Solving estimation problems

Level 3 Mathematical Studies

Introduction

This booklet is a guide on how to approach and solve estimation problems.

Approaching estimation problems

It is **not** anticipated that estimation questions should require the use of sophisticated mathematical techniques. The aim of these questions is to allow students to demonstrate that they can:

- make appropriate assumptions. Often students will be expected to make reasonable assumptions and a range of values will be acceptable for solutions
- use basic mathematical formulae. Some of these will be formulae that the students are expected to know, for example formulae for area and volume of familiar shapes and solids. Others may be found in the formula booklet or might be identified in the pre-release material. The formulae sheet is in section 7 of our specification
- use their knowledge of units for quantities like length, area and volume
- scale quantities up and down to meet the requirements of the problem.

This booklet contains two examples of questions that might be set on estimation in an examination. The four bullet points above are explored in the context of these examples. The aim is to provide teachers with examples that they can use with their students.

Some of the questions that are set will make use of pre-release material. This is issued to the students before the exam, so that they can think about the context that might be considered in the question. Here in the **Building a house** example, information is given about the materials used to build a house. The actual question is about the slightly simpler context of the building a garage which is related to the pre-release material but different from it. Teachers can use the pre-release material to explore how they would approach other questions that could be asked about the **Building a house** context.

Building a house

Preliminary material

External walls of houses are often built using cavity walls. Cavity walls are two walls with a gap in between. In more modern houses this gap will be filled with insulation and in older houses it is just an empty space. The house shown below has the outer cavity wall made from bricks and the inner cavity wall made from concrete blocks.

The dimensions of a typical brick are:

height	6.5 cm
length	21.5 cm
width	10.25 cm

The dimensions of a typical concrete block are:

height	22.5 cm
length	45 cm
width	10 cm

Question

Use **Building a house** on page 2 of this document.

The double garage shown below is built using a cavity wall system.

The outer cavity wall is made with bricks.

The inner cavity wall is made with concrete blocks.

Estimate the number of bricks needed to build this garage.

Possible solution

Estimates of the dimensions of the garage

max height	5 m
min height	2.5 m
width	6 m
length	8 m

Area of each side wall = $2.5 \times 8 = 20\ m^2$

Area of each end wall = $\frac{1}{2} \times 6 \times 2.5 + 6 \times 2.5 = 22.5\ m^2$

Total area of walls = $40 + 45 = 85\ m^2$

Area of doors = $2 \times 2 \times 2 + 2 \times 0.5 \times 1 = 9\ m^2$

Area of exposed bricks = $85 - 9 = 76\ m^2 = 76 \times 100^2 = 760000\ cm^2$

Dimensions of brick with allowance for cement: 22.5 cm and 11.25 cm

Area of one brick and cement = $22.5 \times 11.25 = 253\ cm^2$

$$\text{Number of Bricks Needed} = \frac{760000}{253} \approx 3000 \text{ Bricks}$$

Commentary

Make appropriate assumptions

When approaching the problem it is key for the students to make considered assumptions, some of which may be more refined than others. In this example, a particular set of assumptions are used to illustrate the approach that a particular student might take. However, when examination questions are marked a range of reasonable values will be accepted. For example, when considering the length of the garage it is vital that a car can be driven into the garage, so a value of less than about 5 metres would not be within the reasonable range. A length in excess of about 10 metres would also not be appropriate.

The task **Building a house** actually focuses on the construction of a garage. The key assumptions needed to solve this problem relate to the dimensions of the garage. These can be estimated by thinking about the size of the cars that might be parked in the garage and the fact that people would need to be able to walk in and out of the garage.

When estimating the dimensions of the garage, it is worth noting that the walls will have a thickness which could be estimated at about 0.3 metres. This is a more subtle point which could be considered as a more refined assumption and that would not be expected from all students.

Estimating the width of a car as about 1.5 metres is a starting point for the width of the garage. There will need to be space on each side of a car as it drives in, so a door of width 2 metres would be reasonable. Also the garage is clearly wider than the two doors, so a total of about 6 metres for the width would seem reasonable.

Returning to the idea of parking a car in the garage, the garage should be longer than the length of the car. Given that the maximum length of a car might be about 5 metres, and that there is a need to allow space in front of and behind the parked car as well as for the end walls, suggests that a figure of about 8 metres might be a reasonable length.

As for the height of the garage, it must be such that a person can walk easily through the doors. If the door height is taken as about 2 metres, then the height of the roof will have a minimum of about 2.5 metres. Also the pitch of the roof is such that the maximum height looks to be about double the height at the centre.

In the solution an assumption has been made about the fact that there is a layer of cement between the bricks. This shows evidence of thinking about the problem, but is much less significant than some of the other assumptions.

It is also necessary to make some assumptions about the dimensions of the windows that can be seen in the side wall of the garage.

Use basic mathematical formulae

For this question, formulae for calculating the area of rectangles and triangles are required.

Use their knowledge of units for quantities like length, area and volume

In this solution the dimensions of the garage have been estimated in metres and the dimensions of the bricks have been given in cm. When attempting to solve this problem there is a need to recognise that there are two different units in use and that there is a need to be able to convert between them. In particular to realise that:

$$1 \text{ m}^2 = 10000 \text{ cm}^2$$

Scale quantities up and down to meet the requirements of the problem

Once the total surface area of the brick walls of the garage have been found this area can be divided by the exposed surface area of a brick to estimate the number of bricks that are needed.

Dentists

Question

The population of Bristol is between 430 000 and 440 000

Estimate the number of dentists needed in Bristol.

Possible solution

Assumptions:

- Standard checkup time 10 mins per patient
- 10% of population don't visit dentist
- 45% of population visit every 6 months
- 45% of population visit every 12 months
- Dentists spend 6 hours per day with patients
- Dentists work for 220 days per year
- 20% of those who visit the dentist for a checkup require return visits
- Return visits take on average 20 mins

Check ups

45% of 440 000 = 198000

Time needed for 6 month checkups = 198000 × 10 × 2 = 3960000 mins / year

Time needed for 12 month checkups = 198000 × 10 = 1980000 mins / year

Return visits

20% of 198000 = 39600

Time needed for return visits = 3 × 39600 × 20 = 2376000 mins / year

Total time

Total time = 396000 + 1980000 +2376000 = 8316000 mins / year = 138600 hours / year

Dentists work 6 × 220 = 1320 hours / year

Number of dentists needed = $\frac{138600}{1320} = 105$

It is sensible to estimate that Bristol needs about 100 dentists.

Commentary

Make appropriate assumptions

In this example there is a wide range of acceptable or reasonable assumptions. Making reasonable assumptions is a key stage in solving this problem. Primarily, students need to make assumptions about:

- **How often people visit the dentist.** The example above shows that some will never go to the dentist and that some will visit more frequently than others. A simpler approach would be to assume that everyone goes for a check every 6 months.
- **How many people need return visits for treatment.** This has been estimated as 20% but any estimate in the range 10 – 50% would be reasonable. Some students may not consider return visits but credit would be given to those that make reasonable assumptions about this.
- **How long check-ups and treatment take**. Clearly a check-up does not take very long and treatments will be expected to be longer. The values used here are possible values and others within a reasonable range would be acceptable. For example assuming that a check-up takes 60 mins would not be reasonable.
- **How much time dentists have to treat patients**. The values used here are based on the assumption that dentists work for 220 days and spend 6 hours a day with patients. This is based on the assumption that dentists do not work at weekends and have a reasonable holiday allocation. Also they will need some time each day to deal with administrative and other tasks.

Use Basic mathematical formulae

The main emphasis in this solution has been on the use of percentages.

Use their knowledge of units for quantities like length, area and volume

The emphasis in this example is on the units for time. There is a need to think in minutes and days.

Scale quantities up and down to meet the requirements of the problem

This is a really important aspect of this solution. If you start with the time that a single person might spend at dentist appointments, you can scale up to the time needed for a complete city (Bristol). You can then divide this by the amount of time one dentist spends with patients to estimate the number of dentists needed in Bristol.