A-LEVEL

Mathematics

Paper 3

Mark scheme

Specimen

Version 1.2

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

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General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	18
	Total		**1**	
2(a)	Makes clear attempt to use the cosine rule	AO3.1a	M1	$6^2 = 3^2 + 5^2 - 2 \times 3 \times 5\cos\theta$
	Uses trig identity with 'their' cos θ	AO1.1a	M1	$\cos\theta = \frac{3^2 + 5^2 - 6^2}{30} = -\frac{1}{15}$
	Constructs rigorous argument leading to correct result **AG** Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	$\therefore \sin\theta = \sqrt{1 - \left(-\frac{1}{15}\right)^2}$ $\sin\theta = \frac{4\sqrt{14}}{15}$ (**AG**)
(b)	Writes down correct angle	AO2.2a	B1	1.64
(c)	Uses 'their' angle in $\frac{1}{2}r^2\theta$	AO1.1a	M1	$A = \frac{1}{2} \times 5^2 \times 1.64$
	Correct area FT use of incorrect obtuse angle provided both M1 marks awarded in part **(a)** and M1 awarded in **(c)**	AO1.1b	A1F	$= 20.5$ m^2
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Translates proportionality into a differential equation involving $\frac{\mathrm{d}A}{\mathrm{d}t}$, A and a constant of proportionality.	AO3.3	M1	$\frac{\mathrm{d}A}{\mathrm{d}t} \propto A$ $\Rightarrow \frac{\mathrm{d}A}{\mathrm{d}t} = kA$ $\Rightarrow \int \frac{1}{A}\,\mathrm{d}A = \int k\,\mathrm{d}t$ $\Rightarrow \ln A = kt + c$ $\Rightarrow A = \mathrm{e}^{kt+c}$ $\Rightarrow A = B\mathrm{e}^{kt}$ **AG**
	Separates variables	AO1.1a	M1	
	Integrates both of 'their' sides correctly	AO1.1b	A1F	
	Constructs a rigorous mathematical argument that supports use of the given model. **AG** Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips.	AO2.1	R1	
(b)(i)	States correct value of B	AO1.1b	B1	$B = 0.25$ or $B = \frac{1}{4}$
(b)(ii)	Uses $t = 20$ and $A = 0.5$ to find k	AO3.1b	M1	When $t = 20$, $A = 0.5$ $\Rightarrow 0.5 = 0.25\mathrm{e}^{20k}$ $\Rightarrow 20k = \ln 2$ $\Rightarrow k = \frac{1}{20}\ln 2$ $\Rightarrow A = \frac{1}{4}(\mathrm{e}^{\ln 2})^{\frac{t}{20}}$ $\Rightarrow A = 2^{-2} \times 2^{\frac{t}{20}}$ $\Rightarrow A = 2^{\frac{t}{20}-2}$ **AG**
	Finds correct value of k	AO1.1b	A1	
	Substitutes 'their' k to get A in terms of t	AO1.1a	M1	
	Constructs rigorous and convincing argument to show $A = 2^{\frac{t}{20}-2}$ Using correct notation throughout. **AG**	AO2.1	R1	
(b)(iii)	Uses the model to set up correct equation and attempt to find t	AO3.4	M1	$2\pi = 2^{\frac{t}{20}-2}$ $t = 93.03$ days
	Finds correct value of t	AO1.1b	A1	
(c)	States any sensible and relevant limitation of the model that is specified in terms of the pond, area, weed, rate of change or time.	AO3.5b	E1	Model predicts that the area of weed will increase without limit and this is not possible since the area of the pond is 4π
(d)	Any sensible and relevant refinement to the model that is specified in terms of the pond, area, weed, rate of change or time	AO3.5c	E1	Introduce a limiting factor such as fish eating weed or rate of growth decreases as surface area covered
	Total		**13**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Selects a method of integration, which could lead to a correct solution. Evidence of integration by parts OR an attempt at integration by inspection.	AO3.1a	M1	$u = \ln 2x;\quad \frac{dv}{dx} = x^3$ $\frac{du}{dx} = \frac{1}{x};\quad v = \frac{x^4}{4}$ $\left[\frac{x^4}{4}\ln(2x)\right]_1^2 - \int_1^2 \frac{x^3}{4}\,dx$ $\left[\frac{x^4}{4}\ln(2x) - \frac{x^4}{16}\right]_1^2$
	Applies integration by parts formula correctly OR correctly differentiates an expression of the form $Ax^4\ln 2x$	AO1.1b	A1	$=\left(\frac{2^4}{4}\ln(4) - \frac{2^4}{16}\right) - \left(\frac{1}{4}\ln(2) - \frac{1}{16}\right)$ $\frac{31}{4}\ln 2 - \frac{15}{16}$
	Obtains correct integral, condone missing limits.	AO1.1b	A1	so $p = \frac{31}{4}\qquad q = -\frac{15}{16}$
	Substitutes correct limits into ‘their’ integral	AO1.1a	M1	**ALT** $\frac{d}{dx}(x^4\ln 2x) = 4x^3\ln 2x + x^4.\frac{1}{x}$
	Obtains correct p and q FT use of incorrect integral provided both M1 marks have been awarded	AO1.1b	A1F	$\therefore \int_1^2 x^3\ln 2x\,dx = \left[\frac{1}{4}(x^4\ln 2x - \frac{x^4}{4})\right]_1^2$ $=\left(\frac{2^4}{4}\ln(4) - \frac{2^4}{16}\right) - \left(\frac{1}{4}\ln(2) - \frac{1}{16}\right)$ $\frac{31}{4}\ln 2 - \frac{15}{16}$ $p = \frac{31}{4}\qquad q = -\frac{15}{16}$
	Total		**5**	

	Marking Instructions	AO	Marks	Typical Solution
5(a)	Uses binomial expansion, with at least two terms correct, may be un-simplified	AO1.1a	M1	$(1+6x)^{\frac{1}{3}} \approx 1+\frac{1}{3}\cdot 6x+\frac{1}{3}\cdot\frac{-2}{3}\cdot\frac{(6x)^2}{2}$
	Obtains correct simplified answer	AO1.1b	A1	$(1+6x)^{\frac{1}{3}} \approx 1+2x-4x^2$
(b)	Determines the correct value for x and substitutes this into 'their' answer to part **(a)**	AO3.1a	M1	$x=0.03$
	Obtains correct approximation for 'their' answer to part **(a)** FT allowed only if M1 from part **(a)** and M1 from part **(b)** have been awarded	AO1.1b	A1F	$\sqrt[3]{1.18} \approx 1 + 2(0.03) - 4(0.03)^2$ ≈ 1.0564
(c)	Explains the limitation of the expansion found in part **(a)** with reference to $x=\frac{1}{2}$	AO2.4	E1	Although $\left(1+6\times\frac{1}{2}\right)^{\frac{1}{3}}=\sqrt[3]{4}$ $x=\frac{1}{2}$ cannot be used since the expansion is only valid for $\lvert x\rvert<\frac{1}{6}$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
6	Uses partial fractions with linear denominators $\dfrac{6x+1}{6x^2-7x+2}=\dfrac{A}{ax+b}+\dfrac{B}{cx+d}$	AO3.1a	M1	$\dfrac{6x+1}{6x^2-7x+2}=\dfrac{A}{3x-2}+\dfrac{B}{2x-1}$ $A(2x-1)+B(3x-2)=6x+1$
	Obtains correct linear denominators	AO1.1b	B1	$x=\dfrac{2}{3},\ A\left(\dfrac{1}{3}\right)=5$ so $A=15$
	Obtains at least one numerator correct (using any valid method, eg equating coefficients or substitution of values)	AO1.1b	A1	$x=\dfrac{1}{2},\quad B\left(-\dfrac{1}{2}\right)=4$ so $B=-8$ $\displaystyle\int_1^2 \frac{15}{3x-2}-\frac{8}{2x-1}\,\mathrm{d}x$ $=\left[5\ln(3x-2)-4\ln(2x-1)\right]_1^2$
	Obtains partial fractions completely correct	AO1.1b	A1	$=5\ln(4)-4\ln(3)-(5\ln(1)4\ln(1))$ $=10\ln(2)-4\ln(3)$
	Integrates 'their' partial fractions, must include logs $p\ln(ax+b)+q\ln(cx+d)$	AO1.1a	M1	
	'Their' integral correct (ignore limits)	AO1.1b	A1F	
	Substitutes limits into 'their' integral	AO1.1a	M1	
	Correct final answer in correct form **CAO**	AO1.1b	A1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	Finds 2nd derivative and sets up an inequality	AO3.1a	M1	$\frac{dy}{dx}=-2xe^{-x^2}$
	Obtains correct first derivative	AO1.1b	A1	$\frac{d^2y}{dx^2}=-2e^{-x^2}+4x^2e^{-x^2}$
	Obtains second derivative correct from 'their' first derivative	AO1.1b	A1F	
	Deduces correct final inequality (could use set notation)	AO2.2a	A1	$-2e^{-x^2}+4x^2e^{-x^2}<0$ $4x^2-2<0$ $-\frac{\sqrt{2}}{2}<x<\frac{\sqrt{2}}{2}$
(b)	Uses trapezium rule	AO1.1a	M1	$\int_{0.1}^{0.5} e^{-x^2}\,dx \approx \frac{0.1}{2}(e^{-0.01}+e^{-0.25}$ $+2(e^{-0.04}+e^{-0.09}+e^{-0.16}))$ ≈ 0.3611
	Trapezium rule entries all correct	AO1.1b	A1	
	Finds correct value	AO1.1b	A1	
(c)	References area being completely within **concave** section So…	AO2.4	E1	$[0.1, 0.5]\subset\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$ $\therefore$ area is completely within concave section
	Trapezia all fall completely underneath the curve therefore underestimate (only award this mark if previous E1 has been awarded)	AO2.4	E1	Hence trapezia lie below curve and give an under-estimate for the area
(d)	Uses suitable rectangle to obtain over-estimate	AO3.1a	B1	Using a rectangle with the left hand edge the same height as the curve will produce an over-estimate
	Explains that this rectangle lies above the curve	AO2.4	E1	Area of rectangle = $0.4\times e^{-0.1^2} = 0.396...$
	Constructs rigorous mathematical argument about accuracy, which leads to required result Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips.	AO2.1	R1	$\therefore\ 0.36<A<0.40$ So A = 0.4 to 1 dp
	Total		**12**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Circles correct answer	AO1.2	B1	The 2065 households in the village
(b)	Circles correct answer	AO1.2	B1	Simple random
	Total		**2**	
9(a)	Finds value for p	AO1.1a	M1	$p = \frac{1680}{2400} = 0.7$
	Finds correct probability from calculator	AO1.1b	A1	Using $X \sim \text{B}(25,0.7)$, $\text{P}(X = 22) = 0.0243$
(b)	Explains the reason why the model may no longer apply in context	AO3.5b	E1	It is likely that all the houses (and gardens) will be of similar types, and hence similar owners, so not likely to be independent as binomial model requires.
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	Indicates that 'Semi-skimmed milk' is part of the whole' Skimmed milks' category.	AO2.4	E1	In Figure 2, the vertical axis represents 'Skimmed milks' whereas in Figure 1 the vertical axis represents 'Semi-skimmed milk' that is just one part of the' Skimmed milks' category. Hence, for each region, the recorded values for 'Semi-skimmed milk' will always be lower that the ones for 'Skimmed milks'.
(b)(i)	States: **strong positive** correlation **between** purchases of **'Semi-skimmed milk' and 'Liquid wholemilk, full price**' for most regions.	AO2.5	B1	Regions A, B, D, F, G, H and J indicate strong positive correlation between purchases of 'Semi-skimmed milk' and 'Liquid wholemilk, full price'.
	Identifies regions C and E as outliers	AO3.2b	E1	Regions C and E do not follow the same pattern as the other regions in terms of purchases of skimmed milk and wholemilk or regions C and E can be excluded as they appear to be outliers.
	ALT Allow 1 mark if candidate states: **weak negative** correlation between **consumption of 'Semi-skimmed milk' and 'Liquid wholemilk, full price'.**	AO2.5 AO3.2b	B1 E0	

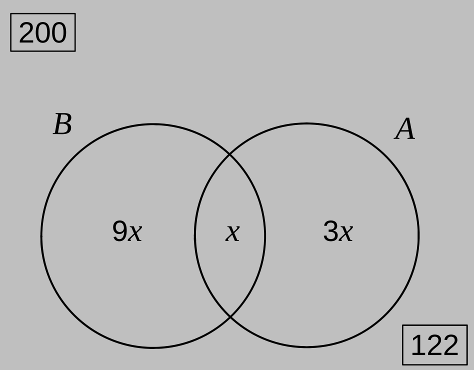
Q	Marking Instructions	AO	Marks	Typical Solution
10(b)(ii)	States that there is evidence of negative correlation between purchases of 'Mineral or spring water' and 'Skimmed milk'.	AO3.1b	B1	Scatter graph indicates evidence of negative correlation between consumption of 'Mineral or spring water' and 'Skimmed milks'.
	The correlation is between water and milk purchases within the regions and not between purchases made by individual people so Bilal's claim is not proven	AO2.3	E1	Bilal's claim is not supported because the correlation in figure 2 is between water and milk purchases within the regions and he is assuming that individuals will follow a similar pattern but the data does not tell us anything about individuals, it refers to average purchases made within the nine regions.
(c)	Identifies London	AO2.2b	B1	London
	Gives valid reason based on knowledge of the LDS	AO2.4	E1	Having studied the LDS I have identified a clear trend: London is a frequent outlier in most categories
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(a)	Finds correct value of k	AO1.1b	B1	$k = \frac{1}{16}$
(b)	Selects relevant probability	AO1.1a	M1	P($\geq$ 2 checkouts staffed) $\frac{3}{16} + k = \frac{3}{16} + \frac{1}{16} = \frac{1}{4}$ **ALT** P($\geq$ 2 checkouts staffed) $= 1 - \frac{3}{4} = \frac{1}{4}$
	Finds correct probability FT 'their' value of k found in part **(a)**	AO1.1b	A1F	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	States both hypotheses correctly for one-tailed test	AO2.5	B1	$X =$ number of Christmas holidays without illness since January 2007 $X \sim B(7, p)$ $H_0 \; p = 0.65$ $H_1 \; p < 0.65$
	States model used (condone 0.009 rather than 0.056) **PI**	AO1.1b	M1	Under null hypothesis, $X \sim B(7, 0.65)$
	Using calculator, 0.056 or better	AO1.1b	A1	$P(X \le 2) = 0.0556$
	Evaluates binomial model by comparing $P(X \le 2)$ with 0.05 **PI**	AO3.5a	M1	0.0556 >0.05
	Infers H_0 accepted PI	AO2.2b	A1	Accept H_0
	Concludes correctly in context. 'not sufficient evidence' or equivalent required	AO3.2a	E1	There is not sufficient evidence that the John's rate of illness has decreased
(b)	States one correct assumption(s) regarding validity of model	AO3.5b	E1	**Assumption 1** The probability of illness remains constant throughout one's life
	States corresponding correct description(s) of likelihood of validity in context	AO2.4	E1	**Validity** Not fully valid, as age has an impact on the immune system **OR**
	States second correct assumption(s) regarding validity of model	AO3.5b	E1	
	States corresponding correct description(s) of likelihood of validity in context	AO2.4	E1	**Assumption 2** Annual results (of illness) are independent of one another
	Max two assumptions with description of validity			**Validity** (Largely) valid. Trials are sufficiently far apart that an illness spanning two Christmases is unlikely. **OR** **Assumption 3** There are only two states, well and ill **Validity** Unclear. Grey area exists. eg does a mild sore throat count as ill?
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)(i)	Finds mean	AO1.1b	B1	Mean $= \bar{X} = \frac{3046.14}{100} = 30.46(14)$
	Finds standard deviation awfw (4.17 – 4.20) Allow $s = \sqrt{\frac{1746.29}{100}} = 4.179$	AO1.1b	B1	Standard deviation $s = \sqrt{\frac{1746.29}{99}} = 4.20$
(ii)	States max range for a normal distribution	AO1.2	M1	$\bar{X} + 3s \approx 43$ $\bar{X} - 3s \approx 18$
	Explains why normal model acceptable	AO2.4	E1	Normal dist model acceptable because data can be regarded as continuous and the range of amounts is within $\bar{X} \pm 3s$
(iii)	Finds probability awfw (0.095 – 0.097)	AO1.1b	B1	$P(X < 25) = 0.0967$
(b)	Finds z from calculator	AO1.1b	B1	$\mathrm{P}\left(Z < \frac{30 - 29.55}{\sigma}\right) = 0.55$ $\frac{30 - 29.55}{\sigma} = 0.1257$ $\sigma = \frac{30 - 29.55}{0.1257} = 3.6$
	Uses Normal model to form equation	AO3.4	M1	
	Finds correct sd Correct accuracy required for this mark Accept 3.5 or 3.7	AO1.1b	A1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)(i)	States both hypotheses using correct language	AO2.5	B1	$H_0 : \mu = 123$ $H_1 : \mu \neq 123$
	Finds test statistic	AO1.1a	M1	Test statistic $= \dfrac{127 - 123}{\dfrac{70}{\sqrt{12144}}}$
	Obtains correct test statistic	AO1.1b	A1	$= 6.30$
	Infers H_0 rejected by comparison of ts with cv	AO2.2b	A1	Critical z values ± 1.96 6.30 > 1.96
	Concludes correctly in context, including 'some evidence'	AO3.2a	E1	Reject H_0 there is evidence (at the 5% level) to suggest the mean expenditure on bread had changed from 2012 to 2013
ALT	States both hypotheses using correct language	AO2.5	B1	$H_0 : \mu = 123$ $H_0 : \mu \neq 123$
	Attempts to find p value for z-test	AO1.1a	M1	From calculator, $P(X < 127) = 3.045 \times 10^{-10}$
	Finds correct p value	AO1.1b	A1	$3.045 \times 10^{-10} < 0.025$
	Infers H_0 rejected by comparison of p with 0.025	AO2.2b	A1	
	Concludes correctly in context, including 'some evidence'	AO3.2a	E1	Reject H_0- there is evidence (at the 5% level) to suggest the mean expenditure on bread had changed from 2012 to 2013
(a)(ii)	Uses Normal model to find critical values PI	AO3.4	M1	$123 \pm 1.96 \times \dfrac{70}{\sqrt{12144}}$
	Obtains correct critical values Correct accuracy required for this mark Disallow integer answers	AO1.1b	A1	min=121.75 and max=124.25
(b)(i)	States valid reason for statement 1 'not supported'	AO2.4	R1	The conclusion implies that the mean changed, not that it increased by a specific amount, so the statement is not supported
	Infers that model/test used would not imply the statement	AO2.2b	R1	
(b)(ii)	States valid reason for statement 2 'not supported'	AO2.4	R1	The conclusion implies that there is evidence that the mean has changed, but expenditure increase may be due to price changes, so statement is not supported
	Infers that model/test used would not imply the statement	AO2.2b	R1	
	Total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
15	Uses conditional probability, either ① or ②	AO3.1b	M1	$\frac{P(A\cap B)}{P(A)}=\frac{1}{4}$ ① $\Rightarrow P(A)=4P(A\cap B)$
	Obtains both equations ① and ② correctly	AO1.1b	A1	$\frac{P(A\cap B)}{P(B)}=\frac{1}{10}$ ② $\Rightarrow P(B)=10P(A\cap B)$
	Evaluates $P(A\cup B)$ correctly **PI**	AO1.1b	B1	$P(A\cup B)=1-\frac{122}{200}=\frac{39}{100}$
	Uses addition law	AO1.1a	M1	$P(A)+P(B)-P(A\cap B)=\frac{39}{100}$ ③
	Combines the **three** equations	AO1.1a	M1	$4P(A\cap B)+10P(A\cap B)-P(A\cap B)=\frac{39}{100}$
	Obtains correct probability, as a fraction or decimal	AO2.2b	A1	$P(A\cap B)=\frac{3}{100}$
ALT	Produces a relevant Venn diagram	AO3.1b	M1	**OR** 
	Labels Venn diagram correctly	AO1.1b	A1	
	Forms correct equation to find x PI	AO1.1b	B1	$9x + x + 3x = 200 - 122$
	Combines terms	AO1.1a	M1	$13x = 78$
	Solves equation	AO1.1a	M1	$x = 6$
	Obtains correct probability	AO2.2b	A1	$P(A\cap B)=\frac{6}{200}$ or 0.03
	Total		**6**	
	TOTAL		**100**	