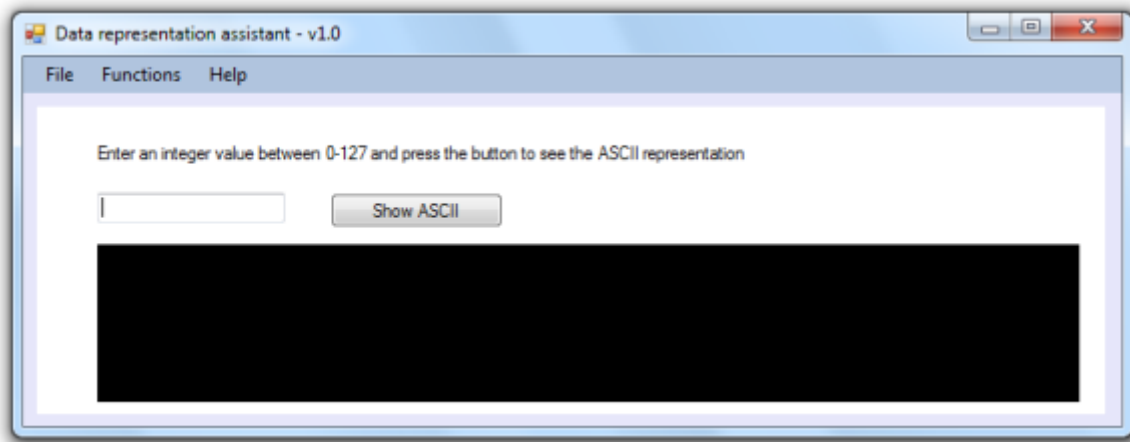
Enter a value between 0 and 127 into the text box as highlighted.
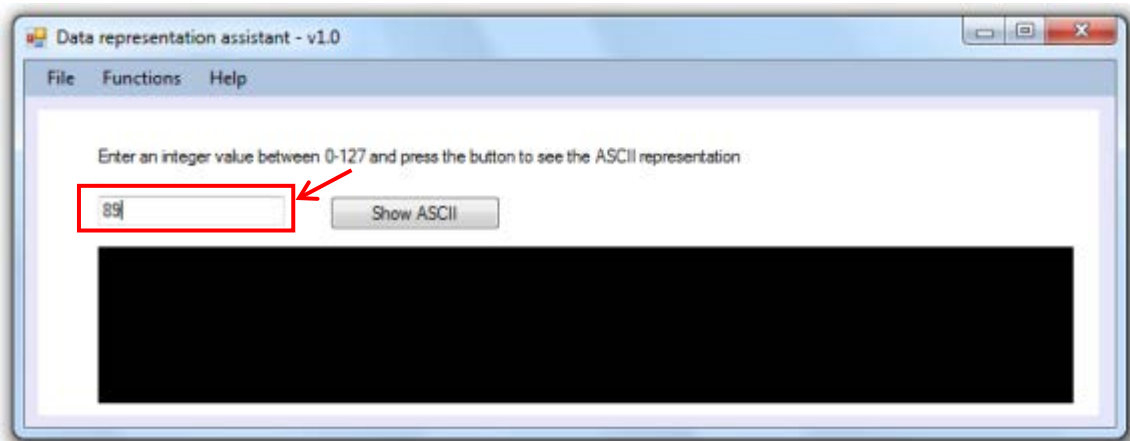
Press the button **Show ASCII** and to see the character value rendered and displayed in the black panel.
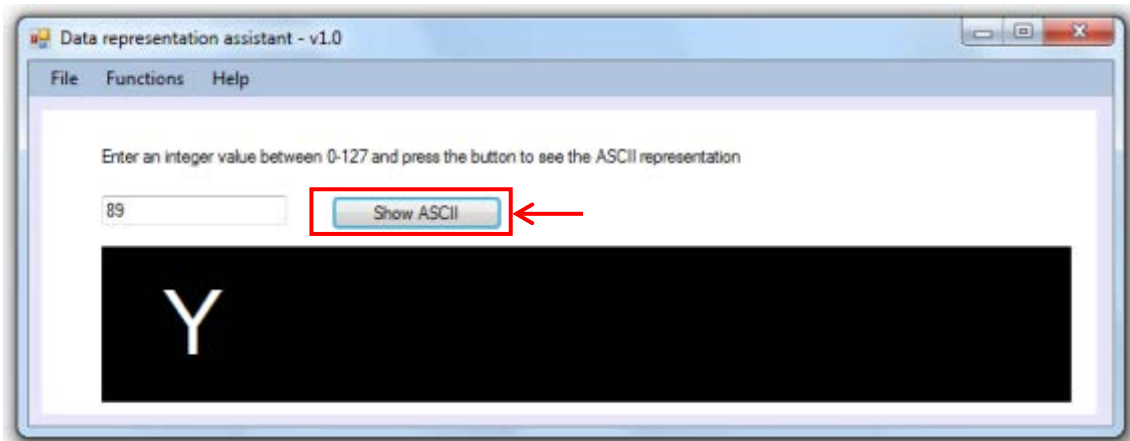
Menu   **Functions → *Show Unicode***

Select this option to display the dialogue below.
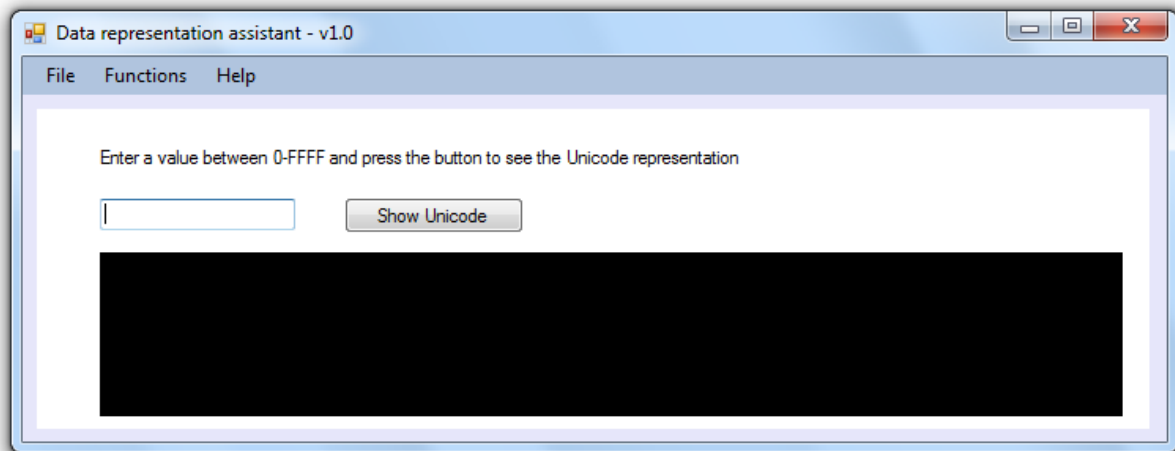
Enter any value between 0000 and FFFF* into the text box as highlighted.
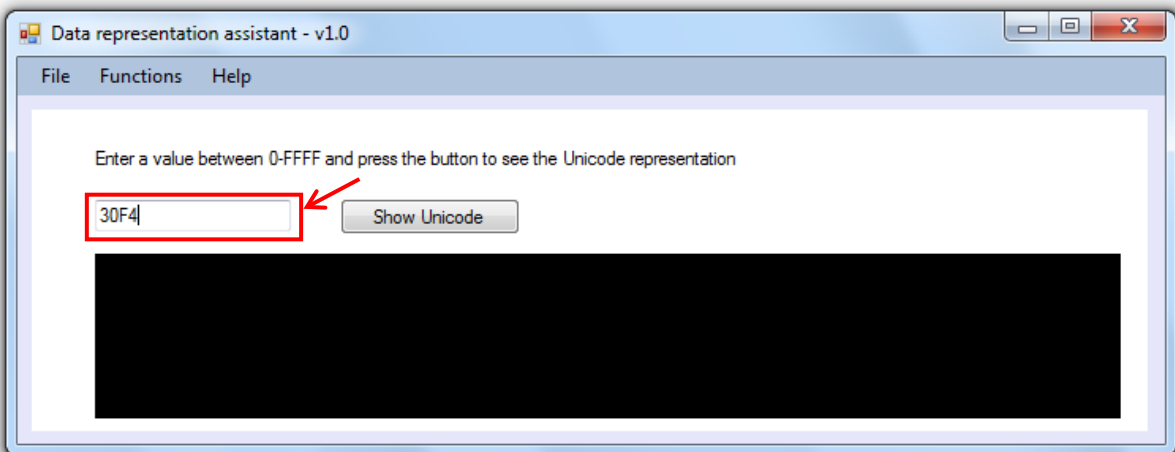
Press the button **Show Unicode** and to see any valid Unicode character* displayed in the black panel.
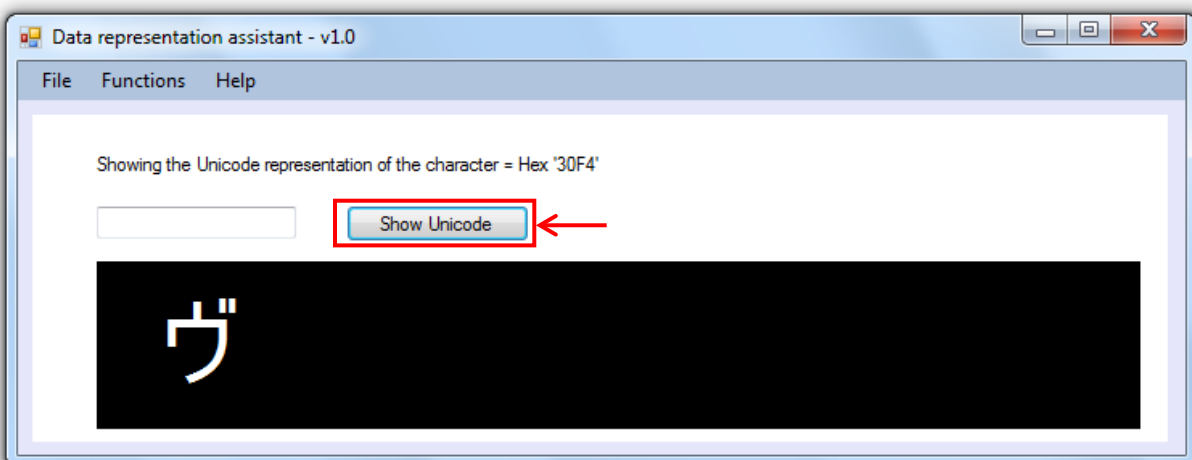
* Note: Not all numbers in the stated numeric range are valid Unicode characters; tutor should use discretion when displaying. For an up-to-date reference look here - http://unicode.org/charts/

Menu   Functions → *Binary calculator*

Select this option to display the dialogue below.
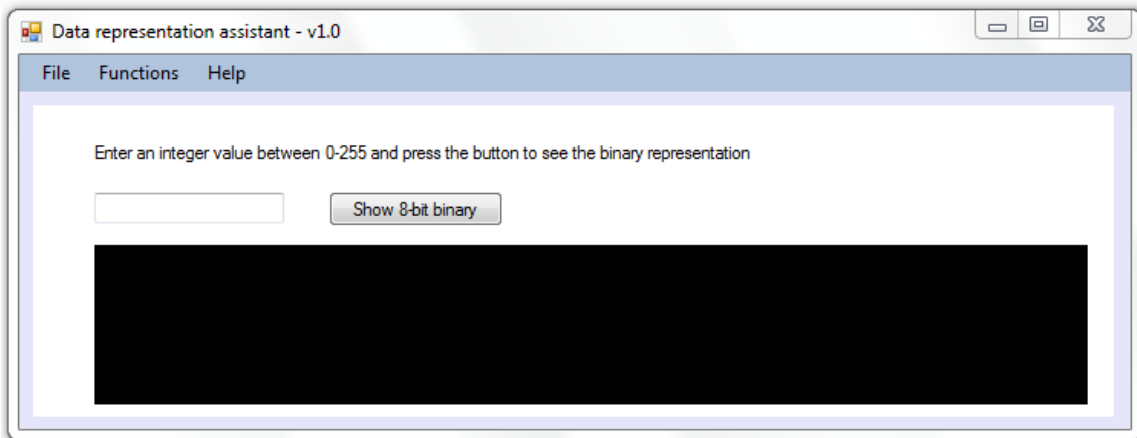
Enter any value between 0 and 255 into the text box as highlighted.
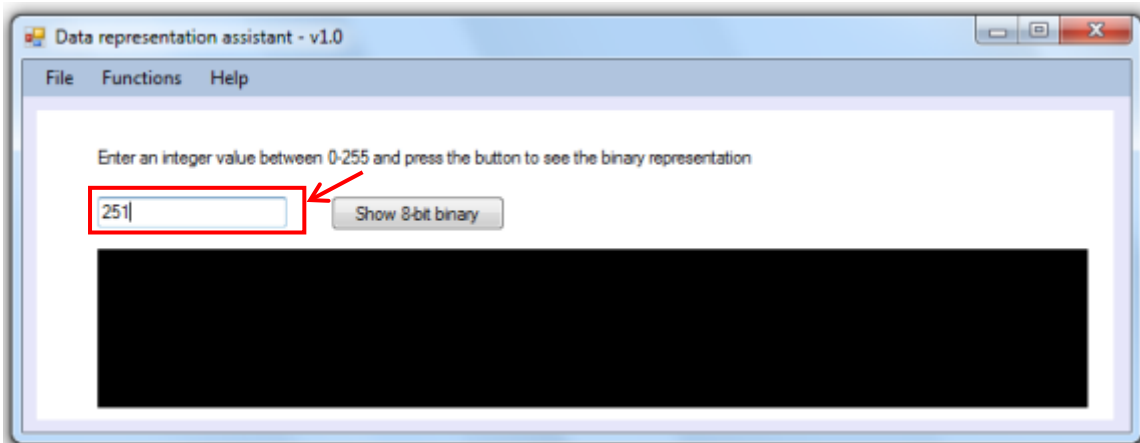
Press the button **Show 8-bit binary** and to see the result which will be displayed in the black panel.
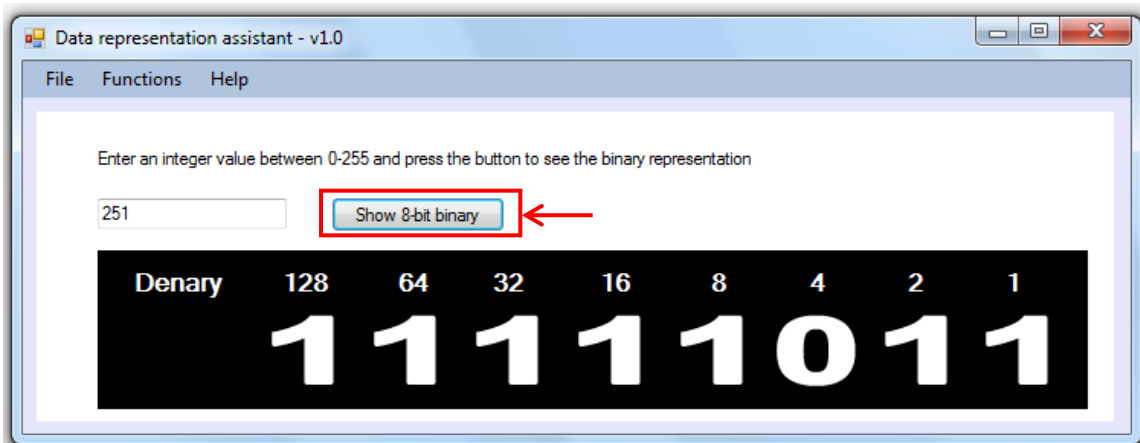
Binary Bingo!

Student name:.. **Total score:**.....................

Write the binary number in the boxes below.									Correct?
Round 1									

Write the binary number in the boxes below.									Correct?
Round 2									

Write the binary number in the boxes below.									Correct?
Round 3									

Write the binary number in the boxes below.									Correct?
Round 4									

Write the binary number in the boxes below.									Correct?
Round 5									

Hex colours! (Extension task)

Real world examples: RGB colour models

Looking back at what we have just learned about hex in particular, you frequently see web-programming examples using hex values.

This web page explains how hex values are used to define the way text appears on a screen:

rapidtables.com/web/color/RGB_Color.htm

Activity

Navigate to the page: index.html and open the page using a web browser.
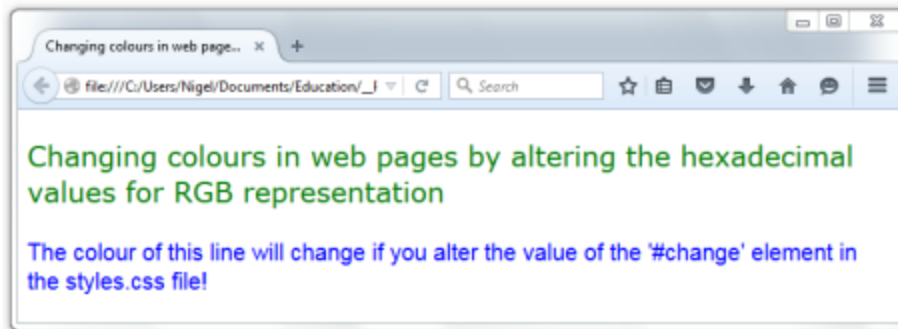
Then open the page: style.css using Notepad (right-click on the file and choose this as the editor).

Edit the RGB colour of the '#change' attribute in the CSS file. Use any of the values given on the next page. This will influence the colour of the font in the second line displayed in the page: index.html.
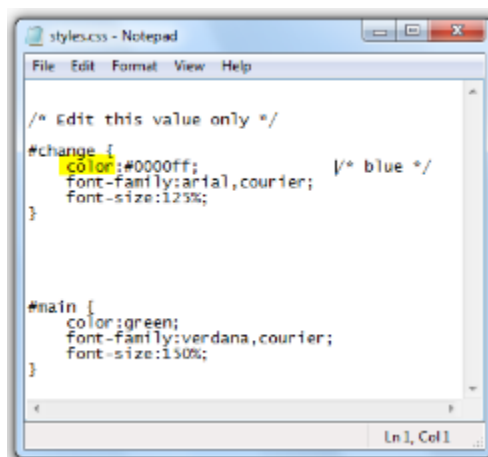
Once you have identified the colours, write them into the blank boxes on the next sheet, or colour the boxes in.

Colour 1

F				F				7				F				5				0			
1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	0	1	0	0	0	0

Colour 2

F				F				A				5				0				0			
1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0

Colour 3

0				0				8				0				8				0			
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0

Colour 4

C				0				C				0				C				0			
1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0

Colour 5

2				2				8				B				2				2			
0	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	0	0	1	0	0	0	1	0

Colour 6

0				0				B				F				F				F			
0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Colour 7

A				0				5				2				2				D			
1	0	1	0	0	0	0	0	0	1	0	1	0	0	1	0	0	0	1	0	1	1	0	1

Colour 8

B				C				8				F				8				F			
1	0	1	1	1	1	0	0	1	0	0	0	1	1	1	1	1	0	0	0	1	1	1	1

Hex colours! (Extension task) answers

Colour 1	Coral
Colour 2	Orange
Colour 3	Teal
Colour 4	Silver
Colour 5	Forest green
Colour 6	Deep sky blue
Colour 7	Sienna
Colour 8	Rosy brown