

Please write clearly, in block capitals.

Centre number

--	--	--	--	--

Candidate number

--	--	--	--

Surname

Forename(s)

Candidate signature

A-level MATHEMATICS

Paper 3

Exam Date

Morning

Time allowed: 2 hours

Materials

For this paper you must have:

- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

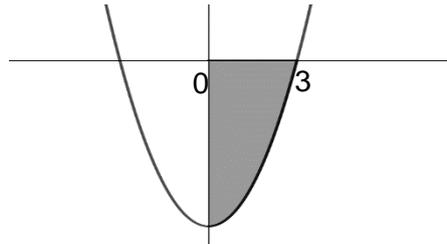
Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet. You do not necessarily need to use all the space provided.

Section A

Answer **all** questions in the spaces provided.

- 1** The graph of $y = x^2 - 9$ is shown below.



Find the area of the shaded region.
Circle your answer.

[1 mark]

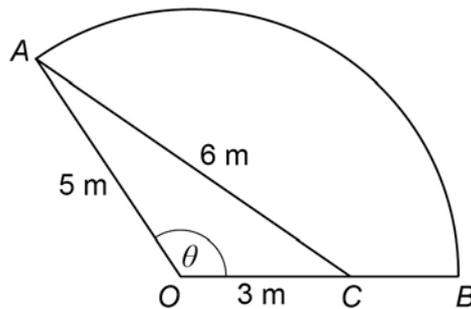
-18

-6

6

18

- 2** A wooden frame is to be made to support some garden decking. The frame is to be in the shape of a sector of a circle. The sector OAB is shown in the diagram, with a wooden plank AC added to the frame for strength. OA makes an angle of θ with OB .



- 2 (a)** Show that the exact value of $\sin \theta$ is $\frac{4\sqrt{14}}{15}$

[3 marks]

2 (b) Write down the value of θ in radians to 3 significant figures.

[1 mark]

2 (c) Find the area of the garden that will be covered by the decking.

[2 marks]

3 (b) (iii) How many days does it take for the weed to cover half of the surface of the pond?

[2 marks]

3 (c) State one limitation of the model.

[1 mark]

3 (d) Suggest one refinement that could be made to improve the model.

[1 mark]

-
- 5 (a)** Find the first three terms, in ascending powers of x , in the binomial expansion of $(1 + 6x)^{\frac{1}{3}}$

[2 marks]

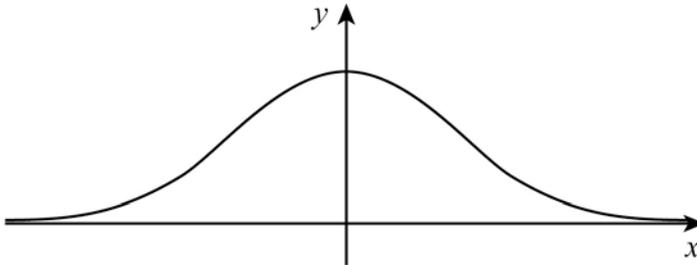
- 5 (b)** Use the result from part **(a)** to obtain an approximation to $\sqrt[3]{1.18}$ giving your answer to 4 decimal places.

[2 marks]

- 5 (c)** Explain why substituting $x = \frac{1}{2}$ into your answer to part **(a)** does not lead to a valid approximation for $\sqrt[3]{4}$.

[1 mark]

- 7 The diagram shows part of the graph of $y = e^{-x^2}$



The graph is formed from two convex sections, where the gradient is increasing, and one concave section, where the gradient is decreasing.

- 7 (a) Find the values of x for which the graph is concave.

[4 marks]

- 7 (c)** Explain with reference to your answer in part **(a)**, why the answer you found in part **(b)** is an underestimate.

[2 marks]

Section B

Answer **all** questions in the spaces provided.

- 8** Edna wishes to investigate the energy intake from eating out at restaurants for the households in her village.

She wants a sample of 100 households.

She has a list of all 2065 households in the village.

Ralph suggests this selection method.

“Number the households 0000 to 2064. Obtain 100 different four-digit random numbers between 0000 and 2064 and select the corresponding households for inclusion in the investigation.”

- 8 (a)** What is the population for this investigation?

Circle your answer.

[1 mark]

Edna and Ralph	The 2065 households in the village	The energy intake for the village from eating out	The 100 households selected
----------------	------------------------------------	---	-----------------------------

- 8 (b)** What is the sampling method suggested by Ralph?

Circle your answer.

[1 mark]

Opportunity	Random number	Continuous random variable	Simple random
-------------	---------------	----------------------------	---------------

9 A survey has found that, of the 2400 households in Growmore, 1680 eat home-grown fruit and vegetables.

9 (a) Using the binomial distribution, find the probability that, out of a random sample of 25 households in Growmore, exactly 22 eat home-grown fruit and vegetables.

[2 marks]

9 (b) Give a reason why you would **not** expect your calculation in part **(a)** to be valid for the 25 households in Gifford Terrace, a residential road in Growmore.

[1 mark]

10 Shona calculated four correlation coefficients using data from the Large Data Set.

In each case she calculated the correlation coefficient between the masses of the cars and the CO₂ emissions for varying sample sizes.

A summary of these calculations, labelled A to D, are listed in the table below.

	Sample size	Correlation coefficient
A	3827	0.088
B	3735	0.246
C	24	0.400
D	1250	- 1.183

Shona would like to use calculation A to test whether there is evidence of positive correlation between mass and CO₂ emissions.

She finds the critical value for a one-tailed test at the 5% level for a sample of size 3827 is 0.027

10 (a) (i) State appropriate hypotheses for Shona to use in her test.

[1 mark]

Question 10 continues on the next page

10 (a) (ii) Determine if there is sufficient evidence to reject the null hypothesis.

Fully justify your answer.

[1 mark]

10 (b) Shona's teacher tells her to remove calculation D from the table as it is incorrect.

Explain how the teacher knew it was incorrect.

[1 mark]

10 (c) Before performing calculation B, Shona cleaned the data. She removed all cars from the Large Data Set that had incorrect masses.

Using your knowledge of the large data set, explain what was incorrect about the masses which were removed from the calculation.

[1 mark]

- 11** Terence owns a local shop. His shop has three checkouts, at least one of which is always staffed.

A regular customer observed that the probability distribution for N , the number of checkouts that are staffed at any given time during the spring, is

$$P(N = n) = \begin{cases} \frac{3}{4} \left(\frac{1}{4}\right)^{n-1} & \text{for } n = 1, 2 \\ k & \text{for } n = 3 \end{cases}$$

- 11 (a)** Find the value of k .

[1 mark]

- 11 (b)** Find the probability that a customer, visiting Terence's shop during the spring, will find at least 2 checkouts staffed.

[2 marks]

- 13** In the South West region of England, 100 households were randomly selected and, for each household, the weekly expenditure, £ X , per person on food and drink was recorded.

The maximum amount recorded was £40.48 and the minimum amount recorded was £22.00

The results are summarised below, where \bar{x} denotes the sample mean.

$$\sum x = 3046.14 \qquad \sum (x - \bar{x})^2 = 1746.29$$

- 13 (a) (i)** Find the mean of X

Find the standard deviation of X

[2 marks]

- 13 (a) (ii)** Using your results from part **(a)(i)** and other information given, explain why the normal distribution can be used to model X .

[2 marks]

-
- 13 (a) (iii)** Find the probability that a household in the South West spends less than £25.00 on food and drink per person per week.

[1 mark]

- 13 (b)** For households in the North West of England, the weekly expenditure, £ Y , per person on food and drink can be modelled by a normal distribution with mean £29.55

It is known that $P(Y < 30) = 0.55$

Find the standard deviation of Y , giving your answer to one decimal place.

[3 marks]

Turn over for the next question

-
- 14 (b)** The 2013 survey revealed that the mean expenditure per adult, per week on alcohol was 324p.

The mean expenditure per adult per week on alcohol for 2009 was 307p.

A test was carried out on the following hypotheses relating to mean expenditure per adult per week on alcohol in 2013.

$$H_0 : \mu = 307$$

$$H_1 : \mu \neq 307$$

This test resulted in the null hypothesis, H_0 , being rejected.

State, with a reason, whether the test result supports the following statements:

- 14 (b) (i)** the mean UK expenditure on alcohol per adult per week increased by 17p from 2009 to 2013;

[2 marks]

- 14 (b) (ii)** the mean UK consumption of alcohol per adult per week changed from 2009 to 2013.

[2 marks]
