A-LEVEL

Further Mathematics

Discrete

Mark scheme

Specimen

Version 1.1

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

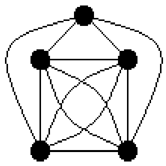
Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

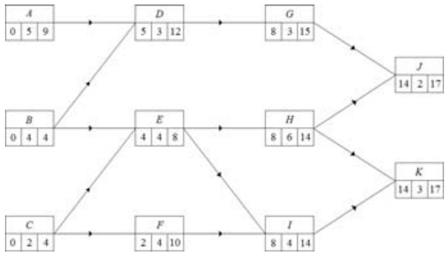
Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	9
	Total		**1**	

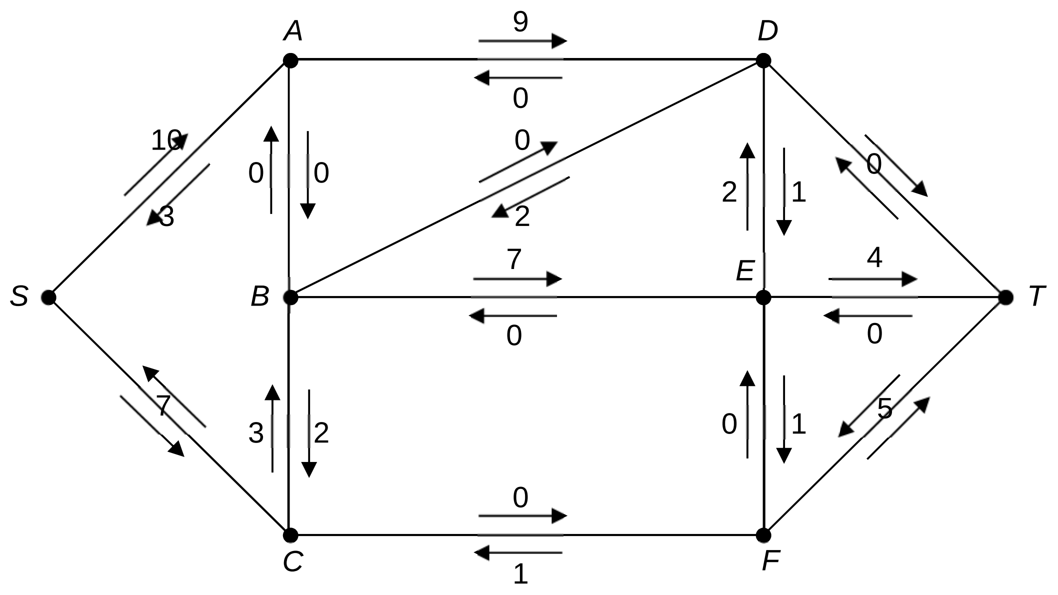
Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Finds the correct earliest start time for each activity	AO1.1b	B1	
	Finds the correct latest finish time for each activity	AO1.1b	B1	
3(b)	Evaluates the effects of reducing the duration of one of the activities on the project completion time	AO3.1b	M1	Reducing activity *E* duration to 1 hour reduces the project completion time to 14 hours, whereas all other activities reduce the project completion time to 15 hours or more
	Compares the effects of reducing the duration of each of the critical activities on the project completion time	AO3.1b	M1	
	Correctly deduces the activity which should have its duration reduced to one hour, from correct reasoning	AO2.2a	R1	Activity *E* clearly identified
3(c)	Correctly identifies a limitation in the context of the project	AO3.5b	B1	Time between one activity ending and the next activity starting is not taken into account, as workers may need to travel to a different location
	Explains how the limitation that has been identified affects project in time or monetary terms	AO2.4	E1	Not taking into account time. The travelling time will cause subsequent activities to be delayed, increasing the project completion time
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Translates problem into that of finding a minimum spanning tree by listing or drawing 4 labelled arcs (Condone A for Alvanley etc)	AO3.1b	M1	
	Finds correctly 4 arcs of the minimum spanning tree by listing or drawing	AO1.1b	A1	Alvanley to Helsby: 750 Elton to Ince: 1250 Alvanley to Dunham: 2000 Dunham to Elton: 2500
	Determines correctly the total minimum length of cable required, complete with unit	AO1.1b	B1	750 + 1250 + 2000 + 2500 = 6500 m
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Shows that the set is closed under the operation * (must show that under modulo 6, $a * b$ can only result in a member of the given set)	AO2.1	R1	As all answers to $a * b$ are reduced modulo 6 they are in the given set and thus the set is closed under *.
	Clearly identifies the identity element	AO1.1b	B1	Identity element = 2
	Finds and states the inverse of each element (PI) FT from 'their' identity	AO1.1b	B1	0 and 4 are inverses of each other 1 and 3 are inverses of each other 2 and 5 are self-inverse elements
	Shows associativity between elements of the set under the operation *	AO2.1	R1	$a*(b*c) = a+(b+c+4)+4$ is shown to equal $(a*b)*c = (a+b+4)+c+4$
	States correct conclusion that *G* is a group under the operation * Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips.	AO2.1	R1	As *G* satisfies each of the four group axioms under the binary operation *, *G* is a group
5(b)	Identifies two correct subgroups. Condone inclusion of $\{0,1,2,3,4,5\}$	AO1.1b	B1	$\{2\},\{0, 2, 4\},\{2, 5\}$
	Identifies all three proper subgroups and no others included	AO1.1b	B1	

Q	Marking Instructions	AO	Marks	Typical Solution
5(c)	Identifies the generator of *G* OR generates every element of the group *K* (PI)	AO3.1a	B1	$G = (\langle 1 \rangle, *)$ OR $K = (\{3,9,13,11,5,1\}, \times_{14})$
	Finds correctly a one-to-one mapping between each element of *G* and *K* (condone use of equal sign for this mark)	AO1.1b	B1	*G* *K* $1 \mapsto 3$ $0 \mapsto 9$ $5 \mapsto 13$ $4 \mapsto 11$ $3 \mapsto 5$ $2 \mapsto 1$
	Deduces that *G* is isomorphic to *K* with a concluding statement using the correct mathematical language, having used the correct notation throughout.	AO2.2a	E1	As there is a one-to-one mapping between the elements of *G* and the elements of *K*, $G \cong K$
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)(i)	Explains correctly using maximum flow into and minimum flow out of *A*	AO2.4	E1	Max flow into vertex *A* = 7 Min flow out of vertex *A* = 4 + 3 = 7 As flow into *A* = flow out of *A*, both *AD* and *AB* must be at the lower capacity
6(a)(ii)	Explains correctly using maximum flow into and minimum flow out of *E*	AO2.4	E1	*AD* = 4 from **(a)(i)**, so *DE* = 1 and *DT* = 3. Since flow out of vertex *E* is at least 2, and *DE* = 1, *BE* must be at its upper capacity of 1.
6(b)	Explains correctly the statement using a cut in the network	AO2.4	E1	Min flow across the cut {*S*, *A*, *B*, *C*}/{*D*, *E*, *G*, *T*} = *AD* + *BE* + min(*BG*) + min(*CG*) = 4 + 1 + 5 + 2 = 12 > 11

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="3">6(c)(i)</td><td>Finds a potential increase and decrease for each arc with a value on the two arrows for each arc, with SA, SC, AB, BC and CF correct.</td><td>AO1.1a</td><td>M1</td><td rowspan="2">See diagram below</td></tr>
<tr><td>Determines correctly the potential increase and decrease for each arc with values on all arrows correct.</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td colspan="4">
</td></tr>
<tr><td rowspan="3">6(c)(ii)</td><td>Finds correctly one augmenting path and flow</td><td>AO1.1b</td><td>B1</td><td rowspan="2">

Augmenting Path	Flow
SADEFT	1
SCBET	3
SADBET	1

</td></tr>
<tr><td>Finds correctly two augmenting paths and the flows</td><td>AO1.1b</td><td>B1</td></tr>
<tr><td>Finds correctly three (or four) augmenting paths and the flows, and clearly states the maximum flow in context with units</td><td>AO3.2a</td><td>B1</td><td>Maximum flow through the network is 22 litres per second</td></tr>
</table>

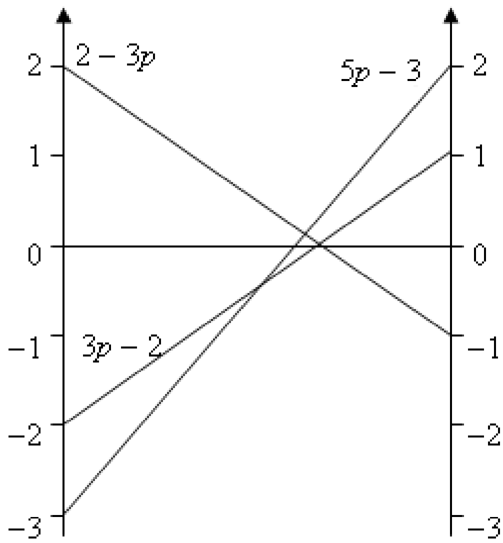
Q	Marking Instructions	AO	Marks	Typical Solution
6(c)(iii)	Constructs a rigorous mathematical proof using the value of a cut and the maximum flow-minimum cut theorem (to achieve this mark, the student must clearly show the cut they are calculating the value of **and** clearly state that the value of this cut is equal to the value of the flow in **(c)(ii)** and then conclude that the flow is maximal by the maximum flow-minimum cut theorem)	AO2.1	R1	{*S*, *A*, *B*, *C*, *D*, *E*}/{*F*, *T*} = 7 + 3 + 9 + 3 = 22 As the flow (22) is equal to the value of the cut (22), the maximum flow is 22 by the maximum flow-minimum cut theorem.
6(c)(iv)	Argues that, as 7 litres per second are initially flowing through node *E*, only a further 2 litres per second can flow through node *E*	AO2.4	R1	Flow through network can only increase by 2 litres per second as *DT* and *CF* are already saturated Therefore restricted capacity node reduces the maximum flow to 17 + 2 = 19 litres per second
	Interprets the impact of the restricted capacity node, concluding that the maximum flow is reduced to 19 litres per second	AO3.A	R1	
	Total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	Formulates correctly one non-trivial inequality	AO3.1b	B1	$3x+2y+z \le 360$
	Formulates correctly a second non-trivial inequality	AO3.1b	B1	$40x+20y+5z \le 2500$
	States correctly all three trivial inequalities	AO1.2	B1	$x \ge 0,\ \ y \ge 0,\ \ z \ge 0$
7(b)(i)	Translates 'their' inequalities into a simplex tableau with two slack variables	AO3.1a	M1	See diagram on the next page
	Identifies a correct pivot from 'their' tableau	AO1.1b	A1	
	Uses the simplex algorithm correctly to modify the 'their' two non-pivot rows	AO1.1a	M1	
	Identifies the correct pivot in 'their' modified tableau	AO1.1b	A1F	
	Uses the simplex algorithm correctly to modify 'their' two non-pivot rows	AO1.1a	M1	
	Makes a correct interpretation of 'their' final tableau in the context of the problem, provided the objective row is non-negative	AO3.2a	E1	To maximise the profit, the company should repair and sell 28 monitors, 0 hard drives and 276 keyboards each month

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="2">7(b)(ii)</td><td>States no further use of the simplex algorithm is required and explains why</td><td>AO2.4</td><td>R1</td><td>The objective row of the final tableau being non-negative numbers.</td></tr>
<tr><td colspan="4">

P	x	y	z	s	t	value
1	–80	–35	–15	0	0	0
0	3	2	1	1	0	360
0	**40**	20	5	0	1	2500
1	0	5	–5	0	2	5000
0	0	0.5	**0.625**	1	–0.075	172.5
0	1	0.5	0.125	0	0.025	62.5
1	0	9	0	8	1.4	6380
0	0	0.8	1	1.6	–0.12	276
0	1	0.4	0	–0.2	0.04	28

</td></tr>
<tr><td>7(b(iii))</td><td>Introduces a new inequality for the hard drive hardware, ensuring that at least some hard drives are required to be repaired and sold</td><td>AO3.4</td><td>E1</td><td>As $y = 0$, enforce some hard drives to be repaired by requiring that, for instance, $y \geq 10$</td></tr>
<tr><td></td><td>Total</td><td></td><td>11</td><td></td></tr>
</table>

Q	Marking Instructions	AO	Marks	Typical Solution
8	Deduces strategy *C* is a dominated strategy	AO2.2a	B1	– 3 = – 3, –4 < –2, 1 < 2, hence strategy B dominates strategy *C*
	Introduces and defines a probability variable	AO3.3	M1	Let John choose strategy *A* with probability p and strategy *B* with probability $1 - p$. If Danielle plays:
	Finds correctly all three expected gain expression for John	AO1.1b	A1	*X*: expected gain for John = $2p - 3(1 - p)$ $= 5p - 3$ *Y*: expected gain for John = $p - 2(1 - p)$ $= 3p - 2$ *Z*: expected gain for John = $-p + 2(1 - p)$ $= 2 - 3p$
	Constructs a graph with at least one vertical axis and plots one of ‘their’ expected gains correctly (PI)	AO1.1a	M1	
	Identifies correctly the optimal point of intersection from ‘their’ graph and finds ‘their’ value of probability variable	AO1.1b	A1	$3p - 2 = 2 - 3p$ $6p = 4$ $p = 2/3$
	Interprets correctly the solution to the problem in the context, giving the optimal mixed strategy for John	AO3.2a	E1	John should play strategy *A* with probability 2/3 and *B* with probability 1/3 (and *C* with probability 0)
	Total		**6**	
	TOTAL		**50**	