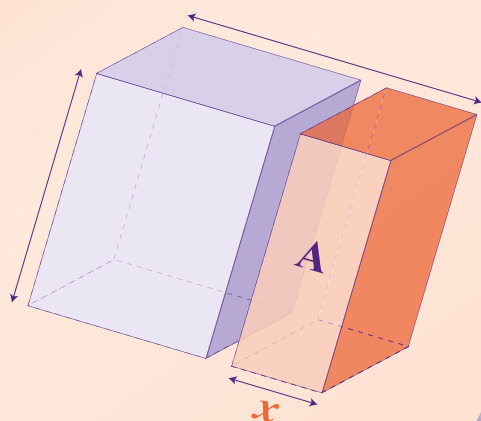


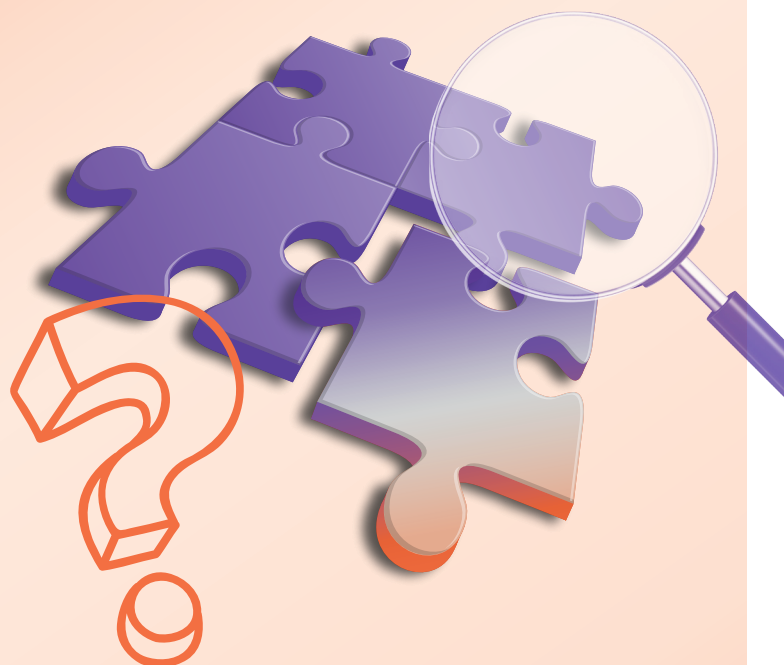
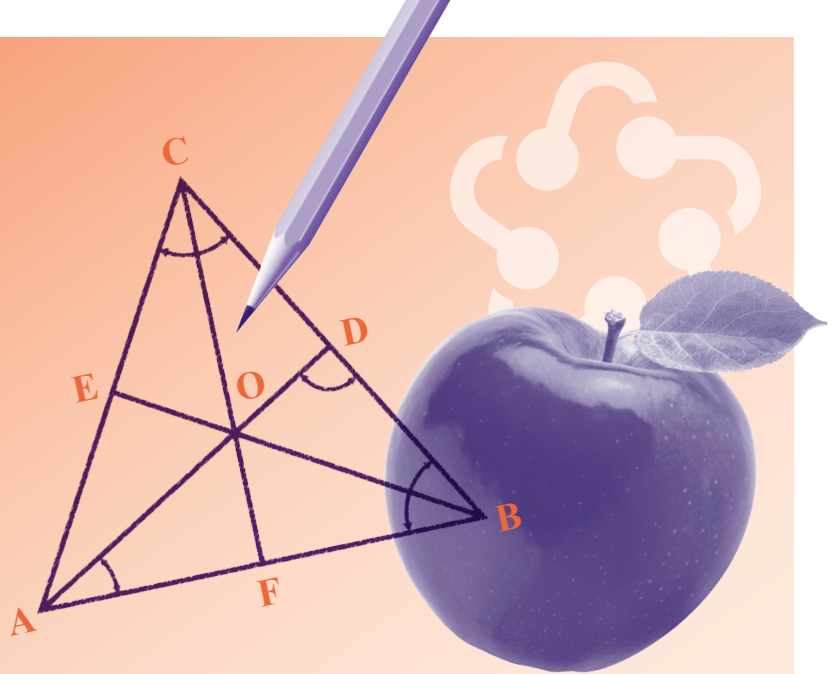
GCSE Maths Focus on:

Problem solving

Build on your students' assessment performance using our self-guided, modular training pack



Pre-reading
booklet



Contents

Contents	Page
Using this pack	4
Summary of activities	5
Problem solving route map	6
Introduction to problem solving	7
Introduction to the problem solving strategies	8
The problem solving strategies	9
Pre-session health check	11
Post-session health check	12

Using this pack

This resource pack is intended to help you deliver a CPD session for teaching colleagues on problem solving questions in GCSE Mathematics. It will give them the opportunity to explore how these questions are assessed, the strategies available for students and implications for the classroom.

Using the completed pre-session health check and provided route map, you'll be able to design a bespoke session to focus learning on the areas your colleagues are less confident teaching.

Before the session

- Ask your colleagues to each complete the Pre-session health check (page 11 in this booklet).
- Use the responses to the Pre-session health check to tailor the training session to the needs of your colleagues. The route map on page 6 will help you plan which activities to use in the session.
- Each colleague should have a copy of the activities and handouts booklets.
- The Post-session health check should be printed for each delegate.

Running the session

- Establish why focusing on problem solving questions is beneficial to students and how these questions are assessed.
- Remind teachers that problem solving questions will be asked at all levels of demand, and that marks are available for progress made and not just complete solutions.
- The presentation will provide guidance and discussion questions to move you through your bespoke session.

After the session

- Ask your colleagues to each complete the Post-session health check (page 12 in this booklet) to ensure the training has been successful. This should have been printed as part of your pre-delivery preparations.
- As a group, discuss how you can support each other to embed the learnings in your teaching.
- Should any weaknesses or concerns persist, you may wish to refer to the *GCSE Mathematics: 90 maths problems solving questions resource* for further suitable development activities.¹

¹ AQA's *GCSE Mathematics: 90 maths problems solving questions resource* can be found here: allaboutmaths.aqa.org.uk/attachments/5592.pdf

Summary of activities

Activity 1

Problem solving can be found anywhere. Even the simplest of questions could be a problem in the right context and this introductory example has been chosen to emphasise this. Make sure to invite colleagues to have a go at this question and the reasons why: this question proved to be a challenge even for the most able students. The mark scheme is provided within the presentation to support discussion.

Activity 2

In this activity we begin to explore some of the possible strategies for approaching problem solving questions. Each of the given strategies is presented with an explanation and an example. All the examples are taken from our legacy Additional Maths qualification. Our Additional Maths papers can still be found on All About Maths.²

Make sure to invite colleagues to look at each strategy in turn and then ask them to think of other problems which could be approached/solved using each strategy.

Activity 3

In this activity some problems have been selected from our *GCSE Mathematics: 90 maths problems solving questions resource* which is also still available on All About Maths.

In groups, invite colleagues to look at each problem in turn and answer the following questions:

- which of the 5 strategies is most likely to be successful here?
- what method did you use to solve it?
- what level of demand is the problem as it stands? (Low Demand Foundation, Foundation, Common, Higher or High Demand Higher)
- what method are your students likely to use?
- how might you use this problem in your teaching?
- how might you simplify or extend this problem?

Should you feel it would be beneficial for the group to complete further activities following the course for consolidation purposes, you may wish to refer to the *GCSE Mathematics: 90 maths problems solving questions resource* for suitable development activities.

² Additional Maths papers can be found here: allaboutmaths.aqa.org.uk/legacymathsQPs

Problem solving route map

Area for development

Do staff know the purpose and value of problem solving?

An introduction to problem solving
Slides 3-6

Activity 1
Shaded squares

Do staff know the ways to approach problem solving questions?

Problem solving strategies
Slides 7-19

Activity 2
The strategies

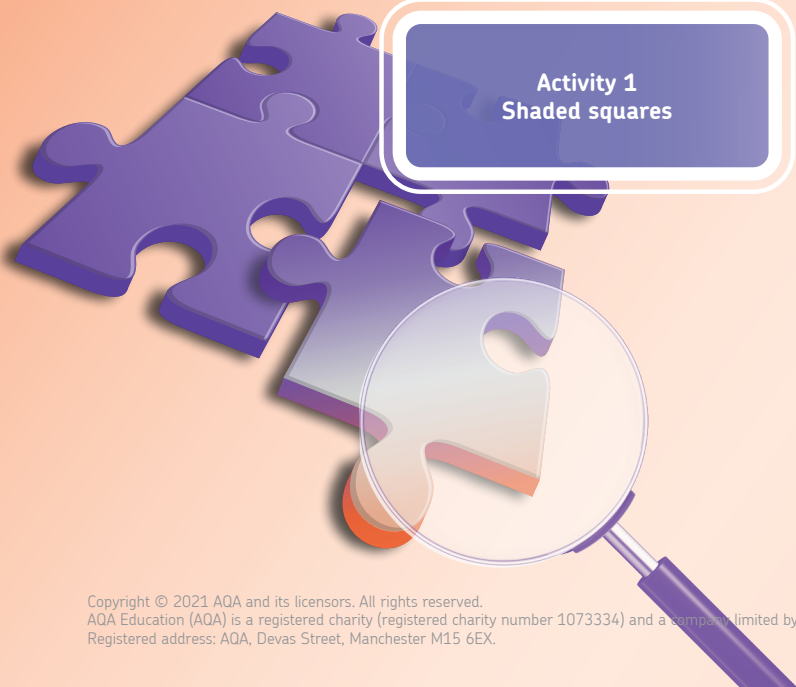
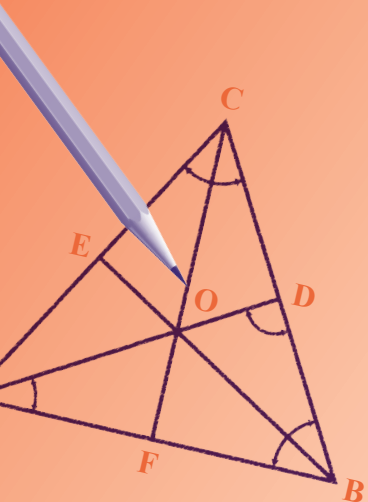
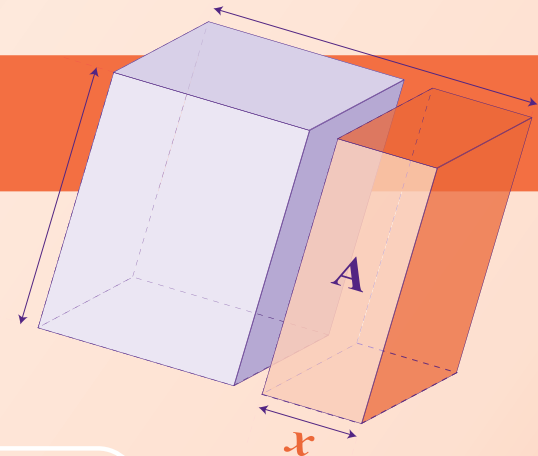
Do staff know how to use common problem solving strategies?

Using the strategies
Slides 20-58

Activity 3
Using the strategies

Handouts Booklet

Further consolidation
*GCSE Mathematics:
90 maths problems solving
questions resource*



Introduction to problem solving

The mathematical processes associated with problem solving have been explicit in all versions of the Mathematics National Curriculum since 1989, and have always been taught as part of preparation for GCSE Mathematics. Nevertheless, the greater emphasis on these processes in the new specification for GCSE Mathematics mean there is a need to have a more direct focus on each of them in preparatory teaching. Specifically, there is:

- more reasoning
- more justification of reasoning in explanations
- more representation of a situation algebraically
- more visualisation
- more ‘unstructured’ questions, lacking the step-by-step build up to a solution that is to be found in many other GCSE questions.

Specifically, 25% of the marks available at Foundation Tier - and 30% at Higher Tier - are allocated to AO3. Some other questions, such as ‘show that’ type questions, could also be considered problems, but would be classified as AO2. Problems which resolve to routine procedures would also have marks allocated to AO1.

Some problems are also dependent on the level they are targeting, for example: a problem set at the beginning of the Foundation Tier may not prove to be a problem at all for Higher Tier students. Therefore, the proportion of questions which could be seen as ‘problem solving’ would be greater than the percentages given above.

Problems are necessarily constrained by the fact that they are in a timed-examination and subject to a mark scheme. A range of strategies can be identified that help with the kinds of problems that are set under such conditions.

Introduction to the problem solving strategies

A student facing a new problem is initially likely to examine it to see if it is like a problem that they have done before. If it is, they will try to use the approach that had worked on the previous occasion. If the problem is not obviously like one they have approached before, then they could consider the features of the problem to decide what might be done that could be helpful. They may ask themselves questions about the problem, as highlighted below.

- Are there examples or 'cases' that it might be helpful to set out systematically?
- Is there a procedural relationship between some elements that could be reversed to find others?
- Are there criteria to apply to possible solutions?
- Are there features with relationships between them that could be drawn out and expressed in some way (eg linguistically, diagrammatically, symbolically and algebraically)?
- Is there some recognisable mathematical approach that could be followed through, extended or applied?

Progress on the problems in GCSE Mathematics is likely to be made through one or another of those possible approaches. As a result, they represent the five strategies with particular relevance to the examination. These are identified below.

1. To set out cases systematically, and identify how many there are of relevant types.
2. To work backwards from a value given in the problem:
 - a. where the inverse is familiar and so just has to be applied, but may have to be sustained over a number of steps
 - b. where the inverse is unfamiliar and so has to be worked out 'from first principles'.
3. To find one or more examples that fit a condition for the answer, and see whether those examples fit with the other conditions in the situation, making adjustments until they do.
4. To look for and represent relationships between elements of the situation, and then act on them to see if any are useful.
5. To find features of the situation that can be acted on mathematically, and see where using them takes you (operating incrementally, yet speculatively).

However, it should be noted that these labels are for the convenience of reference, and should not necessarily be used in the classroom context. To use labels like this in class is to invite students to misunderstand these possibilities for action as distinct sequences of activity that can be learned and applied as procedures. Problem solving is not a matter of identifying a problem as a 'type 3' and solving it by applying the 'find an example to fit' routine. Even though the problems outlined in this resource are broadly classified by which of these five strategies most readily lends itself to the given problem, this is not the same as identifying problem types, since there are almost invariably different ways of tackling any problem. The purpose of the classification is to enable this resource to be used in a systematic way. The five identified strategies are effective approaches to the kind of problems found in GCSE Mathematics, and by associating each problem with a particular strategy. This resource offers a structure for rehearsing and developing techniques to enable success in the GCSE Mathematics exams.

The problem solving strategies

Strategy 1: Set out cases

A problem of this type requires students to work systematically, identifying all relevant cases and being exhaustive so as not to inadvertently exclude or perhaps double count any such cases. Problems like this have always been a part of GCSE Mathematics, but in the latest reform this strategy is now explicitly referenced within the Specification (N5: Apply systematic listing strategies). At Higher Tier, the product rule for counting is also included. This means that students should be familiar with the method, and it may only become a problem where its application is less obvious.

Strategy 2: Work back

This is a common way to ask a problem in GCSE Mathematics and requires the solver to follow a process in the reverse of the usual way it is normally presented. Again, it is explicitly referred to in the Specification in the context of inverse operations: indeed subtraction may be thought of the inverse of addition, and fractions or division as the inverse of multiplication. As such, where the inverse process is more familiar, the problem may become more routine and may not necessarily have AO3 marks allocated.

Strategy 3: Finding an example that fits

This type of problem will usually require students to find an example that fits a constraint in the question and then require them to test their example on the other constraints or conditions in the situation. Unless they are lucky (or intuitive) the first time, they will usually then need to make adjustments to their example to find the right one for all the conditions. Whilst being systematic here will usually help students get to the right answer eventually, unless they think carefully about their first example and all the constraints they may waste a lot of time in an exam situation.

Strategy 4: Find key relationships

For this to be a problem, the key relationship will not usually be obvious and students will have to look for them and use them to see if they are useful in the situation. These problems will frequently feature diagrams and it will usually be a good strategy to add any extra information to the diagram so that any key relationships may be more directly observed.

Strategy 5: Find mathematical features

Again, this approach is likely to be useful on problems with diagrams where adding and labelling the diagram with any additional mathematical features is likely to prove a helpful strategy for breaking down the problem.

Data rich problems

Less a strategy and more a common type of problem, these questions feature a lot of data which may intimidate students. In these problems, students will need to break down and organise the data so they are able to plan a way through. Once the data is organised, the way forward may be more obvious, or perhaps one of the other strategies may need to be employed in order to find links between the data and the answer required.

Pre-session health check

Grade the area of development statements according to your confidence where 0 is not confident at all and 5 is very confident.

Hand back to your Head of Department.

Area of development	Grading 0-5	Reasons/notes/previous training
I am aware of the prevalence of problem solving questions in GCSE Mathematics exams.		
My teaching involves student engagement with, and effective practice of, problem solving questions.		
I understand how and when to include different strategies for approaching problem solving in my teaching.		
I know what I need to do to help improve students' ability to respond appropriately to problem solving questions on different topics.		

Post-session health check

Grade the area of development statements according to your confidence where 0 is not confident at all and 5 is very confident.

Area of development	Grading 0-5	Reasons/notes
I am aware of the prevalence of problem solving questions in GCSE Mathematics exams.		
My teaching involves student engagement with, and effective practice of, problem solving questions.		
I understand how and when to include different strategies for approaching problem solving in my teaching.		
I know what I need to do to help improve students' ability to respond appropriately to problem solving questions on different topics.		

Notes

Contact us

T: 0161 957 3852

E: maths@aqa.org.uk

aqa.org.uk