Formulae for A-level Mathematics

AS Mathematics (7356)
A-level Mathematics (7357)

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For the new specifications for first teaching from September 2017.

This booklet of formulae is required for all AS and A-level Mathematics exams.

There is a larger booklet of formulae and statistical tables for all AS and A-level Further Mathematics exams.

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Contents

Pure mathematics

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{Q})$$

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a+(n-1)d]$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Trigonometry: small angles

For small angle θ, measured in radians:

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

Trigonometric identities

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

Differentiation

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan x$	$\sec^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int u \frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v \frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$\mathbf{f(x)}$	$\int \mathbf{f(x)\,dx}$
$\tan x$	$\ln\lvert\sec x\rvert + c$
$\cot x$	$\ln\lvert\sin x\rvert + c$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Numerical integration

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Mechanics

Constant acceleration

$s = ut + \frac{1}{2}at^2$ $\quad\quad$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$s = vt - \frac{1}{2}at^2$ $\quad\quad$ $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

$v = u + at$ $\quad\quad$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$s = \frac{1}{2}(u + v)t$ $\quad\quad$ $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$v^2 = u^2 + 2as$

Probability and statistics

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B|A)$

Standard deviation

$$\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

Discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x(1 - p)^{n-x}$	np	$np(1 - p)$

Sampling distributions

For a random sample of n observations from $\mathrm{N}(\mu, \sigma^2)$:

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim \mathrm{N}(0, 1)$$

End of formulae

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