Formulae and statistical tables for A-level Mathematics and A-level Further Mathematics

AS Mathematics (7356)
A-level Mathematics (7357)
AS Further Mathematics (7366)
A-level Further Mathematics (7367)

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For the new specifications for first teaching from September 2017.

This booklet of formulae and statistical tables is required for all AS and A-level Further Mathematics exams.

Students may also use this booklet in all AS and A-level Mathematics exams. However, there is a smaller booklet of formulae available for use in AS and A-level Mathematics exams with only the formulae required for those examinations included.

Further copies of this booklet are available from:
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Statistical tables

Pure mathematics

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{Q})$$

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a + (n-1)d]$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Trigonometry: small angles

For small angle θ, measured in radians:

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\tan x$	$\sec^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x)-\mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0} \frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int u \frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v \frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$\mathbf{f(x)}$	$\int \mathbf{f(x)\,d}x$
$\tan x$	$\ln\lvert\sec x\rvert + c$
$\cot x$	$\ln\lvert\sin x\rvert + c$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\left\lvert\tan\left(\frac{1}{2}x\right)\right\rvert + c$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\left\lvert\tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right\rvert + c$
$\sec^2 x$	$\tan x + c$
$\tanh x$	$\ln\cosh x + c$
$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right) + c \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2 + x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + c$
$\dfrac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\dfrac{x}{a}\right)$ or $\ln\{x + \sqrt{x^2 - a^2}\} + c \quad (x > a)$
$\dfrac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}\left(\dfrac{x}{a}\right)$ or $\ln\{x + \sqrt{x^2 + a^2}\} + c$
$\dfrac{1}{a^2 - x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a + x}{a - x}\right\rvert = \dfrac{1}{a}\tanh^{-1}\left(\dfrac{x}{a}\right) + c \quad (\lvert x\rvert < a)$
$\dfrac{1}{x^2 - a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x - a}{x + a}\right\rvert + c$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Numerical integration

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b - a}{n}$

Complex numbers

$[r(\cos\theta + \mathrm{i}\sin\theta)]^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$

The roots of $z^n = 1$ are given by $z = \mathrm{e}^{\frac{2\pi k\mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n - 1$

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

The matrices for rotations (in three dimensions) through an angle θ about one of the axes are:

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ for the x-axis

$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ for the y-axis

$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for the z-axis

Summations

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Maclaurin's series

$$\mathrm{f}(x) = \mathrm{f}(0) + x\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \quad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r\frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r\frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

Vectors

The resolved part of **a** in the direction of **b** is $\dfrac{\mathbf{a.b}}{|\mathbf{b}|}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, then

- the straight line through A with direction vector $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ has equation

  $\dfrac{x - a_1}{b_1} = \dfrac{y - a_2}{b_2} = \dfrac{z - a_3}{b_3} = \lambda$ (Cartesian form)

  or

  $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ (vector product form)

- the plane through A and parallel to **b** and **c** has vector equation
  $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

Area of a sector

$A = \frac{1}{2}\int r^2\,d\theta$ (polar coordinates)

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh^{-1} x = \ln\{x + \sqrt{x^2 - 1}\}$ $(x \geqslant 1)$

$\sinh^{-1} x = \ln\{x + \sqrt{x^2 + 1}\}$

$\tanh^{-1} x = \frac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$ $(|x| < 1)$

Conics

	Ellipse	Parabola	Hyperbola
Standard form	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
Parametric form	$x = a\cos\theta$ $y = b\sin\theta$	$x = at^2$ $y = 2at$	$x = a\sec\theta$ $y = b\tan\theta$
Asymptotes	none	none	$\dfrac{x}{a} = \pm\dfrac{y}{b}$

Further numerical integration

The mid-ordinate rule: $\int_a^b y \, \mathrm{d}x \approx h(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \ldots + y_{n-\frac{3}{2}} + y_{n-\frac{1}{2}})$

where $h = \dfrac{b-a}{n}$

Simpson's rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{3}h\left\{(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right\}$

where $h = \dfrac{b-a}{n}$ and n is even

Numerical solution of differential equations

For $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x)$ and small h, recurrence relations are:

Euler's method: $y_{n+1} = y_n + h\mathrm{f}(x_n), \quad x_{n+1} = x_n + h$

For $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$:

Euler's method: $y_{r+1} = y_r + h\mathrm{f}(x_r, y_r), \quad x_{r+1} = x_r + h$

Improved Euler method: $y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r), \quad x_{r+1} = x_r + h$

Arc length

$s = \int \sqrt{1 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2} \, \mathrm{d}x$ (Cartesian coordinates)

$s = \int \sqrt{\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2} \, \mathrm{d}t$ (parametric form)

Surface area of revolution

$S_x = 2\pi \int y \sqrt{1 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2} \, \mathrm{d}x$ (Cartesian coordinates)

$S_x = 2\pi \int y \sqrt{\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2} \, \mathrm{d}t$ (parametric form)

Mechanics

Constant acceleration

$s = ut + \frac{1}{2}at^2$ $\quad\quad$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$s = vt - \frac{1}{2}at^2$ $\quad\quad$ $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

$v = u + at$ $\quad\quad$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$s = \frac{1}{2}(u + v)t$ $\quad\quad$ $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$v^2 = u^2 + 2as$

Centres of mass

For uniform bodies:

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere, radius r: $\frac{3}{8}r$ from centre

Hemispherical shell, radius r: $\frac{1}{2}r$ from centre

Circular arc, radius r, angle at centre 2α: $\frac{r\sin\alpha}{\alpha}$ from centre

Sector of circle, radius r, angle at centre 2α: $\frac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\frac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h: $\frac{1}{3}h$ above the base on the line from centre of base to vertex

Probability and statistics

Probability

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

$$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B|A)$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

Discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $\mathrm{Po}(\lambda)$	$\mathrm{e}^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ

Sampling distributions

For a random sample $X_1, X_2, \ldots, X_n$ of n independent observations from a distribution having mean μ and variance σ^2:

$\bar{X}$ is an unbiased estimator of μ, with $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2, where $S^2 = \frac{\sum\left(X_i - \bar{X}\right)^2}{n-1}$

For a random sample of n observations from $\mathrm{N}(\mu, \sigma^2)$:

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim \mathrm{N}(0, 1)$$

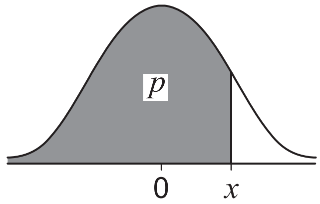
$$\frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$$

Distribution-free (non-parametric) tests

Contingency tables: $\sum \frac{\left(O_i - E_i\right)^2}{E_i}$ is approximately distributed as χ^2

TABLE 1 Percentage points of the student's t-distribution

The table gives the values of x satisfying $\mathrm{P}(X \leqslant x) = p$, where X is a random variable having the student's t-distribution with v degrees of freedom.

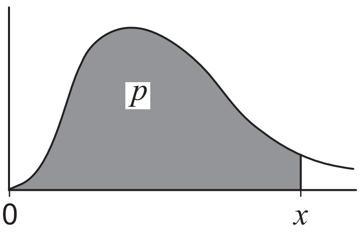


p / v	0.9	0.95	0.975	0.99	0.995
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.121	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763

p / v	0.9	0.95	0.975	0.99	0.995
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
65	1.295	1.669	1.997	2.385	2.654
70	1.294	1.667	1.994	2.381	2.648
75	1.293	1.665	1.992	2.377	2.643
80	1.292	1.664	1.990	2.374	2.639
85	1.292	1.663	1.998	2.371	2.635
90	1.291	1.662	1.987	2.368	2.632
95	1.291	1.661	1.985	2.366	2.629
100	1.290	1.660	1.984	2.364	2.626
125	1.288	1.657	1.979	2.357	2.616
150	1.287	1.655	1.976	2.351	2.609
200	1.286	1.653	1.972	2.345	2.601
∞	1.282	1.645	1.960	2.326	2.576

TABLE 2 Percentage points of the χ^2 distribution

The table gives the values of x satisfying $P(X \leqslant x) = p$, where X is a random variable having the χ^2 distribution with v degrees of freedom.



p / v	0.005	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	0.995	p / v
1	0.00004	0.0002	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	**1**
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597	**2**
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838	**3**
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860	**4**
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750	**5**
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548	**6**
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278	**7**
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955	**8**
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589	**9**
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188	**10**
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757	**11**
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300	**12**
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819	**13**
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319	**14**
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801	**15**
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267	**16**
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718	**17**
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156	**18**
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582	**19**
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997	**20**
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401	**21**
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796	**22**
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181	**23**
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559	**24**
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928	**25**
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290	**26**
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645	**27**
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993	**28**
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336	**29**
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672	**30**
31	14.458	15.655	17.539	19.281	21.434	41.422	44.985	48.232	52.191	55.003	**31**
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328	**32**
33	15.815	17.074	19.047	20.867	23.110	43.745	47.400	50.725	54.776	57.648	**33**
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.996	56.061	58.964	**34**
35	17.192	18.509	20.569	22.465	24.797	46.059	49.802	53.203	57.342	60.275	**35**
36	17.887	19.223	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581	**36**
37	18.586	19.960	22.106	24.075	26.492	48.363	52.192	55.668	59.892	62.883	**37**
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181	**38**
39	19.996	21.426	23.654	25.695	28.196	50.660	54.572	58.120	62.428	65.476	**39**
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766	**40**
45	24.311	25.901	28.366	30.612	33.350	57.505	61.656	65.410	69.957	73.166	**45**
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490	**50**
55	31.735	33.570	36.398	38.958	42.060	68.796	73.311	77.380	82.292	85.749	**55**
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952	**60**
65	39.383	41.444	44.603	47.450	50.883	79.973	84.821	89.177	94.422	98.105	**65**
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215	**70**
75	47.206	49.475	52.942	56.054	59.795	91.061	96.217	100.839	106.393	110.286	**75**
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321	**80**
85	55.170	57.634	61.389	64.749	68.777	102.079	107.522	112.393	118.236	122.325	**85**
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299	**90**
95	63.250	65.898	69.925	73.520	77.818	113.038	118.752	123.858	129.973	134.247	**95**
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169	**100**

TABLE 3 Critical values of the product moment correlation coefficient

The table gives the critical values, for different significance levels, of the product moment correlation coefficient, r, for varying sample sizes, n.

One tail Two tail	10% 20%	5% 10%	2.5% 5%	1% 2%	0.5% 1%	One tail Two tail
n						n
4	0.8000	0.9000	0.9500	0.9800	0.9900	**4**
5	0.6870	0.8054	0.8783	0.9343	0.9587	**5**
6	0.6084	0.7293	0.8114	0.8822	0.9172	**6**
7	0.5509	0.6694	0.7545	0.8329	0.8745	**7**
8	0.5067	0.6215	0.7067	0.7887	0.8343	**8**
9	0.4716	0.5822	0.6664	0.7498	0.7977	**9**
10	0.4428	0.5494	0.6319	0.7155	0.7646	**10**
11	0.4187	0.5214	0.6021	0.6851	0.7348	**11**
12	0.3981	0.4973	0.5760	0.6581	0.7079	**12**
13	0.3802	0.4762	0.5529	0.6339	0.6835	**13**
14	0.3646	0.4575	0.5324	0.6120	0.6614	**14**
15	0.3507	0.4409	0.5140	0.5923	0.6411	**15**
16	0.3383	0.4259	0.4973	0.5742	0.6226	**16**
17	0.3271	0.4124	0.4821	0.5577	0.6055	**17**
18	0.3170	0.4000	0.4683	0.5425	0.5897	**18**
19	0.3077	0.3887	0.4555	0.5285	0.5751	**19**
20	0.2992	0.3783	0.4438	0.5155	0.5614	**20**
21	0.2914	0.3687	0.4329	0.5034	0.5487	**21**
22	0.2841	0.3598	0.4227	0.4921	0.5368	**22**
23	0.2774	0.3515	0.4132	0.4815	0.5256	**23**
24	0.2711	0.3438	0.4044	0.4716	0.5151	**24**
25	0.2653	0.3365	0.3961	0.4622	0.5052	**25**
26	0.2598	0.3297	0.3882	0.4534	0.4958	**26**
27	0.2546	0.3233	0.3809	0.4451	0.4869	**27**
28	0.2497	0.3172	0.3739	0.4372	0.4785	**28**
29	0.2451	0.3115	0.3673	0.4297	0.4705	**29**
30	0.2407	0.3061	0.3610	0.4226	0.4629	**30**
31	0.2366	0.3009	0.3550	0.4158	0.4556	**31**
32	0.2327	0.2960	0.3494	0.4093	0.4487	**32**
33	0.2289	0.2913	0.3440	0.4032	0.4421	**33**
34	0.2254	0.2869	0.3388	0.3972	0.4357	**34**
35	0.2220	0.2826	0.3338	0.3916	0.4296	**35**
36	0.2187	0.2785	0.3291	0.3862	0.4238	**36**
37	0.2156	0.2746	0.3246	0.3810	0.4182	**37**
38	0.2126	0.2709	0.3202	0.3760	0.4128	**38**
39	0.2097	0.2673	0.3160	0.3712	0.4076	**39**
40	0.2070	0.2638	0.3120	0.3665	0.4026	**40**
41	0.2043	0.2605	0.3081	0.3621	0.3978	**41**
42	0.2018	0.2573	0.3044	0.3578	0.3932	**42**
43	0.1993	0.2542	0.3008	0.3536	0.3887	**43**
44	0.1970	0.2512	0.2973	0.3496	0.3843	**44**
45	0.1947	0.2483	0.2940	0.3457	0.3801	**45**
46	0.1925	0.2455	0.2907	0.3420	0.3761	**46**
47	0.1903	0.2429	0.2876	0.3384	0.3721	**47**
48	0.1883	0.2403	0.2845	0.3348	0.3683	**48**
49	0.1863	0.2377	0.2816	0.3314	0.3646	**49**
50	0.1843	0.2353	0.2787	0.3281	0.3610	**50**
60	0.1678	0.2144	0.2542	0.2997	0.3301	**60**
70	0.1550	0.1982	0.2352	0.2776	0.3060	**70**
80	0.1448	0.1852	0.2199	0.2597	0.2864	**80**
90	0.1364	0.1745	0.2072	0.2449	0.2702	**90**
100	0.1292	0.1654	0.1966	0.2324	0.2565	**100**

End of formulae

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