GCSE
MATHEMATICS

Hub school network meetings

Accessibility and exam questions resource booklet

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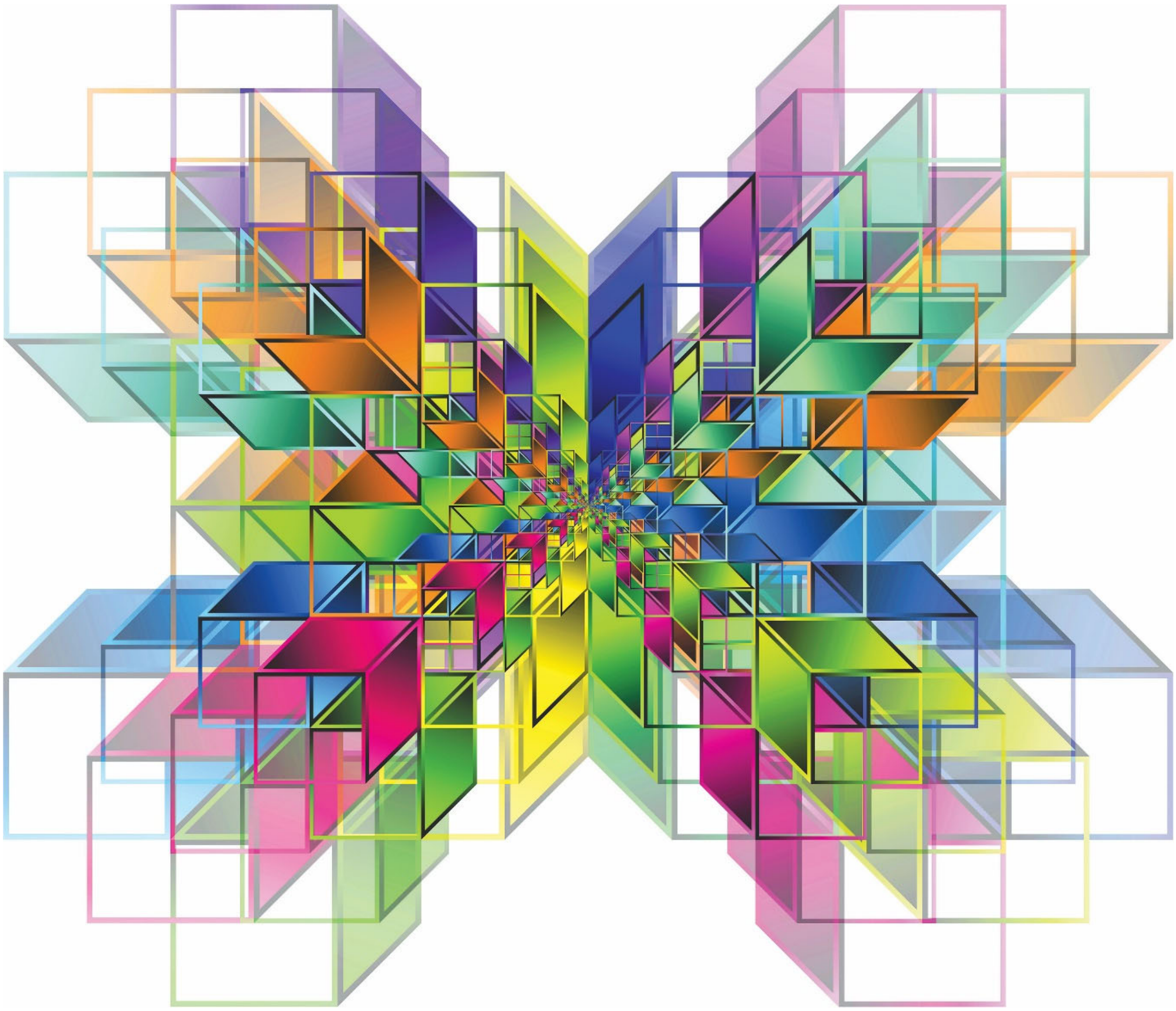


Contents

1F, Q21(a) and 21(b)

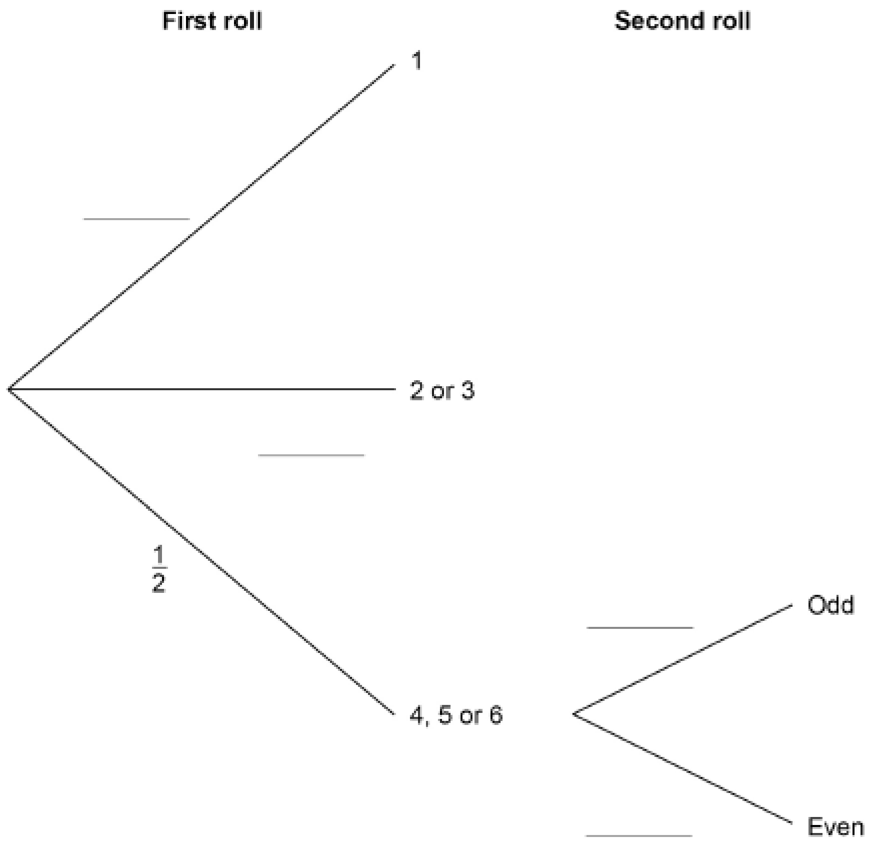
21 Anna plays a game with an ordinary, fair dice.

If she rolls 1 she wins.
If she rolls 2 or 3 she loses.
If she rolls 4, 5 or 6 she rolls again.

When she has to roll again,
if she rolls an odd number she wins
if she rolls an even number she loses.

21 (a) Complete the tree diagram with the four missing probabilities.

[2 marks]



Question	Answer	Mark	Comments
21(a)	$\frac{1}{6}$ on '1' and $\frac{1}{3}$ or $\frac{2}{6}$ on '2 or 3' and $\frac{1}{2}$ on each of 'Odd' and 'Even'	B2	oe fraction, decimal or percentage B1 $\frac{1}{6}$ on '1' and $\frac{1}{3}$ or $\frac{2}{6}$ on '2 or 3' or $\frac{1}{2}$ on each of 'Odd' and 'Even' or all correct unsimplified probabilities with one or more simplification errors eg $\frac{3}{6}$ on 'Odd' simplified to $\frac{1}{3}$
	Additional Guidance		
	Accept decimals or percentages rounded or truncated correctly to at least 2 significant figures		
	Only withhold a mark for simplification errors if B2 would otherwise be awarded		
	Ignore extra branches added		
	Ignore attempts to work out combined probabilities to the right of the tree diagram		
	If an answer line is blank, the student may have written their answer elsewhere on the branch		

21 (b) Is Anna more likely to win or to lose?

You **must** work out the probability that she wins.

[4 marks]

Question	Answer	Mark	Comments
21(b)	**Alternative method 1: P(1) + P(4, 5 or 6) × P(Odd)**		
	$\frac{1}{2}$ × their $\frac{1}{2}$ or $\frac{1}{4}$	M1	oe
	their $\frac{1}{4}$ + their $\frac{1}{6}$	M1dep	oe
	(P(win) =) $\frac{10}{24}$ or $\frac{5}{12}$	A1ft	oe ft their tree diagram
	Lose (and P(Lose) = $\frac{14}{24}$ or $\frac{7}{12}$ oe)	A1ft	ft correct decision for their $\frac{5}{12}$ (and their $\frac{7}{12}$) with M2 scored
	Alternative method 2: 1 – P(2 or 3) – P(4, 5 or 6) × P(Even)		
	$\frac{1}{2}$ × their $\frac{1}{2}$ or $\frac{1}{4}$	M1	oe
	their $\frac{1}{4}$ + their $\frac{1}{3}$ or P(lose) = $\frac{7}{12}$	M1dep	oe ft their tree diagram
	(P(win) =) $\frac{10}{24}$ or $\frac{5}{12}$	A1ft	oe ft their tree diagram
	Lose (and P(Lose) = $\frac{14}{24}$ or $\frac{7}{12}$ oe)	A1ft	ft correct decision for their $\frac{5}{12}$ (and their $\frac{7}{12}$) with M2 scored
	Additional Guidance is on the following page		

Question	Answer	Mark	Comments

21(b) cont	**Additional Guidance**	
	Check the tree diagram for working	
	Any 'their' or ft probability must be > 0 and < 1 for marks to be awarded	
	For the second A1ft, the ft can be from an incorrect tree (which may score 4 marks) or an arithmetic error (which scores 3 marks, M1M1A0A1ft)	
	Accept equivalent fractions or decimals within calculations and equivalent fractions, decimals or percentages for final probabilities	
	Accept decimals or percentages rounded or truncated correctly to at least 2 significant figures	
	Condone $\frac{1}{2} \times$ their $\frac{1}{2}$ as part of a longer, incorrect multiplication eg $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}$	M1M0A0A0
	Condone decimals used within fractions eg P(Win) = $\frac{2.5}{6}$	at least M1M1A1
	For the method marks, condone incorrect mathematical notation eg $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} + \frac{1}{6} = \ldots$	at least M1M1 (may go on to score 3 or 4 marks)
	For the second A1ft, if the student gives a value for P(Lose), their P(Win) + their P(Lose) must equal 1 However, allow a comparison to $\frac{1}{2}$ unless it is clearly an incorrect value for P(Lose)	

Question 21

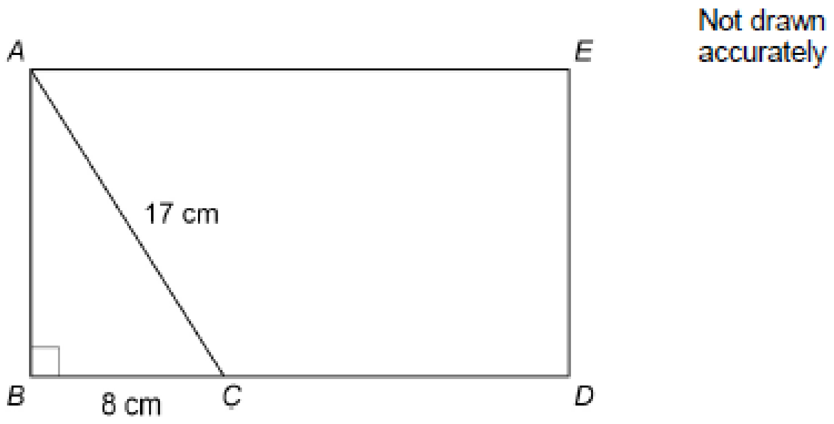
In part (a), the tree diagram was not done very well but in part (b) there were actually very few correct solutions at all. This is a familiar topic but it was done very badly.

2F, Q22

22 The diagram shows rectangle *ABDE* and right-angled triangle *ABC*.

AC = 17 cm

BC = 8 cm



BC : *CD* = 1 : 2

Work out the area of rectangle *ABDE*.

[4 marks]

Answer ______________________ cm^2

Question	Answer	Mark	Comments
22	**Alternative method 1**		
	8^2 or 64 and 17^2 or 289	M1	
	$\sqrt{17^2 - 8^2}$ or $\sqrt{225}$ or 15	M1dep	oe implies M2 may be seen on diagram
	8 × 3 × their 15 or 24 × their 15	M1dep	dep on M2 oe eg (8 + 16) × their 15 or 0.5 × 8 × their 15 × 6
	360	A1	SC2 [448.8, 456]
	Alternative method 2		
	cos $C = \frac{8}{17}$ or C = [61.9, 62]	M1	may be seen on diagram
	17 × sin their [61.9, 62] or [14.9, 15.1]	M1dep	may be seen on diagram oe eg 8 × tan their [61.9, 62]
	8 × 3 × their [14.9, 15.1] or 24 × their [14.9, 15.1] or [357.6, 362.4]	M1dep	dep on M2 oe eg (8 + 16) × their [14.9, 15.1] or 0.5 × 8 × their [14.9, 15.1] × 6
	360	A1	SC2 [448.8, 456]
	Alternative method 3		
	sin $A = \frac{8}{17}$ or A = [28, 28.1]	M1	may be seen on diagram
	17 × cos their [28, 28.1] or [14.9, 15.1]	M1dep	may be seen on diagram oe eg 8 ÷ tan their [28, 28.1]
	8 × 3 × their [14.9, 15.1] or 24 × their [14.9, 15.1] or [357.6, 362.4]	M1dep	dep on M2 oe eg (8 + 16) × their [14.9, 15.1] or 0.5 × 8 × their [14.9, 15.1] × 6
	360	A1	SC2 [448.8, 456]

Alternative method and Additional Guidance continued on the next page

Question	Answer	Mark	Comments

22 cont	**Alternative method 4**		
	$\cos C = \frac{8}{17}$ or C = [61.9, 62]	M1	may be seen on diagram
	$\frac{1}{2} \times 8 \times 17 \times \sin$ their [61.9, 62] or [59.9, 60.1]	M1dep	oe
	6 × their [59.9, 60.1] or [357.6, 362.4]	M1dep	oe
	360	A1	SC2 [448.8, 456]
	Additional Guidance		
	15 without a contradictory value for *AB* scores the first two marks on Alt method 1, even if not subsequently used		M1M1
	$\sqrt{17^2 + 8^2}$		M1M0
	3rd M1 is for the total area and may be calculated in various ways eg using a trapezium + a triangle		
	3rd M1 is for the total area so further work will lose the mark eg 360 seen followed by 360 – 60, answer 300		M1M1M0A0
	May use sine rule or cosine rule but must reach *AB* = ... to award the second M1 in Alt 2 or 3		

Question 22

Most students did not realise that they needed to use Pythagoras' theorem to work out the height of the triangle. Many used 17 cm or 17 – 8 = 9 cm as the height. Those who did work out the height correctly were usually able to give a fully correct solution. Additionally, students, who used Pythagoras' theorem incorrectly by adding the squares of the sides, often went on to gain credit for the special case in the scheme.

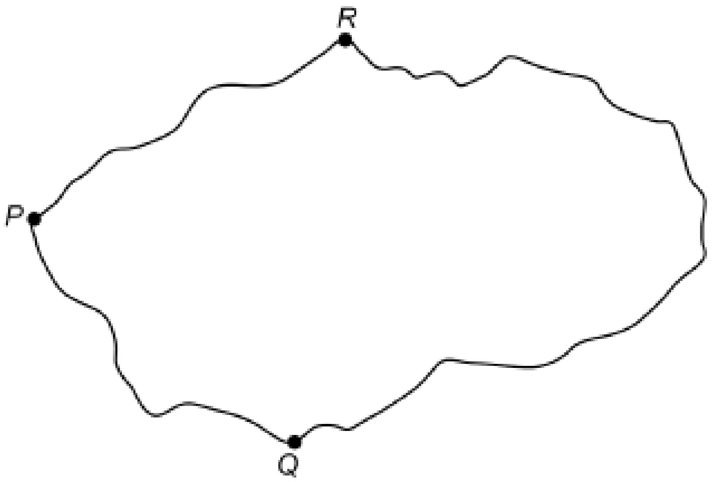
3F, Q25

25 Towns P, Q and R are connected by roads PQ, PR and QR.

PR is 10 km longer than PQ.

QR is twice as long as PR.

The total length of the three roads is 170 km



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Work out the length of PQ.

[4 marks]

Answer ______________________ km

Question	Answer	Mark	Comments
25	**Alternative method 1 – PQ as the unknown**		
	$x + 10$ or $2(x + 10)$	M1	any unknown
	$x + x + 10 + 2(x + 10) = 170$	M1dep	oe any consistent unknown x + their two expressions (with at least one correct) = 170
	$4x + 30 = 170$	M1dep	oe $4x = 140$ must be correct
	35	A1	
	Alternative method 2 – PR as the unknown		
	$x - 10$ or $2x$	M1	any unknown
	$x + x - 10 + 2x = 170$	M1dep	oe any consistent unknown x + their two expressions (with at least one correct) = 170
	$4x - 10 = 170$ or $x = 45$	M1dep	oe $4x = 180$ must be correct
	35	A1	
	Alternative method 3 – QR as the unknown		
	$\frac{x}{2}$ or $\frac{x}{2} - 10$	M1	any unknown
	$x + \frac{x}{2} + \frac{x}{2} - 10 = 170$	M1dep	oe any consistent unknown x + their two expressions (with at least one correct) = 170
	$2x - 10 = 170$ or $x = 90$	M1dep	oe $2x = 180$ must be correct
	35	A1	

Mark scheme for Question 25 continues on next page

Question	Answer	Mark	Comments
25 cont	**Alternative method 4 – trial and improvement with addition of three lengths**		
	A correctly evaluated trial with a difference of 10 (km) between the two shorter lengths and the longest length twice the length of the middle length	M1	may be seen as a subtraction of three numbers from 170
	A different correctly evaluated trial with a difference of 10 (km) between the two shorter lengths and the longest length twice the length of the middle length	M1dep	may be seen as a subtraction of three numbers from 170
	35, 45 and 90	A1	
	35	A1	
	Alternative method 5 – trial and improvement with subtraction from 170		
	A correctly evaluated trial of two lengths subtracted from 170 with a difference of 10 (km) between the two lengths or one length twice the length of the other	M1	
	A different correctly evaluated trial of two lengths subtracted from 170 with a difference of 10 (km) between the two lengths or one length twice the length of the other	M1dep	
	35, 45 and 90	A1	
	35	A1	

Additional Guidance is on the next page

	Additional Guidance	
25 cont	If the student attempts more than one method, mark each method and award the highest mark	
	Alt 1 $PQ + PQ + 10 + 2(PQ + 10) = 170$	M1M1
	Alt 1 $PQ + PQ + 10 + 2PR = 170$	M1
	Alt 2 x, $x + 10$ and $2x$ seen on diagram, $4x + 10 = 170$	M1M1M0A0
	Alt 4 35 + 45 + 90 with no choice made	M1M1A1A0
	Alt 4 170 – 30 – 40 – 80 = 20	M1
	Alt 4 170 – 30 – 40 – 60 = 40 incorrect number is doubled	M0
	Alt 5 170 – 30 – 60 = 80	M1

Question 25

This question was not very well answered and there were a large number of non-attempts. Setting up and solving an equation often led to the correct answer. However, a very large majority of students did not use an algebraic approach. The most common approach was to use trial and improvement with the addition of three lengths; however these were often not evaluated. The most common error involved trials where the length of *PQ* was doubled rather than *PR*, with *PQ* = 40, *PR* = 50 and *QR* = 80 often seen.

1H, Q20

20 *A*, *B* and *C* are points on a circle.

CD is a tangent.

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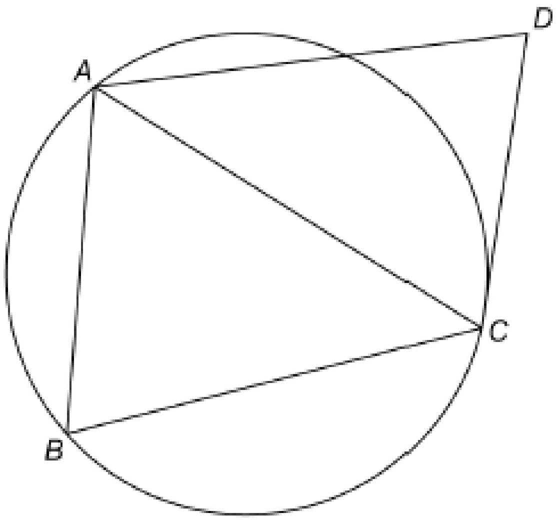


20 (a) Assume that triangle *ABC* is isosceles with *AC* = *BC*

Prove that *AB* is parallel to *DC*.

[4 marks]

Question	Answer	Mark	Comments
20(a)	**Alternative method 1: shows that *BAC* = *ACD* and alternate angles**		
	ACD = *ABC*	M1	accept both with same letter on diagram
	ABC = *BAC*	M1	accept both with same letter on diagram
	BAC = *ACD* and alternate segment (theorem) with M2 awarded	M1dep	dep on M2
	Other two correct reasons given with M3 awarded	A1	eg (base angles of) isosceles triangle and alternate angles
	Alternative method 2: shows that *ABC* + *BCD* = 180 and co-interior angles		
	ACD = *ABC*	M1	accept both with same letter on diagram
	ABC = *BAC*	M1	accept both with same letter on diagram
	BCD = 180 – (*BAC* + *ABC*) + *ACD* and *ABC* + *BCD* = 180 and alternate segment (theorem) with M2 awarded	M1dep	oe dep on M2
	Other two correct reasons given with M3 awarded	A1	eg (base angles of) isosceles triangle and (co-)interior angles or allied angles
	The mark scheme for question 20(a) continues on the next page		

Question	Answer	Mark	Comments
20(a) (cont)	**Alternative method 3: line from midpoint of *AB* to *C* is perpendicular to *AB* and *CD***		
	Let *M* be the midpoint of *AB* and *MC* is perpendicular to *AB*	M1	any letter
	MC is perpendicular to *CD*	M1	
	AB and *CD* are both perpendicular to *MC* with M2 awarded	M1dep	oe dep on M2
	Three correct reasons given with M3 awarded	A1	eg (perpendicular bisector of) isosceles triangle and *MC* goes through the centre of the circle and tangent is perpendicular to radius
	Additional Guidance		
	Other correct methods can be found by extending one or more of the lines. For example, by extending *BC* it is possible to use corresponding angles as a proof instead of alternating angles. This should be reflected in the reasons required for the last mark		
	In the scheme, *ACD* (for example) means angle *ACD* and not triangle *ACD*		
	Accept equality of angles indicated by labelling with the same letter, but not by arcs		
	Accept (angle) *B* for angle *ABC* Do not accept (angle) *A* for angle *BAC* or (angle) *C* for angle *ACB* unless intention is clear from annotation of the diagram		
	For the third mark in alternative method 2, accept algebraic expressions for angles if clearly marked on the diagram		
	Do not award marks for an argument based only on assumed values of angles, but ignore 60° marked on diagram, which is for (b)		
	Ignore an angle marked at *ADC*		
	Ignore incorrect statements that do not affect the proof eg *ACD* is an isosceles triangle (but not used in proof)		

20 (b) In fact, triangle *ABC* is equilateral.

Tick the two boxes for the statements that **must** be correct.

[1 mark]

☐ *AB* is parallel to *DC*

☐ *AC* bisects angle *BCD*

☐ *AC* bisects angle *BAD*

Question	Answer	Mark	Comments
20(b)	✓ *AB* is parallel to *DC* ✓ *AC* bisects angle *BCD* ☐ *AC* bisects angle *BAD*	B1	
	Additional Guidance		

Question 20

Very few students scored more than 2 marks in part (a) with most working on the assumption that *ABCD* was a parallelogram. Those who made some progress often failed to give any reasons for their statements. Students should give a reason for each calculation or statement in geometric proof questions.

Less than a quarter of all students were successful in part (b). Many did not select the first statement, even though it had been given in part (a). Although they had been told to tick two boxes, a minority of students only ticked one.

2H, Q20

20 Expressions for consecutive triangular numbers are

$$\frac{n(n+1)}{2} \quad \text{and} \quad \frac{(n+1)(n+2)}{2}$$

Prove that the sum of two consecutive triangular numbers is always a square number.

[4 marks]

Question	Answer	Mark	Comments
20	**Alternative method 1**		
	$\frac{n^2+n}{2}$ or $\frac{n^2+2n+n+2}{2}$ or $\frac{n^2+3n+2}{2}$	M1	may be seen in stages eg n^2+n followed by $\frac{n^2+n}{2}$
	$\frac{n^2+n}{2}$ and $\frac{n^2+2n+n+2}{2}$ or $\frac{n^2+n}{2}$ and $\frac{n^2+3n+2}{2}$	M1dep	may be seen in stages eg n^2+n followed by $\frac{n^2+n}{2}$ and n^2+3n+2 followed by $\frac{n^2+3n+2}{2}$ implies M2
	$\frac{2n^2+4n+2}{2}$ or n^2+2n+1 with M2 seen	A1	oe single fraction with terms collected eg $\frac{4n^2+8n+4}{4}$
	n^2+2n+1 and $(n+1)^2$ with M2A1 seen	A1	allow $(n+1)(n+1)$ for $(n+1)^2$
	Alternative method 2		
	$\frac{n+1}{2}(n+n+2)$	M1	oe eg $(n+1)\left(\frac{n}{2}+\frac{n+2}{2}\right)$
	$\frac{n+1}{2}(2n+2)$ or $\frac{n^2+n}{2}+\frac{n^2+n}{2}+\frac{2n+2}{2}$ with M1 seen	M1dep	
	$\frac{2n^2+4n+2}{2}$ or n^2+2n+1 with M2 seen	A1	oe single fraction with terms collected eg $\frac{4n^2+8n+4}{4}$
	n^2+2n+1 and $(n+1)^2$ with M2A1seen	A1	allow $(n+1)(n+1)$ for $(n+1)^2$

Mark scheme and Additional Guidance continue on the next two pages

Question	Answer	Mark	Comments
20 cont	**Alternative method 3**		
	$\frac{n+1}{2}(n + n + 2)$	M1	oe eg $(n + 1)\left(\frac{n}{2}+\frac{n+2}{2}\right)$
	$\frac{n+1}{2}(2n + 2)$ with M1 seen	M1dep	oe eg $\frac{(n+1)(2n+2)}{2}$
	$(n + 1)^2$ with M2 seen	A2	A1 $2(n + 1)\ \frac{n+1}{2}$ or $\frac{2(n+1)^2}{2}$ allow $(n + 1)(n + 1)$ for $(n + 1)^2$

Additional Guidance is on the next page

	Additional Guidance	
20 cont	Only substituting in values of n	M0M0A0A0
	Consistently using a different letter to n can score up to M1M1A1A1	
	Using two different letters consistently within the two fractions (eg n replaced by x in the first equation and n replaced by y in the second equation) can score a maximum of M1M1A0A0 unless recovered to the same letter	
	Multiplying fractions instead of adding can score a maximum of M2A0	
	For M marks condone eg $n2$ for $2n$ etc	
	$n^2 + n /2$ and $n^2 + 3n + 2/2$ recovered to $\frac{2n^2+4n+2}{2}$ and/or $n^2 + 2n + 1$ and/or $(n + 1)^2$	M1M1A0A0
	$n^2 + n /2$ and $n^2 + 3n + 2/2$ not recovered	M0M0A0A0
	$n^2 + n$ and $n^2 + 3n + 2$ recovered to $\frac{2n^2+4n+2}{2}$ and/or $n^2 + 2n + 1$ and/or $(n + 1)^2$	M1M1A0A0
	$n^2 + n$ and $n^2 + 3n + 2$ not recovered	M0M0A0A0
	Equating to n^2 in working can score a maximum of M1M1A0A0 (equating to eg x^2 can score up to M1M1A1A1)	
	$1n$ is allowed for n throughout	
	Alts 2 and 3 $\frac{n+1}{2}(2n + 2)$ with M1 seen scores M2 If they attempt to expand $(n + 1)(2n + 2)$ use Alt 2 If they attempt to expand $\frac{1}{2}(2n + 2)$ use Alt 3	

Question 20

Although some excellent proofs were seen, this question was not well answered. Many only substituted in values for n. Errors were made in expansions and it was quite common for students to incorrectly process the sum of the two algebraic fractions.

3H, Q27

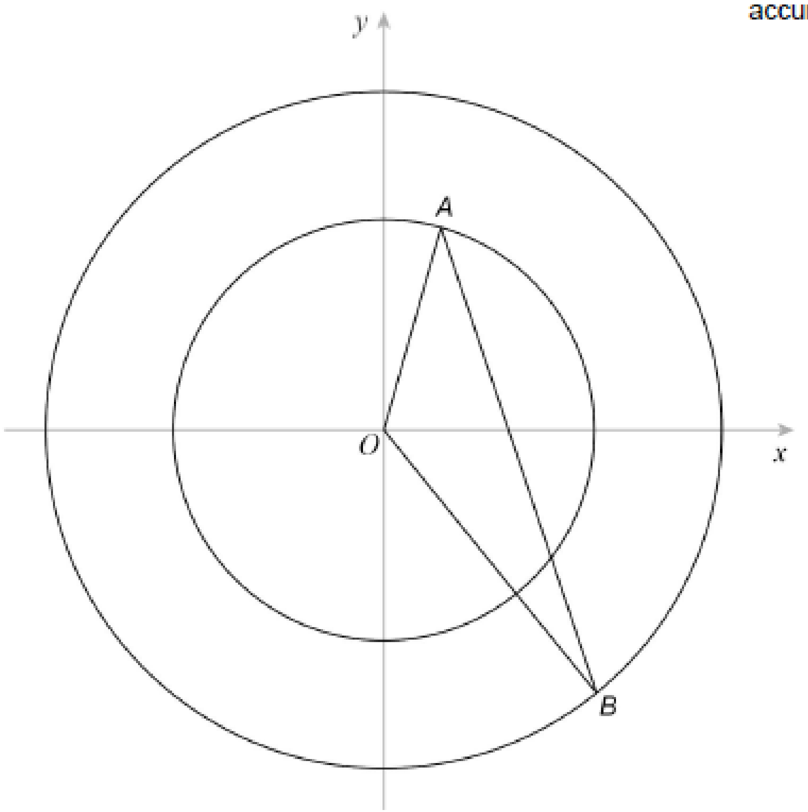
27 In this question, all lengths are in centimetres.

A is a point on a circle, centre *O*.

B is a point on a different circle, centre *O*.

AB = 20

Not drawn accurately



The equation of the larger circle is $x^2 + y^2 = 144$

radius of smaller circle : radius of larger circle = 4 : 5

Work out the size of angle *AOB*.

[5 marks]

Answer ______________________ degrees

Question	Answer	Mark	Comments
27	$\sqrt{144}$ or 12	B1	radius of larger circle may be seen on diagram
	$\frac{4}{5}$ × their 12 or 9.6	M1	their 12 must be a value may be seen on diagram
	(cos *AOB* =) $\frac{\text{their } 12^2 + \text{their } 9.6^2 - 20^2}{2 \times \text{their } 12 \times \text{their } 9.6}$ or $\frac{144 + 92.16 - 400}{230.4}$ or $-\frac{32}{45}$ or –0.71…	M1dep	oe
	$\cos^{-1}$ their $-\frac{32}{45}$	M1dep	dep on M2
	135.(…)	A1	
	Additional Guidance		
	144 $\frac{4}{5}$ × 144 = 115.2 (cos *AOB* =) $\frac{144^2 + 115.2^2 - 20^2}{2 \times 144 \times 115.2}$	B0 M1 M1M0A0	
	12 seen, but a different value used for the radius of the larger circle cannot score B1M1		
	$x + y = 12$ seen, but $x = 6$ used to find radius $OA = 4.8$	B0M1	

Question 27

This question had one of the lowest numbers of completely correct solutions, although more than half of the students gained some marks. It was very clear that if the cosine rule had been recalled and used correctly, then the proportion of fully correct answers would have been significantly higher. It was disappointing to see responses where the student understood the method they needed to use (cosine rule) but were unable to make any further correct progress. Many students correctly stated the radius of the larger circle to be 12 but a significant number were not able to interpret the equation of the circle. Common errors included: using 144 as the radius for the larger circle; using 80 and 64 as the radii of the circles; using right-angled triangle trigonometry; and incorrectly rearranging the cosine rule to find the angle *AOB*. Students should be reminded that if their calculator shows an error when trying to find the angle using the cosine rule, they should go back and check their working.

Contact us

Our friendly team will be happy to support you between 8am and 5pm, Monday to Friday.

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