SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

AS
FURTHER MATHEMATICS

Paper 1

Exam Date Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A reflection is represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

State the equation of the line of invariant points.

Circle your answer.

[1 mark]

$x = 0$ $y = 0$ $y = x$ $y = -x$

2 Find the mean value of $3x^2$ over the interval $1 \leq x \leq 3$

Circle your answer.

[1 mark]

$8\frac{2}{3}$ 10 13 26

3 Find the equations of the asymptotes of the curve $x^2 - 3y^2 = 1$

Circle your answer.

[1 mark]

$y = \pm 3x$ $\quad$ $y = \pm\frac{1}{3}x$ $\quad$ $y = \pm\sqrt{3}\,x$ $\quad$ $y = \pm\frac{1}{\sqrt{3}}x$

Turn over for the next question

4 $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 1 & k \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

4 (a) Find the value of k for which matrix **A** is singular.

[1 mark]

4 (b) Describe the transformation represented by matrix **B**.

[1 mark]

4 (c) (i) Given that **A** and **B** are both non-singular, verify that $\mathbf{A}^{-1}\mathbf{B}^{-1} = (\mathbf{BA})^{-1}$.

[4 marks]

4 (c) (ii) Prove the result $\mathbf{M}^{-1}\mathbf{N}^{-1} = (\mathbf{NM})^{-1}$ for all non-singular square matrices **M** and **N** of the same size.

[4 marks]

Turn over for the next question

5 The region bounded by the curve with equation $y = 3 + \sqrt{x}$, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through 2π radians about the x-axis.

Use integration to show that the volume generated is $\frac{125\pi}{2}$

[5 marks]

6 (a) Use the definitions of $\sinh x$ and $\cosh x$ in terms of e^{x} and e^{-x} to show that $x = \frac{1}{2} \ln\left(\frac{1+t}{1-t}\right)$ where $t = \tanh x$

[4 marks]

Question 6 continues on the next page

6 (b) (i) Prove $\cosh^3 x = \frac{1}{4}\cosh 3x + \frac{3}{4}\cosh x$

[4 marks]

6 (b) (ii) Show that the equation $\cosh 3x = 13\cosh x$ has only one positive solution.

Find this solution in exact logarithmic form.

[4 marks]

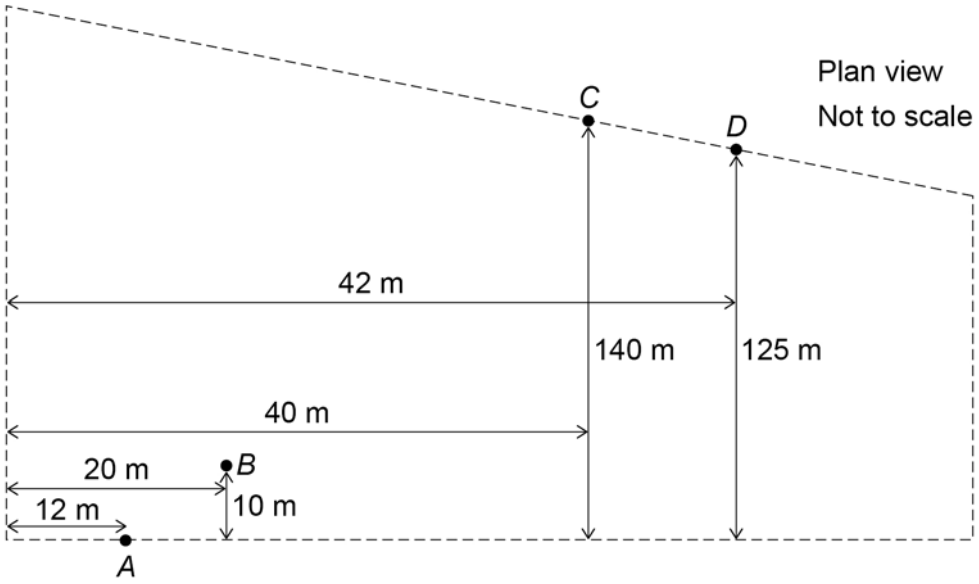
Turn over for the next question

7 A lighting engineer is setting up part of a display inside a large building. The diagram shows a plan view of the area in which he is working.

He has two lights, which project narrow beams of light.

One is set up at a point 3 metres above the point *A* and the beam from this light hits the wall 23 metres above the point *D*.

The other is set up 1 metre above the point *B* and the beam from this light hits the wall 29 metres above the point *C*.



7 (a) By creating a suitable model, show that the beams of light intersect.

[6 marks]

7 (b) Find the angle between the two beams of light.

[3 marks]

7 (c) State one way in which the model you created in part **(a)** could be refined.

[1 mark]

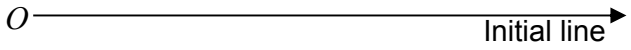
8 A curve has polar equation $r = 3 + 2\cos\theta$, where $0 \leq \theta < 2\pi$

8 (a) (i) State the maximum and minimum values of r.

[2 marks]

8 (a) (ii) Sketch the curve.

[2 marks]



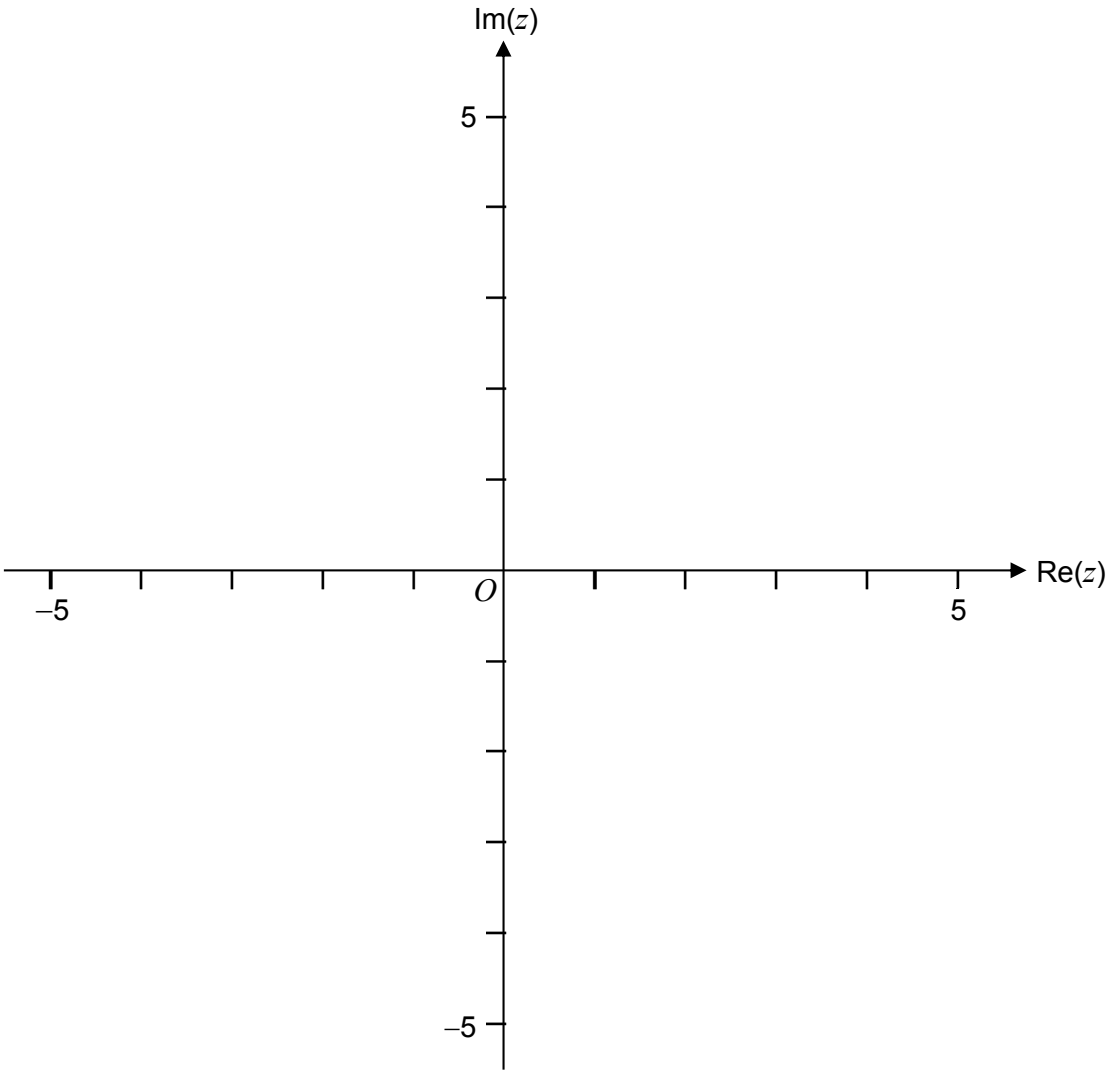
8 (b) The curve $r = 3 + 2\cos\theta$ intersects the curve with polar equation $r = 8\cos^2\theta$, where $0 \leq \theta < 2\pi$

Find all of the points of intersection of the two curves in the form $[r, \theta]$.

[6 marks]

9 (a) Sketch on the Argand diagram below, the locus of points satisfying the equation $|z - 2| = 2$

[2 marks]



9 (b) Given that $|z - 2| = 2$ and $\arg(z - 2) = -\frac{\pi}{3}$, express z in the form $a + b\mathrm{i}$, where a and b are real numbers.

[3 marks]

Turn over for the next question

10 (a) Prove that

$$6 + 3\sum_{r=1}^{n}(r+1)(r+2) = (n+1)(n+2)(n+3)$$

[6 marks]

10 (b) Alex substituted a few values of n into the expression $(n + 1)(n + 2)(n + 3)$ and made the statement:

"For all positive integers n,

$$6 + 3\sum_{r=1}^{n}(r + 1)(r + 2)$$

is divisible by 12."

Disprove Alex's statement.

[2 marks]

Turn over for the next question

11 The equation $x^3 - 8x^2 + cx + d = 0$ where c and d are real numbers, has roots α, β, γ.

When plotted on an Argand diagram, the triangle with vertices at α, β, γ has an area of 8.

Given $\alpha = 2$, find the values of c and d.

Fully justify your solution.

[5 marks]

12 A curve, C_1 has equation $y = f(x)$, where $f(x) = \dfrac{5x^2 - 12x + 12}{x^2 + 4x - 4}$

The line $y = k$ intersects the curve, C_1

12 (a) (i) Show that $(k + 3)(k - 1) \geq 0$

[5 marks]

Question 12 continues on the next page

12 (a) (ii) Hence find the coordinates of the stationary point of C_1 that is a maximum point.

[4 marks]

12 (b) Show that the curve C_2 whose equation is $y = \frac{1}{\mathrm{f}(x)}$, has no vertical asymptotes.

[2 marks]

12 (c) State the equation of the line that is a tangent to both C_1 and C_2.

[1 mark]

END OF QUESTIONS