Assessment Objective 3
GCSE and A-level

Maths summer 18 hub meeting

This meeting will be recorded

Exam boards have an Ofqual requirement to record event audio.

Recordings are kept for one year and not shared as an accompaniment to session resources.

The recording will begin now.

Objectives

- To consider GCSE questions addressing aspects of AO3 (inference, interpretation and evaluation) which students find/found challenging.
- To consider A-level questions addressing aspects of AO3 (problem solving and modelling).
- To consider how we might support students in preparing for and overcoming these challenges.

AQA

Difficult questions: introduction

- The following are all questions from Practice Paper Set 3, summer '17 and November '17 that were not well answered and which test aspects of the Assessment Objectives around inference, interpretation and evaluation.
- This area stood out more than any other aspect of the new GCSE as an area that appears to be particularly challenging for students

AQA

Assessment Objective 1

Use and apply standard techniques

Students should be able to:

- accurately recall facts, terminology and definitions
- use and interpret notation correctly
- accurately carry out routine procedures or set tasks requiring multi-step solutions.

Weighting: F 50% H 40%

Assessment Objective 2

Reason, interpret and communicate mathematically

Students should be able to:

- make deductions and inferences and draw conclusions from mathematical information
- construct chains of reasoning to achieve a given result
- interpret and communicate information accurately
- present arguments and proofs
- assess the validity of an argument and critically evaluate a given way of processing information.

Weighting: F 25% H 30%

7

Assessment Objective 3

Solve problems within mathematics in other contexts

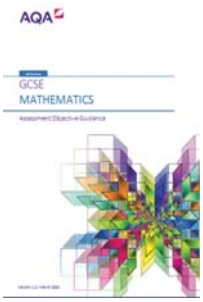
Students should be able to:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
- make and use connections between different parts of mathematics
- interpret results in the context of the given problem
- evaluate methods used and results obtained
- evaluate solutions to identify how they may have been affected by assumptions made.

Weighting: F 25% H 30%

8

Assessment Objective guidance



This document is available to download and is really useful in 'drilling down' to how the AOs might be met in questions.

9

AQA

Assessment Objective guidance

AO3.3

Interpreting results in the context of the given problem. This can be used in questions where the student has to decide whether to round up or truncate to give an integer answer that is appropriate for that particular problem. It can also be used where the student obtains one or more numerical answers and then has to decide which is cheaper / faster or who won a race. Some questions have an extra step, for example to interpret an amount of money as the number of extra weeks of saving required and this would fit this strand. This will frequently appear in multi-strand questions with AO3.1

AQA

Assessment Objective 3

Solve problems within mathematics in other contexts

Students should be able to:

3.1 translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes

3.2 make and use connections between different parts of mathematics

3.3 interpret results in the context of the given problem

3.4 evaluate methods used and results obtained

3.5 evaluate solutions to identify how they may have been affected by assumptions made.

Weighting: F 25% H 30%

June 3F Q21, 3H Q11-AO3.3

Purple paint is made by mixing red paint and blue paint in the ratio 5 : 2

Yan has 30 litres of red paint and 9 litres of blue paint.

What is the **maximum** amount of purple paint he can make?

[3 marks]

AQA

June 3F Q18-AO3.3

A shop sells two brands of battery.

Brand A
Pack of 8
Price £3.60

Brand B
Pack of 6
Price £2.94

One brand A battery powers a toy for 5 hours.

One brand B battery powers the same toy for $5\frac{1}{2}$ hours.

Which brand is better value?

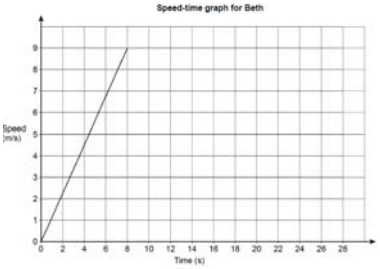
You **must** show your working.

[5 marks]

AQA

November 2H Q24-AO3.3

24 Beth ran a 200 metre race.
Here is a graph of the first 8 seconds of her race.
She completed the race at a constant speed of 9 m/s



Amy completed the race in 27 seconds.

Did Beth finish before Amy?

You **must** show your working.

[3 marks]

AQA

PP3 2F Q16a-AO3.3

The speed of the International Space Station is 27 576 kilometres per hour.

The station travels 42 600 kilometres in one orbit.

Work out the number of **full** orbits the station does in one day.

[3 marks]

AQA

PP3 2H Q6b (2F 22b)-AO3.3

Dev invests £1500 for 2 years.
The compound interest rate is 1.6% per year.

Emma invests £1500 for 2 years.

The interest rate is
1.8% for the first year
1.3% for the second year.

Whose investment is worth more after 2 years?
You **must** show your working.

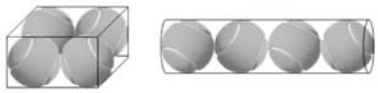
[4 marks]

AQA

PP3 2H Q17-AO3.3

Here are two closed containers.
Four tennis balls just fit in each container.
Each tennis ball has diameter 64 mm

Cuboid **Cylinder**



Which container has the smaller surface area?
You **must** show your working.

[5 marks]

AQA

Assessment Objective guidance

AO3.4a

Evaluating methods used. There must be a method shown in order to evaluate it. It could be a stated method that contains a misconception that the student has to highlight and explain. It could also be used in questions where the student is asked to suggest a way to improve a given method, for example improving an estimate from relative frequency by increasing sample size. In theory, it could also apply to a case where the student is asked to evaluate their own methods but this may be hard to test in practice.

Note the difference between evaluating a **method** and assessing the validity of an argument (just a statement with a reason) which is AO2.5a and evaluating the presentation of information (usually a diagram) which is AO2.5b

AQA

AO2 – Reason, interpret and communicate mathematically

- **1 – Make deductions, inferences and draw conclusions from mathematical information**
- 2.1a - make deductions to draw conclusions from mathematical information
- 2.1b - make inferences to draw conclusions from mathematical information
- **2.2 – Construct chains of reasoning to achieve a given result**
- **2.3 – Interpret and communicate information accurately**
- 2.3a - interpret information accurately
- 2.3b - communicate information accurately.

AQA

AO2 – Reason, interpret and communicate mathematically

- **2.4 – Present arguments and proofs**
- 2.4a - present arguments
- 2.4b - present proofs
- **2.5 – Assess the validity of an argument and critically evaluate a given way of presenting information**
- 2.5a - assess the validity of an argument
- 2.5b - critically evaluate a given way of presenting information.

AQA

Assessment Objective 3

Solve problems within mathematics in other contexts

Students should be able to:

3.1 translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes

3.2 make and use connections between different parts of mathematics

3.3 interpret results in the context of the given problem

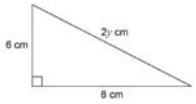
3.4 evaluate methods used and results obtained

3.5 evaluate solutions to identify how they may have been affected by assumptions made.

Weighting: F 25% H 30%

June 2H Q15-A03.4a

15 Sami is trying to work out the exact value of y using Pythagoras' theorem.



Not drawn accurately

Here is her working.

$(2y)^2 = 6^2 + 8^2$

$2y^2 = 36 + 64$

$2y^2 = 100$

$y^2 = 100 \div 2$

$y^2 = 50$

$y = \sqrt{50}$

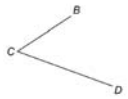
15 (a) What error has she made in her working?

[1 mark]

AQA

November 1F Q21a-A03.4a

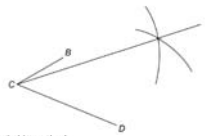
Joe wants to bisect angle *BCD*.



Here is his method.

Use a pair of compasses to draw arcs of the same radius from *B* and *D*.

Draw a straight line from *C* through the intersection of the arcs.



Write down the error in his method.

[1 mark]

AQA

November 2H Q21b-A03.4a

Levi is solving $2x^2 + 5x = 0$

He uses this method.

$2x^2 + 5x = 0$	subtract $5x$ from both sides
$2x^2 = -5x$	divide both sides by x
$2x = -5$	divide both sides by 2
$x = -2.5$	

Evaluate his method and his answer.

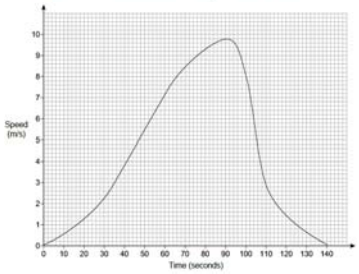
[2 marks]

AQA

PP3 2H Q21a-AO3.4a

21 The graph shows the speed of a skier.

Nick wants to estimate the distance travelled by the skier in 140 seconds.



Does Nick's method give a good estimate?

Tick a box

Yes ☐ No ☐

Give a reason for your answer.

[2 marks]

He works out the area of the triangle with vertices (0, 0), (140, 0) and (90, 9.8)

AQA

Questions or suggestions

AQA

Assessment Objective guidance

AO3.4b

- Evaluating results obtained. This can be tested implicitly in a question where one of the results is impossible within the given constraints or context and needs to be ruled out – this particular result may not be seen but the fact that it is not given as an answer implies the evaluation has taken place.
- However, usually a result will need to be seen (perhaps in a previous part) for the student to evaluate it.
- For example it can be used when deciding whether a previously estimated answer is too high, too low or appropriate. It could also be used in a situation where the student is given a problem and the result and asked whether the result is sensible.
- Note the subtle difference between asking the student to evaluate their result or their method.

AQA

Assessment Objective 3

Solve problems within mathematics in other contexts

Students should be able to:

3.1 translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes

3.2 make and use connections between different parts of mathematics

3.3 interpret results in the context of the given problem

3.4 evaluate methods used and results obtained

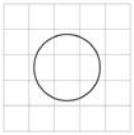
3.5 evaluate solutions to identify how they may have been affected by assumptions made

Weighting: F 25% H 30%

28

June 3F Q13-A03.4b

13 A circle is drawn on a centimetre grid.



13 (b) Grace works out that the area of the circle is more than 9 cm^2

Why must this be wrong?

[1 mark]

29

AQA

November 2F Q13b-A03.4b

Use your calculator to work out the exact value of $\frac{18\,953 \times 437}{11}$

[1 mark]

Answer _____

Use approximations to 1 significant figure to check if your answer to part (a) is sensible.

[3 marks]

30

AQA

Assessment objective guidance

AO3.5

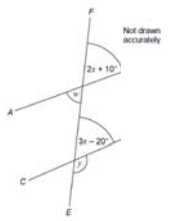
Evaluating solutions to identify how they may have been affected by assumptions made. The assumptions may have been given in the question. It is often useful to give the assumptions in an earlier part that has some problem-solving (often AO3.1) so that a multi-strand task can be claimed. The assumptions used tend to be estimates (weeks worked, weeks in a month/year) or constant speed/rate or straight line/circular travel. The assumption may be generated by the student but this is harder to test in practice.

AO3.1 translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes.

AQA

June 3F Q24b, 3H-AO3.5

AB, *CD* and *EF* are straight lines.



Ava assumes that *AB* and *CD* are parallel.

What answer should she get for the size of angle *y*?

[4 marks]

24 (b) In fact, *AB* and *CD* are **not** parallel angle *w* is 60°

What effect does this have on the size of angle *y*?

Tick a box.

- ☐ *y* is bigger
- ☐ *y* is the same
- ☐ *y* is smaller

Show working to support your answer.

[3 marks]

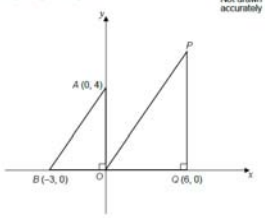
What if…?

AQA

PP3 3F 19b, 3H 6b -AO3.5

Here are two right-angled triangles.

Not drawn accurately



Assume that triangles *AOB* and *PQO* are similar.

Work out the area of triangle *PQO*.

[3 marks]

In fact, *QP* is longer than it would be if the triangles were similar.

How does this affect your answer to part (a)?

[1 mark]

AQA

PP3 1F 23b-A03.5

The air pressure in a tyre measures 7.2 bar.
Air is leaking out at the rate of 0.2 bar per day.

Assume that the air continues to leak at the same rate.

After how many days will the pressure measure 4.8 bar?

[2 marks]

Answer _____

In fact, the rate that the air leaks out increases each day.

How does this affect your answer to part (a)?

[1 mark]

AQA

A few question suggestions

- Where's the algebra?
- Consider the converse
- Assumptions?

- What do I see?
- What do I know?
- What do they want?

- What if?

AQA

AQA
Realising potential

Problem solving beyond GCSE

Maths summer 18 hub meeting

Follow us on Twitter @AQACPD

From L3 Mathematical Studies 2017 Paper 1 Q4

Estimate the number of litres of liquid drunk by the population of a small English town in one month.

State any assumptions that you have made.
You **must** show your working.

[5 marks]

AQA

From L3 Mathematical Studies Paper 1 Q4

			Additional Guidance
Makes an assumption about number of litres per person per day in the range 1 litre to 10 litres (or ml equivalents) **and** assumes a number of days in a month in the range 28 to 31 **and** Makes an assumption about number of people in a small town in the range 1000 to 100000	B3	Must state units eg Minimum for B3 (Assume) 5 litres, 28 days, 15000 people or B2 for 2 correct assumptions (one missing or not in range) eg (Assume) 3 litres, 30 days, 300000 people or B2 for all 3 values within range but not stated as assumptions eg $4 \times 30 \times 10000$ seen gets B2 M1 or B1 Any one correct assumption stated eg drink about 3 litres per day or Multiplication of 3 values with 2 in range and no units eg $12 \times 31 \times 20000$	Ignore any calculations to get the number of litres per day eg 4×300ml glass is 1.5 litres scores B1 for 1.5 litres (even though arithmetic is wrong) The amount of liquid they multiply by must be per person not per household 28 to 31 days can come from various calculations eg 7 days × 4 weeks, $365(.25) \div 12$ Again just award the B1 for a number of days within the range they could use households to estimate population eg small town 2000 houses × 4 people = 8000 population If working in ml they can still gain the method mark but they must convert to litres for the accuracy mark The three values may be multiplied in 2 steps eg litres per day × days in month at one point in their working, then this answer × number of people If they just state a number of litres per month eg 65 litres per month they do not score the marks for assumptions but can score M1 and A1 for multiplying this correctly by their population
Multiplies their 3 values together	M1	This may be done in two steps	Allow rounding at any point eg uses 7 litres and 31 days in a month, $7 \times 31 = 217$ and rounds to 200 or 220
Accurate answer to their calculation	A1ft	ft their 3 values May be rounded	Final answer must be an integer

AQA

From new Level 2 Further Maths Sample Paper 2

21 Show that $(2n + 3)^3 + n^3$ is divisible by 9 for all integer values of n.

[4 marks]

$4n^2 + 6n + 6n + 9$ or $4n^2 + 12n + 9$	M1	allow one error implied by $4n^2 + 12n + k$ or $an^2 + 12n + 9$
$8n^3 + 12n^2 + 24n^2 + 36n + 18n + 9$	M1dep	oe ft their $4n^2 + 6n + 6n + 9$ allow one error
$8n^3 + 36n^2 + 54n + 9$ or $9n^3 + 36n^2 + 54n + 9$	A1	
$9n^3 + 36n^2 + 54n + 9$ and $9(n^3 + 36n^2 + 6n + 1)$	A1	oe eg $(9n^3 + 36n^2 + 54n + 9) \div 9$ $= n^3 + 36n^2 + 6n + 1$ or $9n^3 + 36n^2 + 54n + 9$ and all coefficients are divisible by 9

AQA

From new Level 2 Further Maths Sample Paper 1

19 $f(x) = 2x^3 - 12x^2 + 25x - 11$

Use differentiation to show that $f(x)$ is an increasing function for all values of x.

[4 marks]

$6x^2 - 24x + 25$	M1	allow one error
$6(x^2 - 4x) \ldots$	M1dep	ft their $6x^2 - 24x + 25$ must have 3 term quadratic
$6(x-2)^2 \ldots$	M1dep	ft their $6(x^2 - 4x) \ldots$
$6(x-2)^2 + 1$ and valid argument that this is > 0	A1	

AQA

From new AS Maths Sample Paper 1 Q6

A parallelogram has sides of length 6 cm and 4.5 cm.
The larger interior angles of the parallelogram have size α

Given that the area of the parallelogram is 24 cm^2, find the exact value of $\tan\alpha$

[4 marks]

AQA

From new AS Maths Sample Paper 1 Q6

Marking Instructions	AO	Marks	Typical Solution
Translates given information into an equation by using the formula for the area of triangle or parallelogram to form a correct equation	AO3.1a	M1	$AB \times AD \times \sin\alpha = 24$ hence $6 \times 4.5 \times \sin\alpha = 24$
Rearranges 'their' equation to obtain a correct value of $\sin\alpha$	AO1.1b	A1F	$\sin\alpha = \frac{24}{27} = \frac{8}{9}$
Uses 'their' $\sin\alpha$ value to identify an appropriate right-angled triangle **or** uses identities **and** deduces exact ratio of $\tan\alpha$ – positive or negative Condone only positive ratio seen	AO2.2a	M1	Sides of right angled triangle are 8, 9 and $\sqrt{17}$ Hence $\tan\alpha = \pm\frac{8}{\sqrt{17}}$
Relates back to mathematical context of problem and hence chooses negative ratio – accept any equivalent exact form FT 'their' tan values for obtuse α	AO3.2a	A1F	α is one of the largest angles and must be obtuse hence tangent is negative $\tan\alpha = -\frac{8}{\sqrt{17}} = -\frac{8\sqrt{17}}{17}$
Total		4	

AQA

From Set 2 AS Maths Practice Paper 2 Q5 (draft)

The line joining A (4, –5) to B (18, k) has gradient $\frac{9}{7}$

Find the exact length of AB.

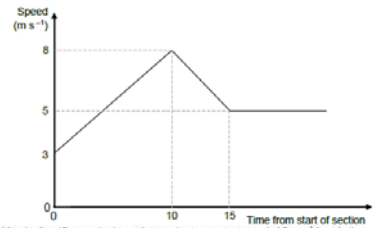
[4 marks]

Selects an appropriate method by finding an expression for the gradient and forming an equation	AO3.1a	M1	Gradient $= \frac{k--5}{18-4} = \frac{k+5}{14}$
Solves the equation to find the correct value of k	AO1.1b	A1	$\frac{k+5}{14} = \frac{9}{7}$ $k = 13$
Substitutes 'their' value of k and applies 'Pythagoras' rule to obtain the required distance	AO1.1a	M1	$\sqrt{18^2+14^2}$ $= \sqrt{520}$
Obtains 'their' correct exact value for the distance AB	AO1.1b	A1F	$= 2\sqrt{130}$

AQA

From AS Maths Sample Paper 1 Q15

The graph shows how the speed of a cyclist varies during a timed section of length 120 metres along a straight track.



(b) After the first 15 seconds, the cyclist travels at a constant speed of 5 m s^{-1} for a further T seconds to complete the 120-metre section.

Calculate the value of T.

[4 marks]

AQA

From AS Maths Sample Paper 1

15 (b) After the first 15 seconds, the cyclist travels at a constant speed of 5 m s^{-1} for a further T seconds to complete the 120-metre section.

Calculate the value of T.

[4 marks]

The problem in (b) can be better understood as a process of using the mathematical model presented in the graph and translating it into a mathematical process of solving a linear equation.

(b)	Identifies $5T$ as the distance travelled after the first 15 seconds	AO3.4	B1	Distance at constant speed = $5T$
	Uses the information given to form an equation to find T (award mark for either trapezium expression separate, totalled or implied)	AO3.1b	M1	Distance in first 15 secs = $\frac{1}{2} \times (3+8) \times 10 + \frac{1}{2} \times (8+5) \times 5$ $= 55 + 32.5 = 87.5$ $5T + 87.5 = 120$
	Correctly calculates the distance for the first 15 secs	AO1.1b	A1	So $T = 6.5$
	Deduces the values of T from the mathematical models applied	AO2.2a	A1	

AQA

Assessment Objectives

AO	Weighting (approx %)		Overarching theme link
	A-level	AS	
AO1	50	60	
AO2	25	20	Mathematical argument, language and proof
AO3	25	20	Mathematical problem solving Mathematical modelling

AQA

Assessment Objectives

		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

AQA

AOs from the specification (AO3)

AO3: Solve problems within mathematics and in other contexts.

Students should be able to:

- translate problems in mathematical and non-mathematical contexts into mathematical processes
- interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations
- translate situations in context into mathematical models
- use mathematical models
- evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.

AQA

AOs from the specification (AO3)

Where questions/tasks targeting this Assessment Objective will also credit students for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding Assessment Objective(s).

AQA

Problem-solving cycle (DfE)

- Specifying the problem.
- Collecting information.
- Processing and representing information.
- Interpreting results.
- Repeating the cycle if necessary.

"Each set of assessment should include questions/tasks where learners are assessed on their ability to solve complete problems in an unstructured manner and which require the use of multiple parts of this cycle."
Ofqual

Some attributes of problem solving questions

- It is not necessary for every problem solving task to exhibit all of the following attributes.
- At least one attribute should generally apply for a task to be regarded as problem solving.

However:
- a question with one of these attributes may not be problem solving
- a problem solving question may be written with none of these attributes.

Problem-solving attributes (Ofqual)

A. Tasks have little or no scaffolding.
B. Tasks provide for multiple representations.
C. The information is not given in mathematical form.
D. Tasks have a variety of techniques that could be used.
E. The solution requires understanding of the processes involved.
F. The task requires two or more mathematical processes.

It is useful to look at the SAMs and try to spot the problem-solving question.

Attributes of problem solving questions

Question 11 AS Sample Paper 1

Chris claims that, "for any given value of x, the gradient of the curve $y = 2x^3 + 6x^2 - 12x + 3$ is always greater than the gradient of the curve $y = 1 + 60x - 6x^2$ ".

Show that Chris is wrong by finding all the values of x for which his claim is **not** true.

[7 marks]

A. **Little or no scaffolding**…

- little guidance will be provided beyond a start and end point
- mathematical processes required are not explicitly stated.

F. **Two or more processes are required, or the solution requires drawing together different parts of mathematics.**

Attributes of problem solving questions

Question 7 AS Sample paper 2

Solve the equation

$$\sin\theta\tan\theta + 2\sin\theta = 3\cos\theta \quad \text{where} \quad \cos\theta \neq 0$$

Give **all** values of θ to the nearest degree in the interval $0° < \theta < 180°$

Fully justify your answer.

[5 marks]

D. **A choice for the student of techniques to be used to solve the problem.**

Attributes of problem solving questions

Question 8 AS Sample paper 2

Prove that the function $f(x) = x^3 - 3x^2 + 15x - 1$ is an increasing function.

[6 marks]

E. **The solution requires understanding of the processes involved, rather than just application of techniques**

19 $f(x) = 2x^3 - 12x^2 + 25x - 11$

Use differentiation to show that $f(x)$ is an increasing function for all values of x.

[4 marks]

Attributes of problem solving questions

Question 15 A-level Sample paper 1

The height x metres, of a column of water in a fountain display satisfies the differential equation $\frac{dx}{dt} = \frac{8\sin 2t}{3\sqrt{x}}$, where t is the time in seconds after the display begins.

(a) Solve the differential equation, given that initially the column of water has zero height.

Express your answer in the form $x = f(t)$

[7 marks]

(b) Find the maximum height of the column of water, giving your answer to the nearest cm.

[1 mark]

C. **Results and/or methods need to be interpreted and/or evaluated** for example in a real world context.

Attributes of problem solving questions

Question 15 A-level Sample paper 3

A sample of 200 households was obtained from a small town.

Each household was asked to complete a questionnaire about their purchases of takeaway food.

A is the event that a household regularly purchases Indian takeaway food.

B is the event that a household regularly purchases Chinese takeaway food.

It was observed that $P(B|A) = 0.25$ and $P(A|B) = 0.1$

Of these households, 122 indicated that they did **not** regularly purchase Indian or Chinese takeaway food.

A household is selected at random from those in the sample.

Find the probability that the household regularly purchases **both** Indian and Chinese takeaway food.

[6 marks]

B. **Provision for multiple representations** such as use of diagrams as well as calculations.

From new AS Maths Sample Paper 1

11 Chris claims that, "for any given value of x, the gradient of the curve $y = 2x^3 + 6x^2 - 12x + 3$ is always greater than the gradient of the curve $y = 1 + 60x - 6x^2$".

Show that Chris is wrong by finding all the values of x for which his claim is **not** true.

[7 marks]

AS Paper 1, question 11
This is a high demand, extended response problem-solving question. There is little scaffolding, and minimal guidance beyond a start and end point.

The question addresses some of the attributes of a problem solving question provided by Ofqual: the mathematical processes required are not explicitly stated, two or more mathematical processes are required, and the solution requires the drawing together of different parts of mathematics.

AQA

From new AS Maths Sample Paper 1

The first M1 mark is for turning the question and, particularly 'gradient', into the process of differentiating. This is a good example of a mathematical problem being turned into a mathematical process.

The second M1 is similar, but this time the process is solving an inequality.

Q	Marking instructions	AO	Marks	Typical Solution
11	Obtains $\frac{dy}{dx}$ for both the given curves – at least one term must be correct for each curve	AO3.1a	M1	$\frac{dy}{dx} = 6x^2 + 12x - 12$ $\frac{dy}{dx} = 60 - 12x$
	States both derivatives correctly	AO1.1b	A1	
	Translates problem into an inequality	AO3.1a	M1	Chris's claim is **incorrect** when $6x^2 + 12x - 12 \leq 60 - 12x$ $2x^2 + 8x - 24 \leq 0$

The actual solving of this inequality should be done directly from the calculator, which will give the correct notation.

AQA

From new AS Maths Sample Paper 1

The final R mark shows the importance of linking the solution back to the context.
When solving a problem we expect to see an explicit reference to the problem in some kind of concluding statement.

Interprets final solution in context of the original question, must refer to Chris's claim	AO3.2a	R1

Chris's claim is incorrect for values of x in the range $-6 \leq x \leq 2$, so he is wrong

AQA

From new AS Maths Sample Paper 2

11 The circle with equation $(x-7)^2+(y+2)^2=5$ has centre C.

11 (a) (i) Write down the radius of the circle.

[1 mark]

11 (a) (ii) Write down the coordinates of C.

[1 mark]

11 (b) The point $P(5, -1)$ lies on the circle.

Find the equation of the tangent to the circle at P, giving your answer in the form $y=mx+c$

[4 marks]

11 (c) The point $Q(3, 3)$ lies outside the circle and the point T lies on the circle such that QT is a tangent to the circle. Find the length of QT.

[4 marks]

This shows how the level of difficulty can increase within a question. The first two parts are accessible to all students – they address recalling facts, terminology and definition – while parts (b) and (c) involve problem-solving.

We could have omitted (a) entirely, but that would risk making the question inaccessible to some students.

AQA

From new AS Maths Sample Paper 2

Q	Marking instructions	AO	Marks	Typical Solution
11(a)(i)	States correct radius CAO	AO1.2	B1	Radius $=\sqrt{5}$
(a)(ii)	States correct centre CAO	AO1.2	B1	C is $(7, -2)$
(b)	Finds gradient of the line through the points P and 'their' C (as found in part (a)) Condone one sign error	AO3.1a	M1	Gradient $CP=\frac{-1-(-2)}{5-7}=-\frac{1}{2}$
	Correct tangent gradient obtained from 'their' CP gradient	AO3.1a	M1	So tangent gradient $=2$
	Uses a correct form for the equation of a straight line with correct coordinates of P and 'their' tangent gradient	AO1.1a	M1	$y-(-1)=2(x-5)$
	States correct final answer in required form ($y=mx+c$) FT from 'their' C found in part (a)	AO1.1b	A1F	$y=2x-11$

Parts (b) and (c) include marks where follow-through will be applied so long as correct a mathematical method is used.

Both parts assess AO3 when a mathematical problem is being translated into processes.

AQA

From new AS Maths Sample Paper 2

(c)	Identifies QTC as a right-angled triangle 'PI'	AO3.1a	M1	QTC is a right-angled triangle so we can use Pythagoras
	Finds QC or QC^2 FT 'their' C found in part (a)	AO1.1b	B1F	$QC^2=(7-3)^2+(-2-3)^2$
	Uses Pythagoras' theorem correctly for 'their' triangle	AO1.1a	M1	$4^2+5^2=(\sqrt{5})^2+QT^2$
	Correct evaluation of length of QT FT 'their' QC and 'their' radius found in part (a)	AO1.1b	A1F	$QT^2=36$ so $QT=6$

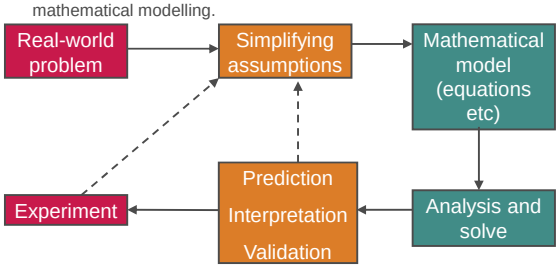
Parts (b) and (c) include marks where follow-through will be applied so long as correct a mathematical method is used.

Both parts assess AO3 when a mathematical problem is being translated into processes.

AQA

The modelling process

- AO3.3, 3.4 and 3.5 all relate specifically to mathematical modelling.
- However it will sometimes be the case that unstructured, extended response, problem solving questions will involve the use of mathematical modelling.



From new A-level Maths Sample Paper 3

3 A circular ornamental garden pond, of radius 2 metres, has weed starting to grow and cover its surface.

As the weed grows, it covers an area of A square metres. A simple model assumes that the weed grows so that the rate of increase of its area is proportional to A.

3 (a) Show that the area covered by the weed can be modelled by

$$A = Be^{kt}$$

where B and k are constants and t is time in days since the weed was first noticed.

[4 marks]

3 (b) When it was first noticed, the weed covered an area of 0.25 m^2. Twenty days later the weed covered an area of 0.5 m^2

3 (b) (i) State the value of B.

[1 mark]

3 (b) (ii) Show that the model for the area covered by the weed can be written as

$$A = 2^{\frac{t}{20}-2}$$

[4 marks]

A-level paper 3, question 3

This 13-mark question is broken up into six parts and applies both modelling and problem solving to Pure maths, rather than in a Mechanics or Statistics context.

AQA

From new A-level Maths Sample Paper 3

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Translates proportionality into a differential equation involving $\frac{dA}{dt}$, A and a constant of proportionality.	AO3.3	M1	$\frac{dA}{dt} \propto A$ $\Rightarrow \frac{dA}{dt} = kA$
	Separates variables	AO1.1a	M1	$\Rightarrow \int \frac{1}{A}\,dA = \int k\,dt$
	Integrates both of 'their' sides correctly	AO1.1b	A1F	$\Rightarrow \ln A = kt + c$ $\Rightarrow A = e^{kt+c}$
	Constructs a rigorous mathematical argument that supports use of the given model. **AG** Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips.	AO2.1	R1	$\Rightarrow A = Be^{kt}$ **AG**
(b)(i)	States correct value of B	AO1.1b	B1	$B = 0.25$ or $B = \frac{1}{4}$
(b)(ii)	Uses $t = 20$ and $A = 0.5$ to find k	AO3.1b	M1	When $t = 20$, $A = 0.5$ $\Rightarrow 0.5 = 0.25e^{20k}$
	Finds correct value of k	AO1.1b	A1	$\Rightarrow 20k = \ln 2$
	Substitutes 'their' k to get A in terms of t	AO1.1a	M1	$\Rightarrow k = \frac{1}{20}\ln 2$ $\Rightarrow A = \frac{1}{4}(e^{\ln 2})^{\frac{t}{20}}$
	Constructs rigorous and convincing argument to show $A = 2^{\frac{t}{20}-2}$ Using correct notation throughout. **AG**	AO2.1	R1	$\Rightarrow A = 2^{-2} \times 2^{\frac{t}{20}}$ $\Rightarrow A = 2^{\frac{t}{20}-2}$ **AG**

AQA

From new A-level Maths Sample Paper 3

3 (b) (ii) Show that the model for the area covered by the weed can be written as

$$A = 2^{\frac{t}{20}-2}$$

[4 marks]

3 (b) (iii) How many days does it take for the weed to cover half of the surface of the pond?

[2 marks]

3 (c) State one limitation of the model.

[1 mark]

3 (d) Suggest one refinement that could be made to improve the model.

[1 mark]

AQA

From new A-level Maths Sample Paper 3

(b)(iii)	Uses the model to set up correct equation and attempt to find t	AO3.4	M1	$2\pi = 2^{\frac{t}{20}-2}$
	Finds correct value of t	AO1.1b	A1	$t = 93.03$ days
(c)	States any sensible and relevant limitation of the model that is specified in terms of the pond, area, weed, rate of change or time.	AO3.5b	E1	Model predicts that the area of weed will increase without limit and this is not possible since the area of the pond is 4π
(d)	Any sensible and relevant refinement to the model that is specified in terms of the pond, area, weed, rate of change or time	AO3.5c	E1	Introduce a limiting factor such as fish eating weed or rate of growth decreases as surface area covered

AQA

From new A-level Maths Sample Paper 2

17 **In this question use $g = 9.81\ \text{m s}^{-2}$.**

A ball is projected from the origin. After 2.5 seconds, the ball lands at the point with position vector $(40\mathbf{i} - 10\mathbf{j})$ metres.

The unit vectors **i** and **j** are horizontal and vertical respectively.

Assume that there are no resistance forces acting on the ball.

17 (a) Find the speed of the ball when it is at a height of 3 metres above its initial position.

[6 marks]

17 (b) State the speed of the ball when it is at its maximum height.

[1 mark]

17 (c) Explain why the answer you found in part **(b)** may not be the actual speed of the ball when it is at its maximum height.

[1 mark]

It's an Ofqual requirement that every set of assessments will have at least one question addressing AO3.3, AO3.4 and AO3.5. We'll take you through the mark scheme on the next page to see what students need to do to answer this successfully.

AQA

From new A-level Maths Sample Paper 2

The first mark could be given at any stage in the solution.

The second mark is for translating the situation in context into a mathematical model.

The third mark is for using correct techniques.

The fourth mark is for using the mathematical model.

The fifth mark is for the correct calculation.

The final mark is awarded for the correct calculation and interpreting the solution to the problem in its original context, rewarding the completion of a complete strategy.

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)	Obtains correct horizontal component of the initial velocity	AO1.1b	B1	$2.5U = 40$ $U = 16$
	Forms equation to find vertical component of initial velocity	AO3.3	M1	$-10 = 2.5V - 0.5 \times 9.81 \times 2.5^2$
	Obtains correct vertical component of initial velocity	AO1.1b	A1	$V = 8.2625$
	Forms equation for vertical component of velocity at height 3 using 'their' derived values for U and V	AO3.4	M1	$v_y^2 = 8.2625^2 + 2\times(-9.81)\times 3$
	Obtains correct component of velocity	AO1.1b	A1	$v_y = 3.067...$
	Correct final speed with units, correct for 'their' U and v_y FT applies only if both M1 marks have been awarded	AO3.2a	A1F	$v = \sqrt{16^2 + 3.067^2} = 16.3\,\text{m s}^{-1}$

AQA

From new A-level Maths Sample Paper 2

Answers must be given to three significant figures with appropriate units to match the value of g in the question.

Part (b) is given for using the model to state 'their' value.

Part (c) is given if the student recognises the limitation of the model used. Whilst the mark scheme limits this to horizontal velocity, a similar argument holds for vertical velocity.

(b)	States 'their' value of horizontal component of the initial velocity from part **(a)**	AO3.4	A1F	16 m s^{-1}
(c)	Explains that horizontal velocity has been assumed to be constant in their model and that this is not likely to be true, with valid reasoning	AO3.5b	E1	It was assumed that there were no resistance forces acting on the ball which is unlikely to be true in reality. The horizontal speed of the ball is likely to vary… air resistance would slow the ball down, wind might speed the ball up

AQA

From new A-level Maths Sample Paper 3

12 During the 2006 Christmas holiday, John, a maths teacher, realised that he had fallen ill during 65% of the Christmas holidays since he had started teaching.

In January 2007, he increased his weekly exercise to try to improve his health.

For the next 7 years, he only fell ill during 2 Christmas holidays.

AQA

From new A-level Maths Sample Paper 3

12 (a) Using a binomial distribution, investigate, at the 5% level of significance, whether there is evidence that John's rate of illness during the Christmas holidays had decreased since increasing his weekly exercise.

[6 marks]

Part (a) is an extended response question that addresses all three AOs, but it is a standard example of a hypothesis test.

AQA

From new A-level Maths Sample Paper 3 continued

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	States both hypotheses correctly for one-tailed test	AO2.5	B1	X = number of Christmas holidays without illness since January 2007 $X \sim B(7, p)$ H_0 $p = 0.65$ H_1 $p < 0.65$
	States model used (condone 0.009 rather than 0.056) **PI**	AO1.1b	M1	Under null hypothesis, $X \sim B(7, 0.65)$
	Using calculator, 0.056 or better	AO1.1b	A1	$P(X \leq 2) = 0.0556$
	Evaluates binomial model by comparing $P(X \leq 2)$ with 0.05 **PI**	AO3.5a	M1	0.0556 >0.05
	Infers H_0 accepted PI	AO2.2b	A1	Accept H_0
	Concludes correctly in context, 'not sufficient evidence' or equivalent required	AO3.2a	E1	There is not sufficient evidence that the John's rate of illness has decreased

It's worth teachers and students familiarising themselves with the marking instructions for hypothesis testing questions. Marks are awarded for 'stating both hypotheses correctly', 'evaluating the model by comparison of relevant probabilities', 'inferring whether H0 should be accepted/rejected' and 'stating a conclusion in context'. This pattern is designed to encourage clear communication from students.

AQA

From new A-level Maths Sample Paper 3

12 (b) State **two** assumptions, regarding illness during the Christmas holidays, that are necessary for the distribution you have used in part **(a)** to be valid.

For **each** assumption, comment, in context, on whether it is likely to be correct.

[4 marks]

Part (b) requires students to identify two necessary assumptions and discuss their likely validity.

AQA

From new A-level Maths Sample Paper 3 continued

(b)	States one correct assumption(s) regarding validity of model	AO3.5b	E1
	States corresponding correct description(s) of likelihood of validity in context	AO2.4	E1
	States second correct assumption(s) regarding validity of model	AO3.5b	E1
	States corresponding correct description(s) of likelihood of validity in context	AO2.4	E1
	Max two assumptions with description of validity		

Assumption 1
The probability of illness remains constant throughout one's life
Validity
Not fully valid, as age has an impact on the immune system

OR

Assumption 2
Annual results (of illness) are independent of one another
Validity
(Largely) valid. Trials are sufficiently far apart that an illness spanning two Christmases is unlikely.

OR

Assumption 3
There are only two states, well and ill
Validity
Unclear. Grey area exists. eg does a mild sore throat count as ill?

AQA

From new A-level Maths Sample Paper 1

12 A sculpture formed from a prism is fixed on a horizontal platform, as shown in the diagram.

The shape of the cross-section of the sculpture can be modelled by the equation $x^2 + 2xy + 2y^2 = 10$, where x and y are measured in metres.

The x and y axes are horizontal and vertical respectively.

Find the maximum vertical height above the platform of the sculpture.

[8 marks]

This is an A-level example of an extended response problem solving question. This is a high-demand question with no scaffolding. It highlights the importance of follow-through marks in determining how many marks a question should be worth and how they should be accessible to students of differing abilities.

AQA

From new A-level Maths Sample Paper 1

This question covers all three AOs.

You can see the first M1 mentioned will not necessarily appear first in the solution, but it might be shown on a diagram.

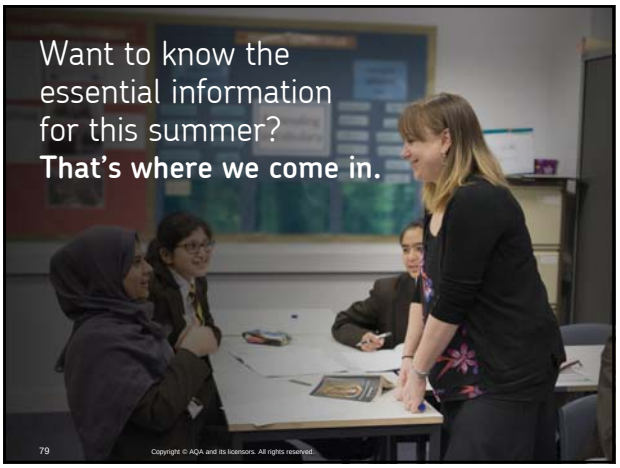
There's a stand-alone mark for identifying the condition for stationary points, because this is an essential part of solving the problem.

The final two marks allow follow-through provided students use a correct method. This use of follow-through marks is an important change from the old specification and meets DfE requirements.

An incorrect answer with correct methods could still gain up to 6 marks out of 8.

Q	Marking instructions	AO	Marks	Typical Solution
12	Finds the difference between the maximum and minimum values of y	AO3.1b	M1	$x^2+2xy+2y^2=10$ $2x+2y+2x\frac{dy}{dx}+4y\frac{dy}{dx}=0$ Highest and lowest points occur when $\frac{dy}{dx}=0$ $\frac{dy}{dx}=0 \Rightarrow x=-y$ $y^2-2y^2+2y^2=10$ $y=2\sqrt{10}$ $\therefore$Height $=\sqrt{10}-(-\sqrt{10})$ $=2\sqrt{10}=6.32$ m
	Uses implicit differentiation	AO1.1a	M1	
	Differentiates correctly	AO1.1b	A1	
	States stationary points occur when $\frac{dy}{dx}=0$	AO2.4	R1	
	Uses $\frac{dy}{dx}=0$ to find x in terms of y (or vice versa)	AO1.1a	M1	
	Finds $x=-y$	AO1.1b	A1	
	Deduces maximum and minimum values of y FT 'their' expression provided all M1 marks have been awarded	AO2.2a	A1F	
	States the height of the sculpture above the platform FT 'their' max and min values for y provided all M1 marks have been awarded	AO2.2a	A1F	

AQA



How did we do?

Please take a moment to complete a brief evaluation form for today's event. Your feedback is very important to us as it helps us improve and plan future training.

You should have been emailed the evaluation form. Please check your inbox (possibly your junk mail folder). If you haven't received it please give your trainer your name, centre name/ number and email address so that we can look into it for you.

Thank you.

AQA

AQA
Realising potential

Thank you

Follow us on Twitter @AQACPD