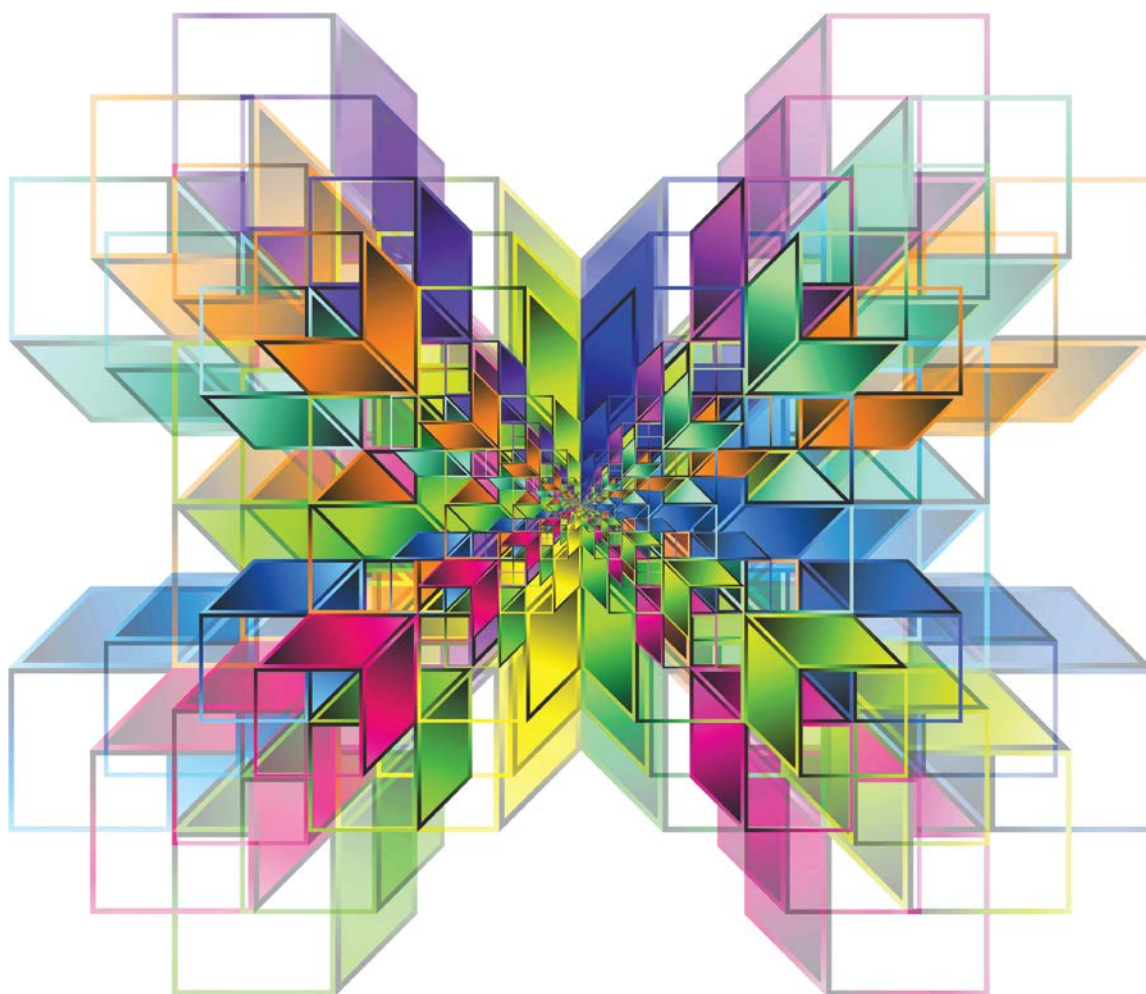


GCSE Maths

Summer hub schools network meeting

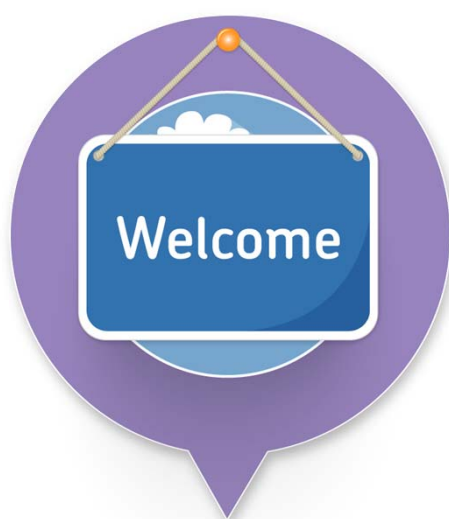
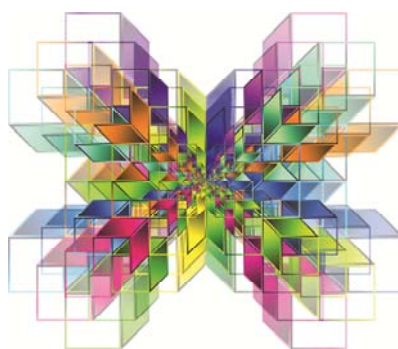
Presentation slides booklet

Published: Summer 2019



GCSE Maths: Hub schools network meeting

Summer 2019



This meeting will be recorded

Exam boards have an Ofqual requirement to record event audio.

Recordings are kept for the lifetime of the specification and not shared as an accompaniment to session resources.

The recording will begin now.

Introduction

We will

- Look at Ratio and Proportion in the GCSE Specification
- Think about a definition of Ratio and Proportion
- Discuss ways of introducing Ratio as a topic
- Look in more detail at each specification reference and its assessment
- Discuss the underlying mathematics of proportional division and equality of ratios
- Look in more detail at how these concepts are applied
- Look in more detail at how they are assessed

The assessment of Ratio and Proportion

16 sections - R1 to R16

This presentation considers

- R1 Units
- R2 Maps and scales
- R3 Fractions
- R4 Ratio notation
- R5 Proportional division
- R6 Multiplicative relationships
- R7 Equality of ratios
- R9 Percentages

Foundation v Higher

- Most of the content is categorised as Foundation
- R2 to R10 all basic Foundation content only
- R1, R11, R12 and R14 have some additional Foundation (but no Higher)
- R13 and R16 have additional and Higher content
- Only R15 has exclusively Higher content

An accessible topic area?

- Looking at the content, most students should be able to grasp it
- A significant part of the GCSE

Topic Area	Foundation Tier (%)	Higher Tier (%)
Number	25	15
Algebra	20	30
Ratio	25	20
Geometry	15	20
Probability and statistics (combined)	15	15

- But clearly underpins a lot of advanced ideas
- Lends itself to setting questions in harder contexts

Assessment of ratio and proportion (June 2018)

- Distributed throughout the papers
- Earliest question is Q3 on Foundation Paper 2
- Latest question is Q26 on Higher Paper 1
- AO1, AO2 and AO3 marks are evenly distributed
- Ratio of AO1 to AO2 to AO3 marks is $\approx 6 : 4 : 11$
- There are 8 common questions
- R10 means that there is scope at all levels for problem solving

Ratio – a definition (discussion)

- A **ratio** is a relationship between two numbers indicating how many times the first number contains the second
- Written as ' a to b ' or ' $a:b$ ', or by giving just the value of their quotient a
- A ratio compares the size, or magnitude, of two numbers
- A ratio that compares quantities of different types is called a rate
- When two ratios are equal, the mathematical statement of that equality is called a proportion
- Proportions are used when three quantities are given, and the fourth quantity is an unknown

Prerequisites of teaching ratio (discussion)

- What is ratio?
- What topics need to have been mastered before ratio and proportion can be taught?
- Where should we start?
- What is the order of topics (how does the topic progress)?

R3 Express quantity as a fraction of another

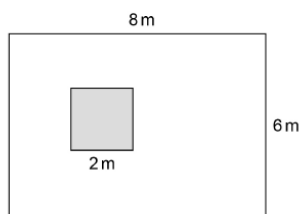
R3

Basic foundation content	Additional foundation content	Higher content only
express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1		

'express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1'

2018 Foundation Paper 1 (R3, A02)

- 7 A rectangular carpet measures 8 m by 6 m
Part of the carpet is covered by a square rug of length 2 m



Not drawn accurately

Show that $\frac{1}{12}$ of the carpet is covered by the rug.

[2 marks]

2018 Paper 1 Foundation Q25 /Higher Q6 (R3, A02)

- 25** The height of Zak is 1.86 metres.
 The height of Fred is 1.6 metres.
- Write the height of Zak as a fraction of the height of Fred.
Give your answer in its simplest form.

[3 marks]

- Needs the correct denominator
- Needs the concept of units cancelling
- Handling decimals in a fraction/ratio/proportion
- A sophisticated understanding is needed

-
- GCSE demands a bit more than just expressing one number as a fraction
 - But it is an important idea
 - Students likely to have met this before getting to ratios
 - But these questions are AO2 and so the concept must be well understood to succeed with them

What do DfE and Ofqual want?

- They wanted to end predictability and rote learning of methods
- The Ofqual assessment objectives are designed to test knowledge, conceptual understanding and application
- There is a requirement from Ofqual for a degree of unpredictability
- Students must be able to think on their feet and there is a limit to what can be achieved through drilling

Thinking about ratio and proportion topics

From its introduction to the type of basic fluency that is required for GCSE:

- manipulatives
- visual representations
- numerical and algebraic methods

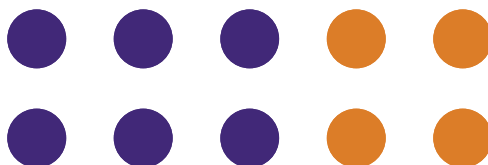
there is an underlying consistency of approach which justifies their grouping in this topic.

Ratio as manipulative



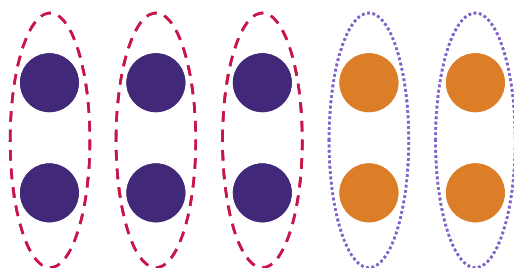
3 : 2

Ratio as manipulative



6 : 4

Ratio as manipulative



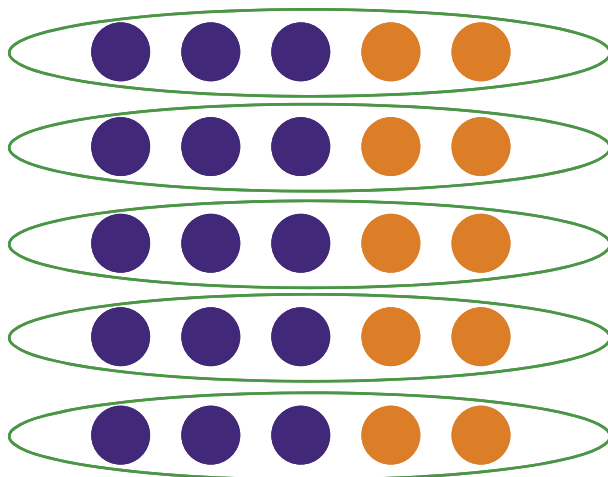
3 : 2

19

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AQA

Ratio as manipulative



3 : 2

20

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What can we ask?



21

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R4 – Ratio Notation

R4

Basic foundation content	Additional foundation content	Higher content only
use ratio notation, including reduction to simplest form		

'use ratio notation, including reduction to simplest form'

22

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2018 Foundation Paper 1 Q22 (R4, A03)

22

Anna plays a computer game.

Each game is a win or a loss.

She wins three quarters of her first 24 games.

She then wins her next 12 games.

For all 36 games, work out the ratio wins : losses

Give your answer in its simplest form.

[3 marks]

Write ratio in the form 1 : n and n : 1

A : B

1 : n

B is n times the size of A

A : B

n : 1

A is n times the size of B

Ratio as a fraction

$$2 : 3$$

$$= \frac{2}{3} : \frac{3}{3}$$

$$= \frac{2}{3} : 1$$

$$= \frac{2}{3}$$

25

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Ratio as a fraction and $n : 1$

$$a : b$$

$$= \frac{a}{b} : 1$$

$$= \frac{a}{b}$$

Ratio written as a fraction:

a is $\frac{a}{b}$ times the size of b

26

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R5 – Proportional Division

R5

Basic foundation content	Additional foundation content	Higher content only
<div>divide a given quantity into two parts in a given part : part or part : whole ratio</div> <div>express the division of a quantity into two parts as a ratio</div> <div>apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)</div>		

Notes: including better value or best-buy problems.

'divide a given quantity into two parts in a given part : part or part : whole ratio express the division of a quantity into two parts as a ratio apply ratio to real contexts and problems'

27

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Fair shares

Anna gets £25. Claire gets 3 times as much. How much does Claire get?

Anna

£25

Claire

£25

£25

£25

28

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As a ratio

Anna : Claire

£25 : £25 £25 £25

1 : 3

Divide £100 in the ratio 1 : 3

1 : 3

:

4 parts

$$£100 \div 4 = £25$$

£25 : £25 £25 £25

2018 Foundation Paper 1 Q24 (R5, A01)

24 Divide 405 in the ratio 4 : 11

[3 marks]



15 parts
 $405 \div 15 = 27$



$$4 \times 27 = 108$$

$$11 \times 27 = 297$$

R6 – Multiplicative Relationships

R6

Basic foundation content	Additional foundation content	Higher content only
express a multiplicative relationship between two quantities as a ratio or a fraction		

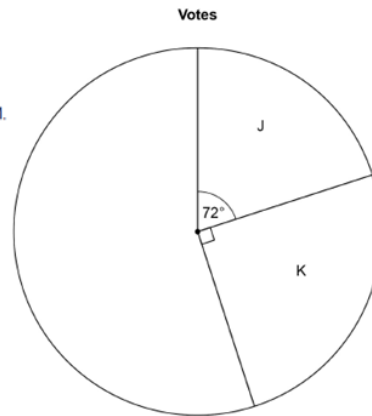
'express a multiplicative relationship between two quantities as a ratio or a fraction'

2018 Foundation Paper 3 Q19a (R6, A01)

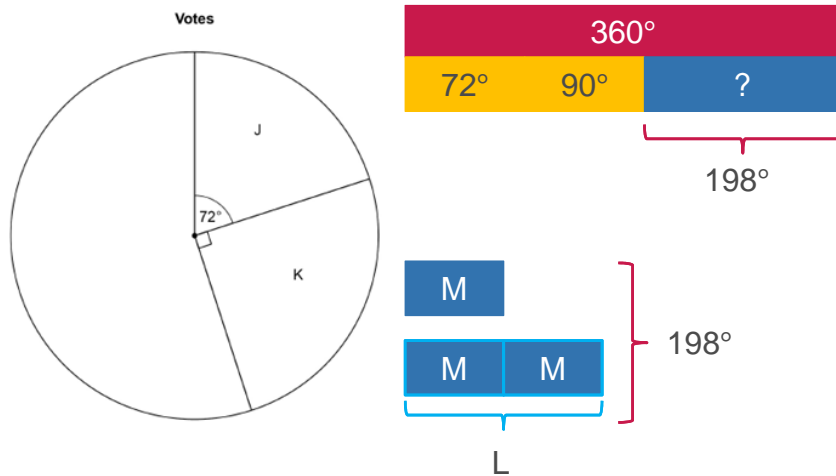
- 19** In an election there were four candidates, J, K, L and M.
Fran is drawing a pie chart to show the results.
The sectors for J and K have been drawn.

- 19 (a)** Twice as many people voted for L as voted for M.
Complete the pie chart.

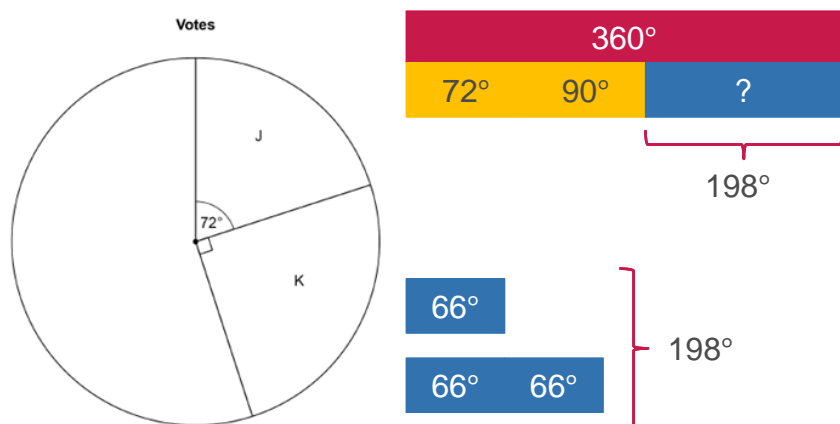
[3 marks]



Twice as many people voted for L as for M



Twice as many people voted for L as for M



35

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2018 Foundation Paper 2H Q3 (R6, A02)

3 y is $1\frac{1}{2}$ times x .

Circle the ratio that is equivalent to $y : x$

[1 mark]

2 : 5

5 : 2

3 : 2

2 : 3

36

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AQA

Thinking about Proportion (discussion)

What is proportion?

- In proportion
- Is proportional to
- As a proportion

Proportion – key concepts

- Equality of ratios
- Multiplicative relationships

Equality of ratios

a is to b as c is to d

$$a : b = c : d$$

$$a : b :: c : d$$

Equality of ratios

$$a : b = c : d$$

$$\Rightarrow \frac{a}{b} : \frac{c}{d}$$

$$\Rightarrow \frac{b}{a} : \frac{d}{c}$$

$$\Rightarrow b : a = d : c$$

$$\times 1.5$$


$$2 : 3$$

$$4.7 : 7.05$$



$$\div 1.5$$

Equality of Ratios

$$a : b = c : d$$

$$\Leftrightarrow a : c = b : d$$

$$\begin{array}{ccc} & 2 : 5.37 & \\ \times 2 \swarrow & & \searrow \div 2 \\ & 4 : 10.75 & \end{array}$$

41

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Equality of ratios

$$a : b = c : d$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow ad = bc$$

$$a : b$$



$$c : d$$

$$2 : 3$$



$$x : 12.42$$

$$3x = 2 \times 12.42$$

$$x = 8.28$$

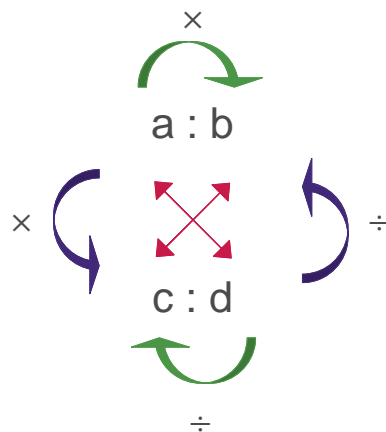
42

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In summary

$$a : b = c : d$$



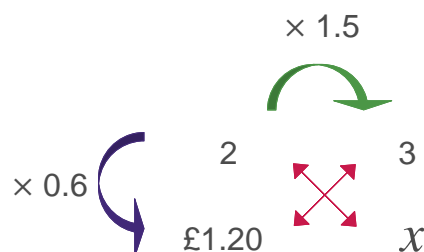
43

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Generalising

In general, any missing value in a multiplicative relationship can be found this way

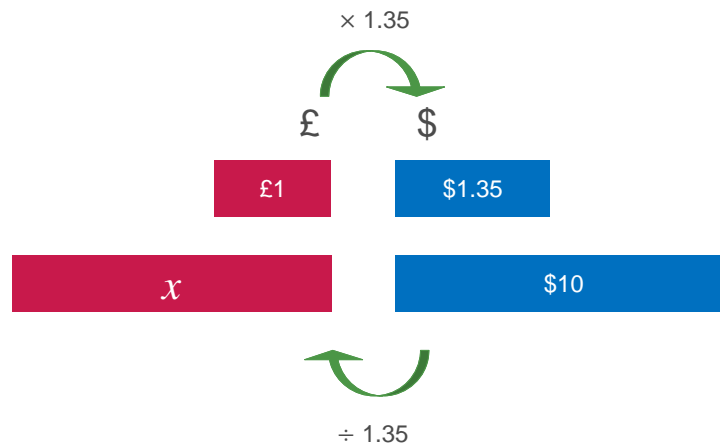


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Visualising - conversion

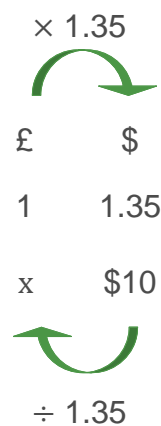


45

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Without the bars...



46

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R7 – Equality of Ratios

R7

Basic foundation content	Additional foundation content	Higher content only
understand and use proportion as equality of ratios		

'understand and use proportion as equality of ratios'

2018 Higher Paper 1 Q14b (R5, A03)

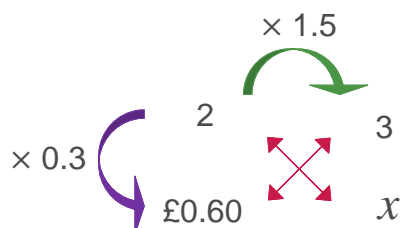
- 14 (b) A different pattern is made using 20 straight lines and 16 arcs.
The straight lines and arcs are made from metal.

20 straight lines cost £12

cost of one straight line : cost of one arc = 2 : 3

Work out the **total** cost of the metal in the pattern.

[3 marks]



2018 Foundation/Higher Paper 2 Q26/8 (R7, A03)

8

Theo starts with savings of £18
James starts with no savings.

Each week from now,

Theo will save £4.50 and James will save £4

In how many weeks will Theo and James have savings in the ratio 15 : 8 ?

[3 marks]

Theo : James

$$\begin{array}{ccc} 15 & & 8 \\ 18 + 4.5n & & 4n \end{array}$$

$$\begin{aligned} 60n &= 36n + 144 \\ n &= 6 \end{aligned}$$

2018 Higher Paper 1H Q14a (R5, A02)

14

Patterns are made using straight lines and arcs.

14 (a)

Pattern A (one row)



Pattern B (two rows)



Lines : Arcs

$$\begin{array}{ccc} 10 & & 9 \\ 5r & & 4r + 4 \end{array}$$

$$\begin{aligned} 45r &= 40r + 40 \\ r &= 8 \end{aligned}$$

More rows are added to Pattern B so that
number of straight lines : number of arcs = 10 : 9

How many rows are added?

Equality of Ratios and Percentages

Equality of ratios and multiplicative reasoning applied to percentages

51

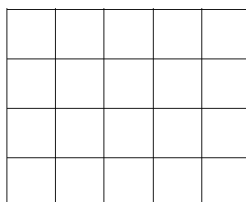
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2018 Foundation Paper 3 Q5b (R9, A02)

5 (b) Shade 10% of this grid.

[1 mark]



R9 - 'define percentage as number of parts per hundred'

52

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2018 Foundation Paper 1 Q19 (R9, A01)

- 19** Circle the percentage that is closest in value to $\frac{1}{3}$ [1 mark]
- 30% 33% 33.3% 33.4%

R9 - 'interpret percentages... as a decimal'

2018 Higher Paper 2 Q4 (R9, A01)

- 4** Work out 40 as a percentage of 10
Circle your answer. [1 mark]
- 4% 25% 300% 400%

R9 - 'express one quantity as a percentage of another ...
work with percentages greater than 100%'

Percentage as a multiplicative relationship

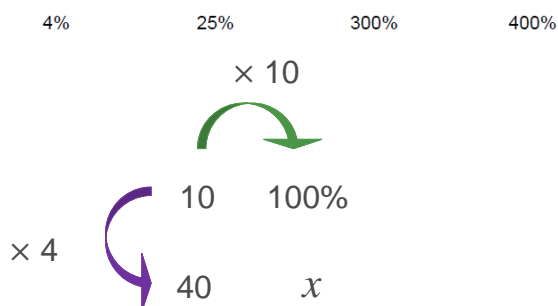
Using the method of equality of ratios for solving:

- find a percentage
- percentage increase
- percentage change (two ways)
- reverse percentage.

2018 Higher Paper 2 Q4 (R9, A01)

- 4 Work out 40 as a percentage of 10
Circle your answer.

[1 mark]



R9

R9

Basic foundation content

define percentage as 'number of parts per hundred'

interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively

express one quantity as a percentage of another

compare two quantities using percentages

work with percentages greater than 100%

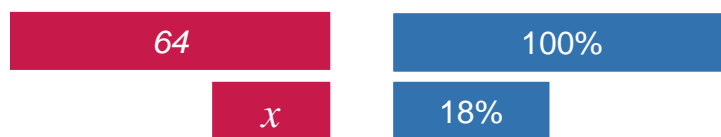
solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics

Notes: see also [N2](#), [N12](#)

'interpret percentages and percentage changes... multiplicatively'

'solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics'

Find 18% of 64



$$\begin{array}{cc} 64 & 100\% \\ x & 18\% \end{array}$$

The diagram shows a proportion with 64 and x on the left, and 100% and 18% on the right. Red arrows cross between the top and bottom rows, indicating the cross-multiplication process.

$$x = \frac{18\% \times 64}{100\%}$$

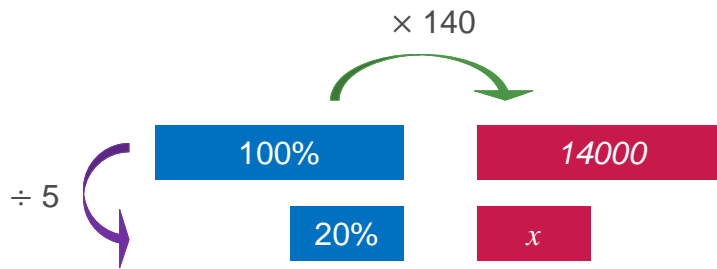
2018 Foundation Paper 1 Q5 (R9, A01)

5

Work out

20% of 14 000

[2 marks]

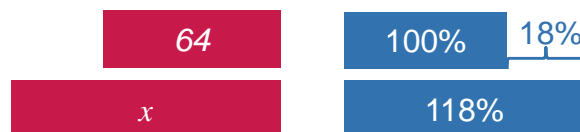


59

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AQA

Increase £64 by 18%



$$\begin{array}{cc} 64 & 100\% \\ x & 118\% \end{array}$$

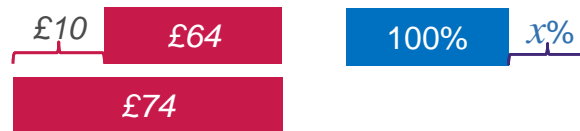
$$x = \frac{118\% \times 64}{100\%}$$

60

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AQA

What is the percentage change from £64 to £74 (method 1)



$$\begin{array}{ccc} \text{£64} & \nearrow & 100\% \\ \text{£10} & \nwarrow & x\% \end{array}$$

$$x = \frac{100\% \times \text{£10}}{\text{£64}}$$

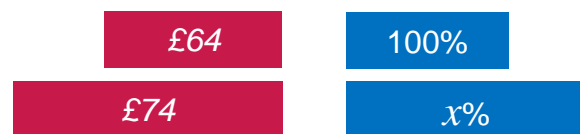
$$x = 15.625\%$$

61

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What is the percentage change from £64 to £74 (method 2)



$$\begin{array}{ccc} \text{£64} & \nearrow & 100\% \\ \text{£74} & \nwarrow & x\% \end{array}$$

$$x = \frac{100\% \times \text{£74}}{\text{£64}} = 115.625\%$$

Therefore the change is 15.625%

62

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2018 Foundation/Higher Paper 2 Q24/Q6 (R9, A03)

6 The table shows information about the population of a city.

Population in 2001	Population in 2011
420 000	480 000

Liam claims,

"From 2011 to 2021 the population of the city will increase by the same percentage as from 2001 to 2011"

He works out,

$$\begin{aligned} \text{population increase from 2001 to 2011} &= 480\,000 - 420\,000 \\ &= 60\,000 \end{aligned}$$

$$\begin{aligned} \text{population in 2021} &= 480\,000 + 60\,000 \\ &= 540\,000 \end{aligned}$$

Does the population of 540 000 match his claim?

You **must** show your working.

[3 marks]

63

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2018 Foundation/Higher Paper 2 Q24/Q6 (R9, A03)



$$\begin{array}{cc} 420000 & \nearrow 100\% \\ & \searrow x\% \\ 60000 & \end{array}$$

$$x = \frac{100\% \times 60\,000}{420\,000}$$

$$x = 14.3\% \text{ to 1 dp}$$

64

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2018 Foundation/Higher Paper 2 Q24/Q6
(R9, A03)



$$\begin{array}{cc} 480\,000 & 100\% \\ x & 114.3\% \end{array}$$

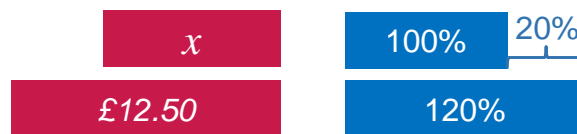
$$\begin{aligned} x &= \frac{114.3\% \times 480\,000}{100\%} \\ &= 548\,640 \end{aligned}$$

65

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Price increased by 25% to £12.50, what was original price?



$$\begin{array}{cc} x & 100\% \\ £12.50 & 120\% \end{array}$$

$$x = \frac{100\% \times £12.50}{120\%}$$

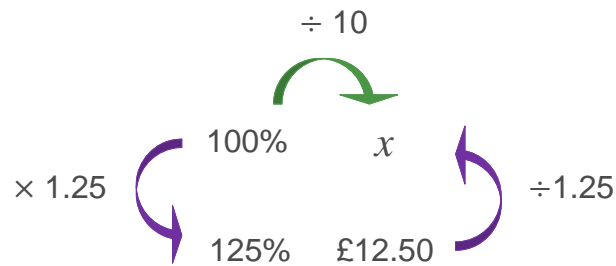
$$x = £10$$

66

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Price increased by 25% to £12.50, what was original price?

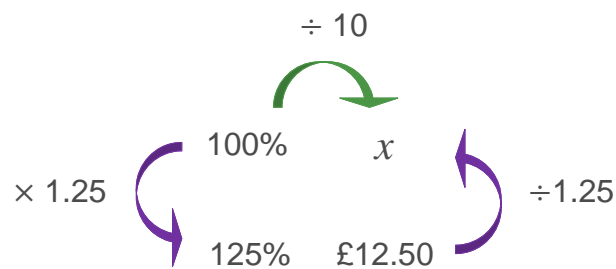


67

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AQA

Price increased by 25% to £12.50, what was original price?



68

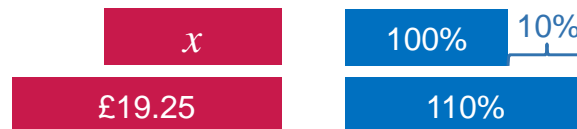
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AQA

2018 Foundation/Higher Paper 3 Q28/Q9 (R9, A03)

- 28 The cost of a ticket increases by 10% to £19.25
Work out the original cost.

[3 marks]



$$\begin{array}{cc} x & 100\% \\ \swarrow \searrow & \swarrow \searrow \\ £19.25 & 110\% \end{array}$$

$$x = \frac{100\% \times £19.25}{110\%} = £17.50$$

R16 – Repeated multipliers and iteration

R16

Basic foundation content	Additional foundation content	Higher content only
	set up, solve and interpret the answers in growth and decay problems, including compound interest	and work with general iterative processes

'set up, solve and interpret the answers in growth and decay problems, including compound interest and work with general iterative processes'

Application to units and scales

- Equality of ratios and multiplicative reasoning applied to units and scales.
- Leading to dimensions and direct proportion.

R1 – changing units

R1

Basic foundation content	Additional foundation content	Higher content only
change freely between related standard units (eg time, length, area, volume/capacity, mass) and compound units (eg speed, rates of pay, prices) in numerical contexts	compound units (eg density, pressure) in numerical and algebraic contexts	

'change freely between related **standard units** (eg time, length, area, volume/capacity, mass) and **compound units** (eg speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts'

R2 – scaling

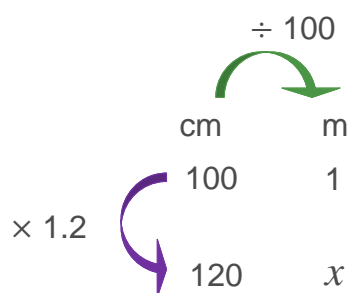
R2

Basic foundation content	Additional foundation content	Higher content only
use scale factors, scale diagrams and maps		

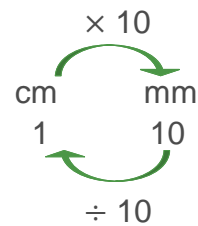
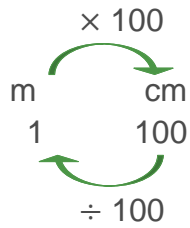
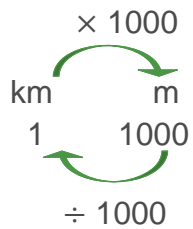
Notes: including geometrical problems.

'use scale factors, scale diagrams and maps
(including geometrical problems)'

Units – 120 cm to m



Repeated conversion in a system



75

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AQA

2018 Foundation Paper 1 Q18 (R2, A01)

18 Circle the ratio which is the same as the scale 1 cm represents 1 km **[1 mark]**

1 : 100

1 : 1000

1 : 10 000

1 : 100 000

76

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AQA

2018 Foundation Paper 3 Q4 (R1, A01)

4 Circle the shortest length.

[1 mark]

1200 cm

0.13 km

110 m

140 000 mm

77

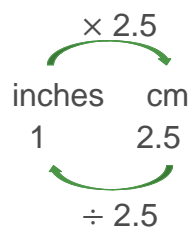
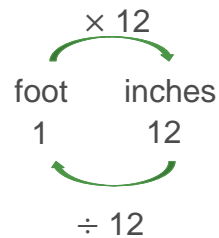
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2018 Foundation Paper 2 Q9 (R1, A03)

9 In this question, use
1 foot = 12 inches
1 inch = 2.5 centimetres
Change 5 feet 8 inches to centimetres.

[3 marks]



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2018 Foundation Paper 3 Q4 (R1, A01)

12 How many minutes are there in $5\frac{1}{4}$ hours?

Circle your answer.

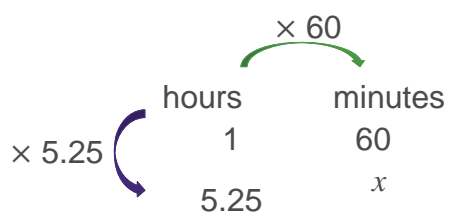
[1 mark]

315

325

515

525

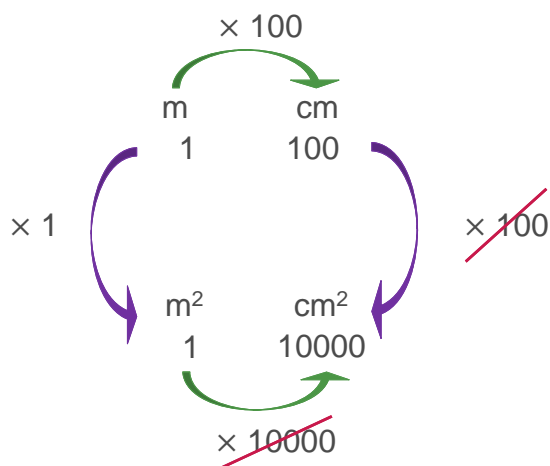


79

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AQA

Changing dimensions

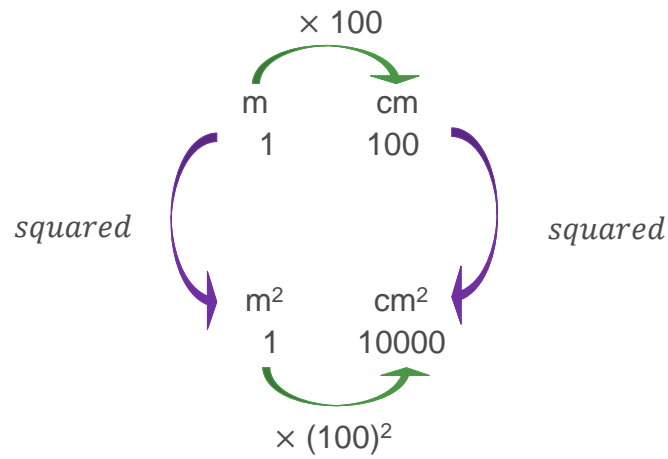


80

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AQA

Changing dimensions



81

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AQA

2018 Higher Paper 1 Q13 (R1/R11, A01)

13

Circle the volume that is the same as 15 cm³

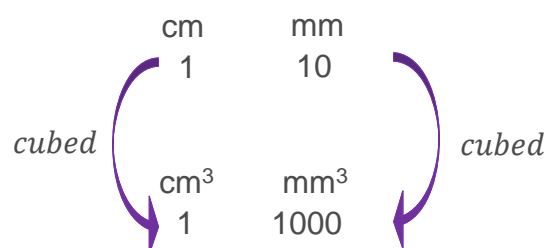
[1 mark]

15 000 mm³

1.5 mm³

0.0015 mm³

150 mm³

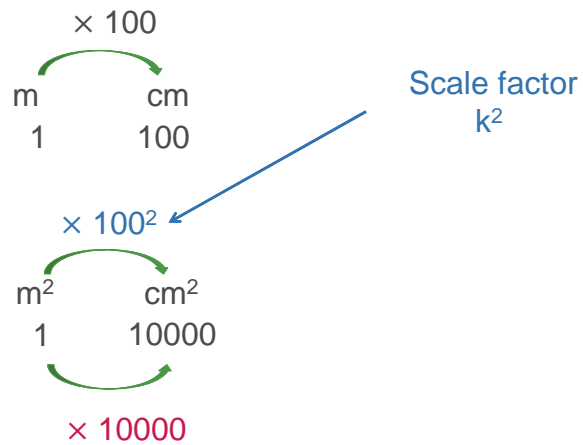


82

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AQA

Changing dimensions

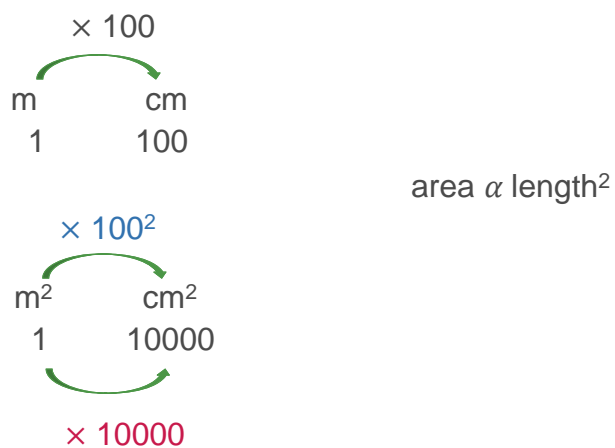


83

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Changing dimensions



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Any questions?



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Thank you

Notes

Notes

Notes

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