AQA Realising potential

Teaching guide: Astrophysics

This teaching guide provides background material for teachers preparing students for the optional topic of Astrophysics (section 3.9) for our A-level Physics specification (7408). It gives more detail on topics that teachers may not be familiar with and should be used alongside the specification. This guide is not a comprehensive set of teaching notes. It has been designed in response to feedback from teachers who taught the previous specification.

Contents

Introduction

The Astrophysics option is intended to give students an opportunity to study an area of Physics that many find fascinating. It includes a study of the main methods of acquiring information, describes how such information is used to analyse and catalogue astronomical objects, and outlines some modern theories on cosmology. The topic provides a grounding in both the practical and theoretical study of astrophysics.

In the first section, students see how some of the optics they studied earlier in the specification are applied to the design of telescopes. The relative merits of the two basic methods – reflecting and refracting – are discussed, together with the influence telescope design has on the quality of the image produced. The section continues by studying how information from some of the other parts of the electromagnetic spectrum can be gathered. Comparisons are made between the devices in terms of their design and application.

In section 2, students have the opportunity to learn how the observed information is used to describe stars. The idea of apparent magnitude is introduced, with its mathematical relationship to absolute magnitude and distance leading to a discussion of the light year and parsec. How the temperature of a star can be measured is then described, before combining absolute magnitude and temperature in the Hertzsprung-Russell (H-R) diagram. The different types of stars – and other astronomical objects – are then put in a context of observable properties rather than theoretical behaviour. The use of supernovae to measure distances, and the implications these measurements have on ideas of dark energy, provide an opportunity for students to experience some of the theories of modern cosmology. This section concludes with black holes and how their 'size' can be calculated.

The final section puts astrophysics into a wider context. The analysis of motion using the Doppler Effect is described, and applied to various situations, including Hubble's Law and the age of the Universe. This allows for a discussion of evidence for the Big Bang. The discovery of Quasars, and some of their properties, is investigated. Finally, the principles behind the discovery of exoplanets are introduced.

Astrophysics is a subject undergoing constant change and development. This course attempts to reflect this by including very recent ideas such as dark energy and the discovery of exoplanets. It is also an opportunity to put scientific theory into the context of discovery, as better techniques provide more information about exotic and fascinating phenomena.

As with the other options and the core A-level topics, students are expected to use their earlier knowledge and understanding in topics which make use of such knowledge and understanding. Each of the sections may be linked with core topics, thereby allowing these topics to be reinforced. For example, the work on resolving power links closely to the work on diffraction and the work on black holes can be related to the topic on gravitational fields.

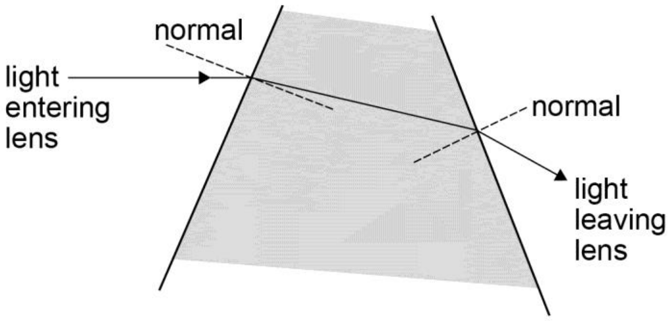
Section 1 Telescopes

a) Astronomical telescope consisting of two converging lenses

The converging lens

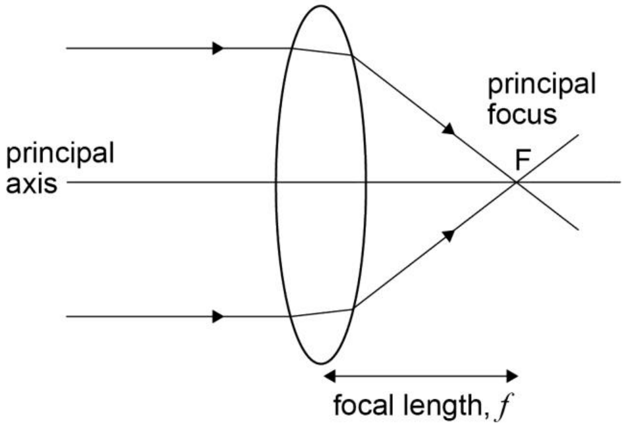
There is some evidence to suggest that the earliest lenses may have been made over 3000 years ago, in order to start fires by focusing sunlight. However, it was not until the 17th century that lenses were being used to make telescopes.

The process of refraction is covered in the core A-level. When light passes from air into the lens, it slows down. Light which is not incident normally gets bent towards the normal to the boundary. As the light leaves the lens and enters the air, it will bend away from the normal as it speeds up again.



A convex lens is designed to have the correct curvature needed to bring parallel light to a point focus. This diagram shows light entering a lens parallel to an axis of symmetry called the principal axis. Such rays of light are called axial rays. The point on the axis where the rays cross is called the principal focus, F.

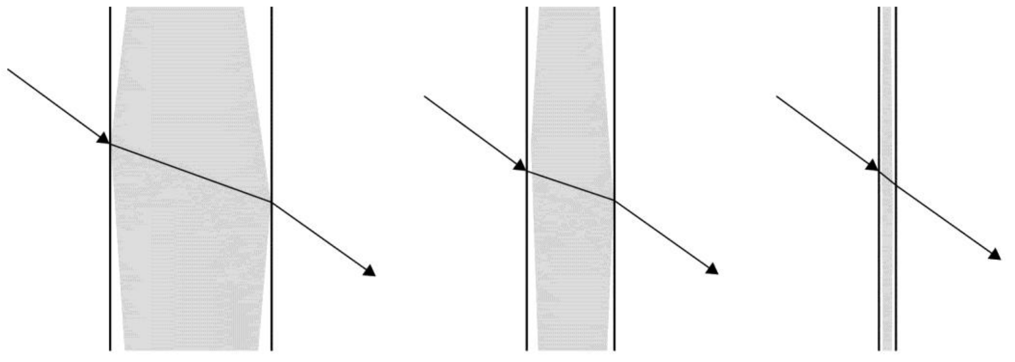
The distance from the centre of the lens to the principal focus is called the focal length, f.



The refraction of the light passing through the lens is shown in this diagram. When drawing ray diagrams, the situation is simplified by assuming that the lens is very thin so that the ray is only shown bending once, in the middle of the lens.

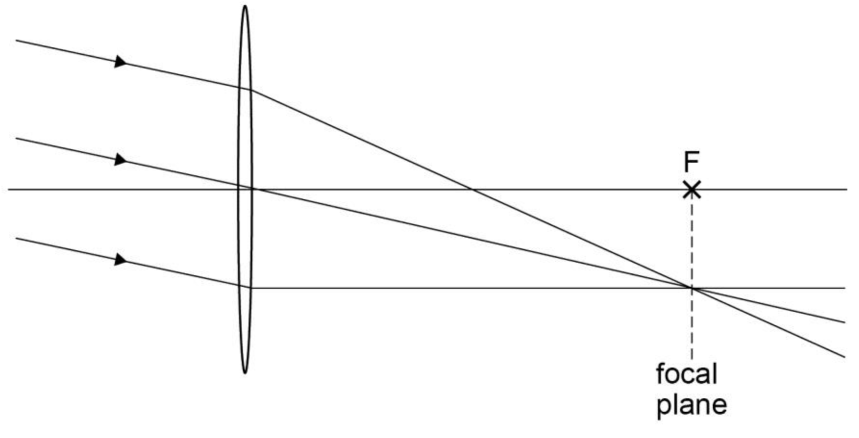
The other advantage of assuming that the lens is thin is that a ray of light passing through the centre of a thin lens (the optical centre) effectively does not bend.

The paths of rays through a thin converging lens



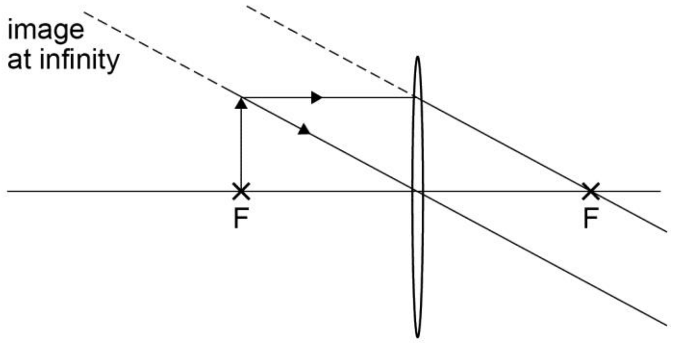
This diagram shows that, as the lens is made thinner, the path of the ray through the centre of the lens is deviated less. This is useful in ray diagrams because the path of this ray can be drawn as a straight line when the lens is thin.

If the parallel rays entering the lens are non-axial, they will be brought to a focus in the focal plane.



Notice that, with this thin lens, the rays are only shown bending in the middle of the lens, and that the central ray passes straight through without bending.

A similar diagram is produced if the rays are drawn for an object placed in the focal plane of a converging lens.

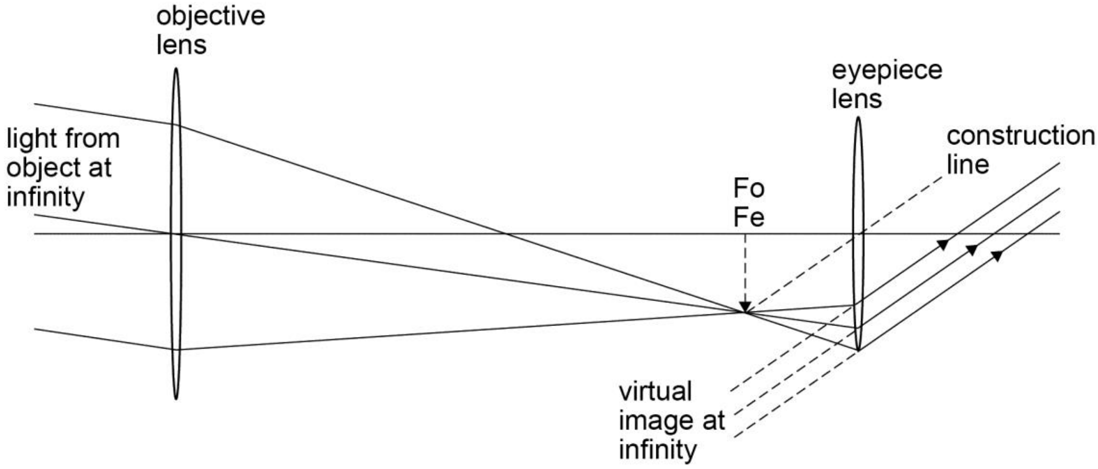


In this situation, the rays leave the lens parallel. The image can only be seen by looking through the lens – it is a virtual image at infinity.

Ray diagram for an astronomical refracting telescope

We can use this behaviour of the light passing through lenses to show how an image would be formed. The refracting telescope consists of two lenses. The first lens (the objective) collects the light from a distant object and brings it to a focus in its focal plane. The second lens (the eyepiece) uses this light to form a magnified image at infinity. This is achieved by placing the lens so that its focal plane coincides with the focal plane of the objective.

The required ray diagram is given below.



The telescope is in normal adjustment because the image is formed at infinity, ie the light rays leave the telescope parallel. This means that the focal points of the two lenses coincide, making the distance between them equal to the sum of the focal lengths.

When drawing a ray diagram in an examination, students should:

- use a pen and ruler
- start by drawing a principal axis, and drawing and labelling the lenses as lines or very thin ellipses
- mark and label the common principal foci

- draw an off-axis ray through the centre of the objective to the eyepiece
- draw an intermediate image from the common principal foci to this ray, at right angles to the principal axis
- draw a construction line from the end of the intermediate image through the centre of the eyepiece
- draw two more rays parallel to the first before the objective
- continue these two rays to the eyepiece, crossing at the end of the intermediate image
- draw the continuation of the three rays from the eyepiece, parallel to the construction line.

The diagram includes the intermediate image (at the point where the two focal points coincide), but this isn't essential.

Common problems with the diagram include:

- axial rays rather than non-axial rays being drawn
- the central ray bending at the objective lens
- the rays bending at the intermediate image (at Fo, Fe)
- the rays not leaving the eyepiece parallel to each other or parallel to the construction line
- the principal foci labelled where the rays cross, rather than on the principal axis.

Another difficulty some students have with this diagram is understanding that the three parallel rays come from the same point on the object, ie the rays may not be parallel in reality – but are too close to being parallel to make any difference.

Angular magnification

The angular magnification is based on the angles subtended by the object and image (clearly, if they are both at infinity, the magnification cannot be based on their lengths). Students need to have an idea of radian measure to help them deal with this. This unit is important – the two angles need to be in the same unit.

The angle, in radians, subtended by an object of height, h, a distance, d, away is given by:

$$\theta = \frac{h}{d}$$

provided h and d are in the same units. An interesting thing to show is that the Sun and the Moon both subtend approximately the same angle at the Earth – this is why solar eclipses are so spectacular. You can use the data to verify this.

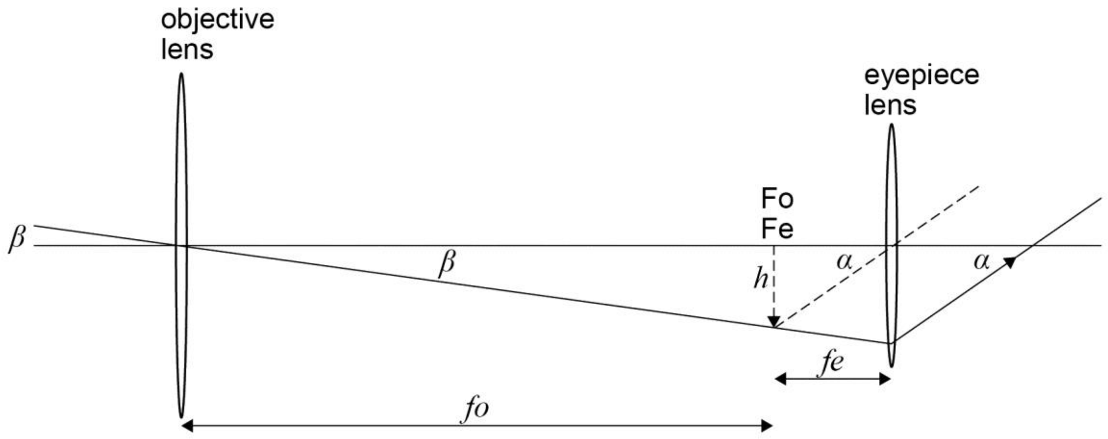
Mean distance to Moon	= 380 000 km
Mean diameter of Moon	= 3500 km
Mean distance to Sun	= 150 000 000 km
Mean diameter of Sun	= 1 400 000 km

The light leaving the telescope causes the image to subtend a bigger angle than the object – it is magnified. The definition of angular magnification is

$$\textit{angular magnification, } M = \frac{\textit{angle subtended by image at eye}}{\textit{angle subtended by object at unaided eye}}$$

Note that M is also used for the absolute magnitude later in the specification, so students need to be sure which M is being referred to in a question.

Looking at a simplified ray diagram



It should be clear from this diagram that the angular magnification is $\frac{\alpha}{\beta}$, and that the image is inverted.

For small angles, $\alpha = \frac{h}{fe}$ and $\beta = \frac{h}{fo}$, so $\frac{\alpha}{\beta} = \frac{fo}{fe}$. Again this derivation is not needed but is straightforward, and it does make use of the idea of the 'small angle approximation' which is always useful to emphasise.

From this formula, and the ray diagram, it is clear that a high magnification telescope requires a long objective focal length and short eyepiece focal length. Refracting telescopes are relatively easy to make in the lab using a couple of lenses, a metre ruler and plasticine and this relationship can be easily investigated.

The equations

$$M = \frac{f_0}{f_e}$$

and

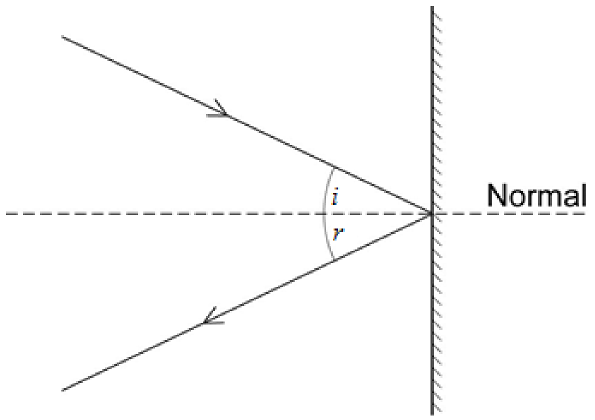
distance from objective to eyepiece = $fo + fe$

can be used together to help design particular telescopes. It is fairly common to see past questions ask about this.

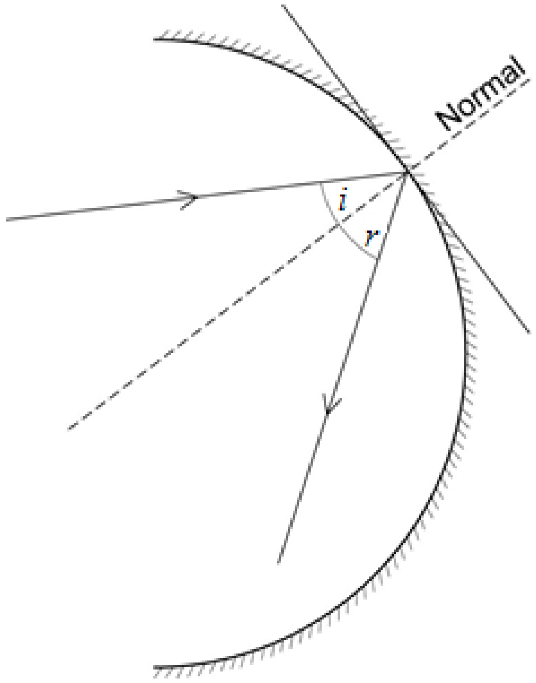
b) Reflecting telescopes

The ideas behind the reflecting telescope start with the simple law:

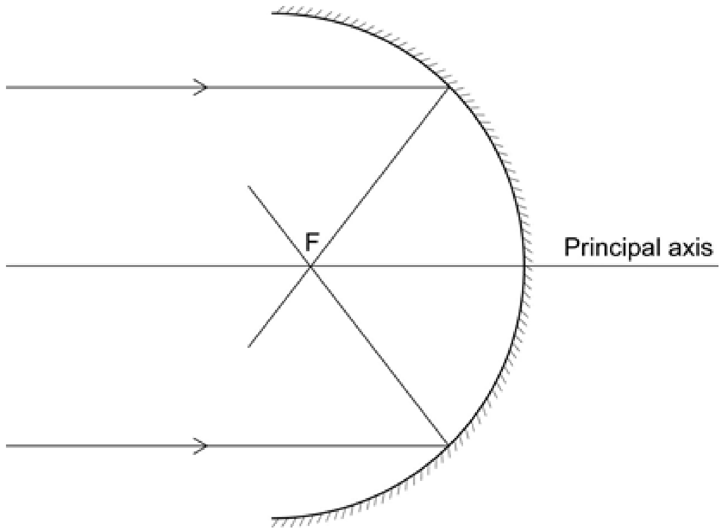
angle of incidence *(i)* = angle of reflection *(r)*



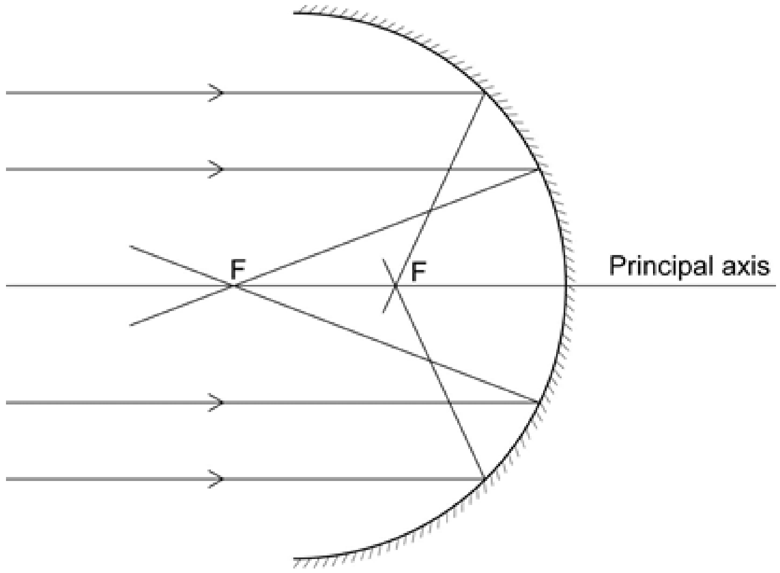
This can be applied to curved surfaces too.



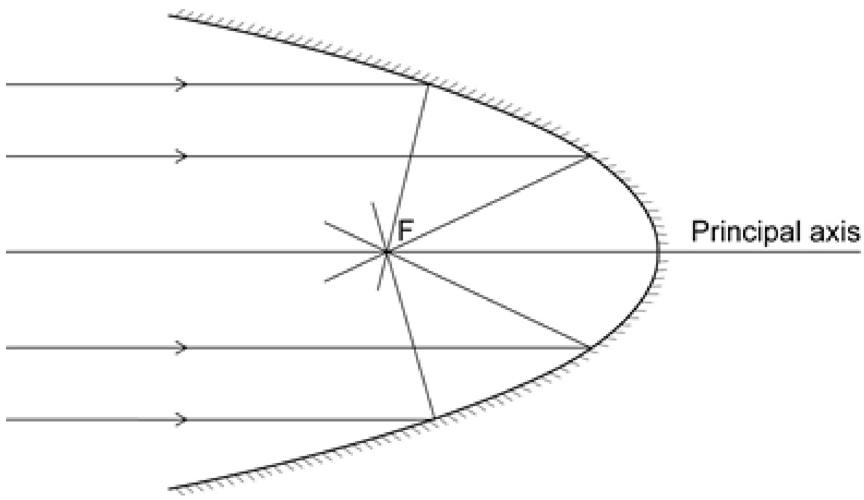
This leads to the idea that a concave mirror will bring parallel light to a point focus. The focal point, F, is taken to be at half the distance to the centre of curvature.



Unfortunately, circular (or spherical) mirrors are not so simple. Off-axial rays are brought to a focus closer to the mirror. This means that the focal point is different for different rays of light. This results in a blurred image and is called spherical aberration. This ray diagram is obtained fairly easily by following the reflection law, having drawn a circular mirror on a piece of paper.



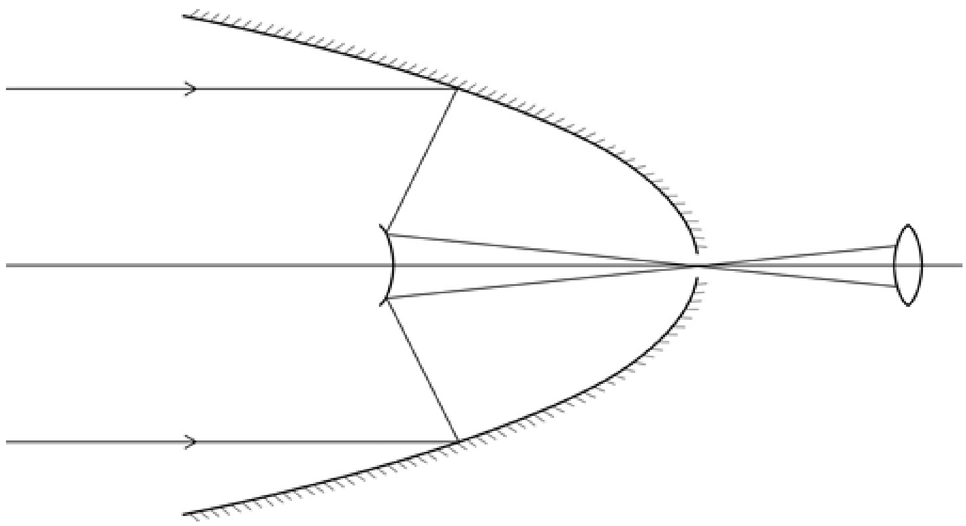
The problem of spherical aberration is overcome using a parabolic mirror (the more mathematical students may recognise a parabola as an ellipse with one focus at infinity).



The Cassegrain telescope

There have been several designs of reflecting telescopes, but the specification requires knowledge only of the Cassegrain arrangement. Parallel light enters the telescope from a distant object. The large parabolic primary reflector is used to collect the light and bring it to a focus.

A secondary convex reflector is used to reflect the collecting light out through a hole in the primary parabolic mirror. The light then enters an eyepiece. The whole arrangement is shown below. Notice that, in contrast with the ray diagram for the refracting telescope, only two rays are required, and that they are drawn initially parallel to the principal axis. The curvature of the primary mirror in this diagram has been exaggerated to emphasise its parabolic nature.



There are several common errors that are worth pointing out:

- students draw the two halves of the primary as two separate mirrors – ie their curve does not look like a continuous parabola
- the secondary reflector is drawn plane, or even concave
- the eyepiece is left out
- the rays are drawn crossing before they hit the secondary mirror.

Although it is difficult to do using simple software, it is worth including the shading in all of these diagrams to show which side is not the reflecting surface.

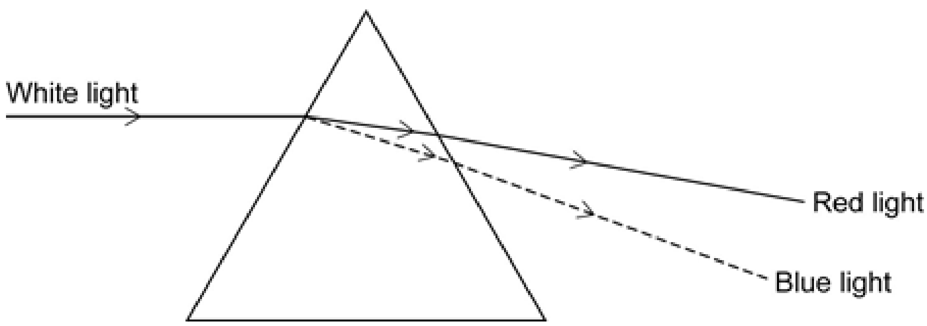
Relative merits of reflectors and refractors

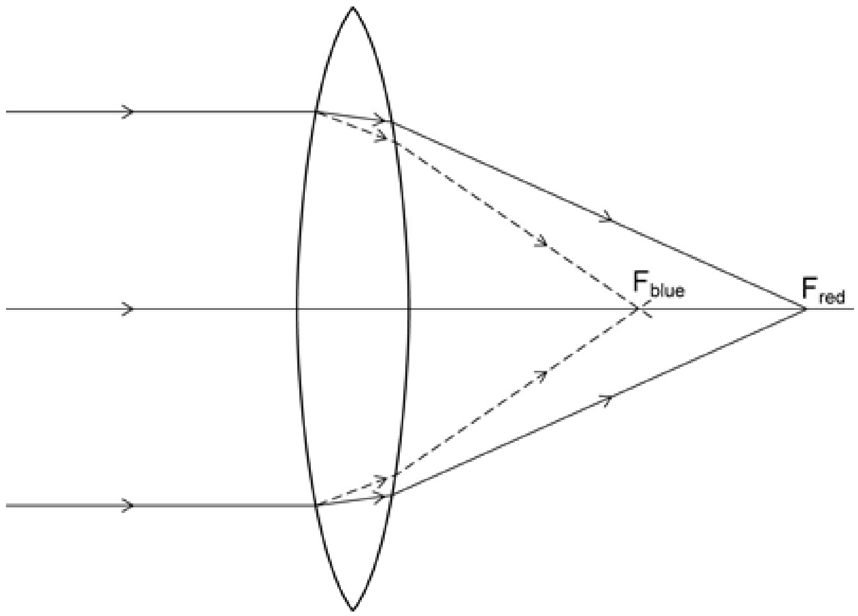
The largest (and therefore best) telescopes are reflectors. There are several reasons for the dominance of reflectors in modern telescopes.

Reflectors can be made much larger than refractors because a mirror can be supported from behind, whereas a lens must be supported at the edge. A large lens is likely to break under its own weight. The advantages of using larger diameter telescopes are discussed in a later section.

There are two important problems associated with telescopes – chromatic aberration and spherical aberration.

Mirrors do not refract light and therefore do not suffer from chromatic aberration. The edge of a lens can be regarded as a slightly curved triangular prism. White light is dispersed – ie split into its different colours – resulting in different focal points for different colours or wavelengths. The focus for blue (or violet) light is closer to the lens than that for red light. This can be seen in these diagrams:





The resulting images tend to have multi-coloured blurred edges.

Spherical lenses suffer from spherical aberration too. It has already been shown that, using a parabolic mirror, this can be eliminated in reflectors.

There are some problems with reflectors. The secondary mirror and the 'spider' (framework) holding it in place both diffract the light as it passes, leading to a poorer quality image. Also, there is some refraction and therefore chromatic aberration eventually, in the eyepiece used to view the final image. Overall, however, it is clear that reflecting telescopes have the edge in design.

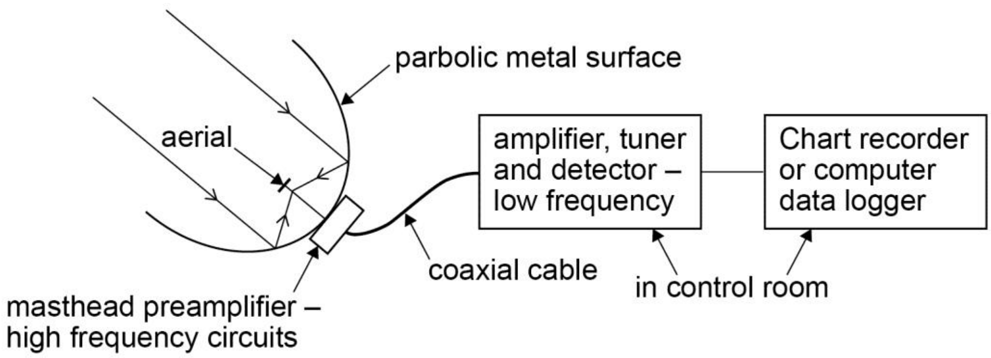
One common misconception associated with the secondary mirror is that it blocks the light (which it does) resulting in the central portion of the image being missing (which is not the case). There will be a slight reduction in the amount of light, but the light rays from a distant source are parallel, so that even light from a source on the mirror axis will reach the secondary and be collected.

It is also worth noting that non-optical differences, such as cost, would not gain credit in the examination.

c) Single dish radio telescopes, I-R, U-V and X-ray telescopes

Ground-based observations are severely affected by the atmosphere for most of the electromagnetic spectrum. Two of the regions of the spectrum least affected are visible wavelengths, dealt with earlier, and radio waves.

Radio telescopes



The details of this diagram will not be examined.

The design of a single dish radio telescope is very similar to that of a reflecting optical telescope. The parabolic metal surface (reflecting dish) reflects the radio waves to an aerial without any spherical aberration. There is no need for a secondary reflector as the aerial can be placed at the focal point. Information is then transmitted for analysis.

As will be seen later, a simple calculation shows that, despite their large size, the resolving power of radio telescopes tends to be quite poor. This can make it hard to identify the source of radio waves. For example, in the early 1960s lunar occultation was needed to identify a particular source of radio waves, and this led to the identification of the first quasar, 3C-273.

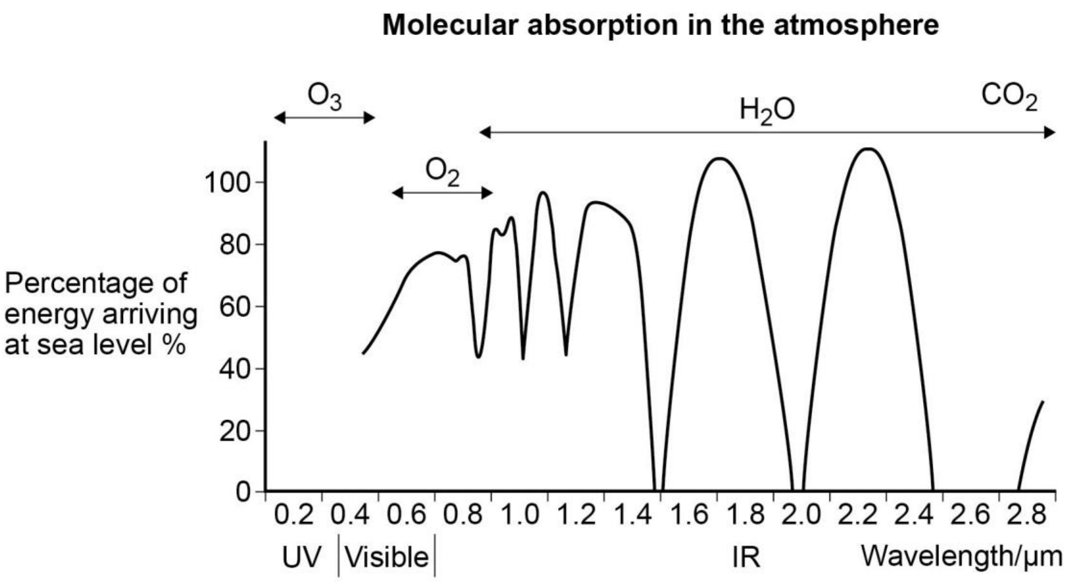
Man-made interference can also cause a big problem for radio astronomy. Radio transmissions, mobile phones, radar systems, global navigation satellites, satellite TV and even microwave ovens all contribute to making the detection of extra-terrestrial objects difficult.

It is difficult to deal with interference from orbiting satellites, but the effect of microwave ovens is minimised by building radio telescopes away from centres of population. Linking telescopes together also helps, as local interference can be removed from individual detectors, enhancing the common signal from the distant sources.

One aspect of the design of radio telescopes is worth mentioning. Because the wavelengths of the radio waves are relatively long, the reflecting dish of the telescope can be made from mesh rather than solid metal, decreasing the weight and cost. Provided the mesh size is smaller than $\lambda/20$, radio waves are reflected by, rather than diffracted through, the mesh.

I-R, U-V and X-ray telescopes

Ground-based telescopes are largely restricted to the visible and radio regions of the electromagnetic spectrum.



This graph shows the effect that water vapour, carbon dioxide, oxygen and ozone have on the IR, visible and U-V spectrum as the radiation passes through the atmosphere. This explains why telescopes detecting these parts of the em spectrum are placed on balloons, in orbit, at the top of high mountains, or in very dry areas (such as the Atacama Desert).

Perhaps surprisingly, X-rays and gamma rays are also absorbed by the atmosphere. Some of the earliest satellites were used to carry X-ray and gamma ray detectors. Detection of these high-energy photons allows analysis of some of the most violent and exotic phenomena in the Universe, and may provide data about its composition, age and expansion rate.

Information about some typical telescopes together with links for further information are given below. A quick internet search would reveal many more.

em spectrum	telescope	position	examples of use
Infrared	UKIRT 3.8 m diameter http://www.ukirt.hawaii.edu/ Spitzer 0.85 m diameter http://spitzer.caltech.edu/	4200 m above sea level on Mauna Kea, Hawaii earth-trailing solar orbit	finding exoplanets observing after-effects of gamma ray bursts, observing nebulae
ultraviolet	GALEX 0.5 m diameter www.galex.caltech.edu/ COS (2.4 m diameter) http://hubblesite.org/	low Earth orbit (ht 700 km) on board the HST low Earth orbit (ht 570 km)	star formation rate in distant galaxies analysing quasars
X-ray	Chandra 'grazing' mirrors http://chandra.harvard.edu/ XMM-Newton 'grazing' mirrors http://xmm.esac.esa.int/	Earth orbit (ht 140 000 km) Earth orbit (ht 30 000 km)	discovering supermassive black holes observing stellar remnants

It is worth noting that the STIS (Space Telescope Imaging Spectograph) analyses wavelengths of light from IR, visible and UV. The light is collected by the 2.4 m Cassegrain reflector on the Hubble Space Telescope. In fact, it is only the X-ray telescopes which require a significantly different design – 'grazing' mirrors, details of which can be found on the Chandra and XMM-Newton websites.

For an interesting comparison of the information obtained from different telescopes, go to en.wikipedia.org/wiki/File:800crab.png. It contains 6 different images of the crab nebula including visible (Hubble), I-R (Spitzer) and X-ray (Chandra).

Although they are not on the specification, it is worth mentioning gamma-ray telescopes at this point. The occurrence of gamma-ray bursts is yet another aspect of astronomy which has caused controversy. Detection of the particles and radiation produced by these gamma rays when they hit the upper atmosphere allows for ground-based observations. It is the acronyms used for the various experiments which students may find interesting too – for example, 'CANGAROO' (Collaboration of Australia and Nippon for a Gamma Ray Observatory in the Outback).

The table shows that many modern telescopes are put into space. There are three main reasons for this

- the absorption of the electromagnetic waves by the atmosphere
- the light pollution and other interference at ground level
- the effect the atmosphere has on the path of the light as it passes through.

Clearly there are disadvantages to putting telescopes into space. It is much more difficult to correct and maintain telescopes in space. Most of the telescopes that have been sent into space in the past 40 years have been decommissioned. This includes Herschell, an I-R telescope in heliocentric orbit, which has been decommissioned as it has run out of the coolant necessary for its operation. (See en.wikipedia.org/wiki/List_of_space_observatories for lists of telescopes and whether they are still working). It should be noted that many telescopes operate beyond their expected time, and modifications can be made to correct faults either by direct intervention (as with the HST) or by modifications of the mission objectives (as with Kepler).

The table on the previous page also includes the diameter of the telescopes. This allows for a comparison of collecting power, and with the wavelength information, the resolving power of the different telescopes.

These are discussed in the next section.

d) Advantages of large diameter telescopes

Larger aperture diameter telescopes are desirable for two main reasons:

- greater collecting power so images are brighter
- better resolving power so images are clearer.

As technology has improved, larger and larger diameter telescopes have been built. As has already been stated, it is easier to build large diameter mirrors than large lenses as the mirrors can be supported from behind but lenses need to be supported at their edges. The largest operational refractor is the 102 cm Yerkes, built in 1895. By contrast, reflecting telescopes with diameters of 10 m or more have been built, using segmented mirrors rather than single mirrors. The planned Giant Magellan Telescope will have a diameter of 24.5 m made up of 7 segments. (See gmto.org/overview/ for more information).

The largest telescopes in the world are radio receivers. The famous Arecibo radio telescope in Puerto Rico has a diameter of 305 m. The Five Hundred Metre Aperture Spherical Telescope (FAST) is currently under construction in China. (fast.bao.ac.cn/en/FAST.html). However, the effective diameter of radio telescopes can be increased even further by linking them together. The VLA, EVLA, VLBA are examples of extended grouped radio telescopes. The Square Kilometre Array (SKA) is under construction and will be in the southern hemisphere extending out over a distance of more than 3 000 km from its central array.

Collecting power

The rate at which useful energy is received by a telescope is its collecting power and depends on the area of the collecting surface. This can usually be taken to be the primary mirror or objective lens, but in general is just called the aperture.

As the area, A, of a circular surface of diameter d is given by $A = \pi (d/2)^2$, the collecting power is proportional to the diameter^2. Even a small pair of binoculars can be used to show the significance of increasing collecting area when viewing faint objects.

With greater collecting power, fainter objects can be seen. In general terms this also means that objects at greater distances can also be seen. The intensity of light from a point source decreases inversely with the square of the distance. If a telescope can be used to see an object at a certain distance, one with double the diameter could be used to see it from twice the distance.

Furthermore, as it takes time to reach us from these objects, larger diameter telescopes are also often described as allowing us to see further back in time.

However, it is worth noting that, as well as increasing aperture size, there are other ways of increasing the amount of useful energy received from a source. This includes increasing exposure times and using very sensitive detectors, such as CCDs.

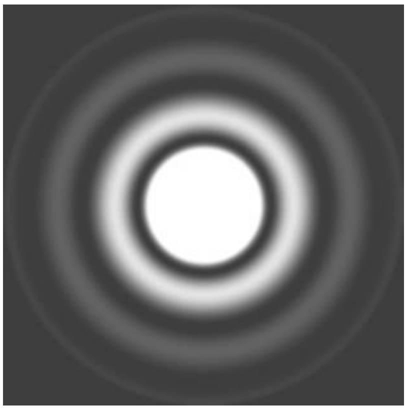
Minimum angular resolution

The other main improvement with larger diameter telescopes is the resolving power, or the minimum angular resolution. This is related to the diffraction produced as light enters the telescope.

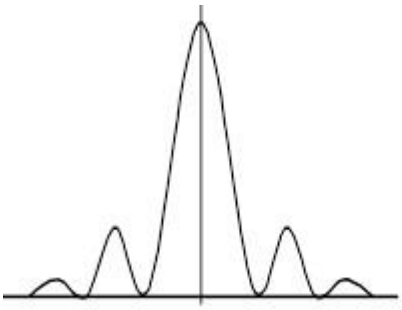
The diffraction pattern produced by a circular aperture

When a wave passes through an aperture or past an obstacle it is diffracted. When monochromatic light passes through a very narrow slit, this diffraction can result in interference fringes.

Rotating that pattern about its centre produces an approximation for the pattern for a circular aperture. The large central maximum is called the Airy Disc, and it is twice as wide as the further maxima in the pattern.



Students should be able to link the picture above with the graph below. The vertical axis is intensity or brightness, and the horizontal axis is angle, θ, or sin θ. For small angles these two are approximately equal, if θ is in radians.



For a single slit, minima in the pattern are at angles given by

$$\sin\theta = \frac{n\lambda}{D}$$

where n is the 'order' of the minimum, λ is the wavelength of the light and D is the width of the slit.

The Rayleigh Criterion

When light from an object enters a telescope it is diffracted, resulting in loss of detail in the image. The aperture of the telescope is assumed to be the same diameter as the objective.

How much detail the telescope can show is called its resolution.

It is easier to consider two objects which are very close together. Is the telescope capable of resolving them into two images, or will the diffraction patterns overlap so much that they will be seen as a single image?

This question is answered by considering the Rayleigh Criterion.

'Two objects will be just resolved if the centre of the diffraction pattern of one image coincides with the first minimum of the other'.

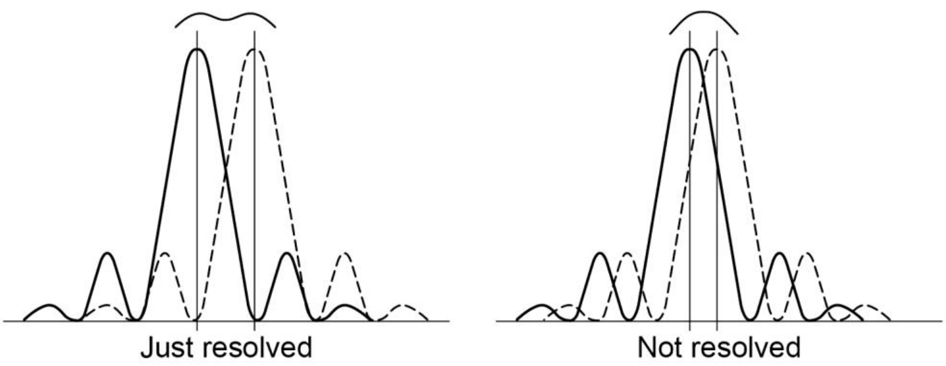
Using the equation from above gives a minimum angular separation, θ, given by:

$$\theta \approx \frac{\lambda}{D}$$

where D is the diameter of the objective. The equation for a circular aperture is actually

$$\theta = 1.22\,\frac{\lambda}{D}$$

which helps explain the "approximately equals" sign.



The problems with resolving power

When using a telescope, the better the resolution, the smaller the angle θ, and therefore the greater the detail which can be seen. In the examination, a question which asks the students to calculate the resolving power in a particular situation requires the student to calculate θ.

This angle is better referred to as the minimum angular resolution of the telescope.

Using the term ‘resolving power’ can cause students several problems.

- Some students expect the resolving power to be bigger for a smaller angle, which is reasonable. To achieve this, they invert their calculated angle. This is not penalised provided it is clear what the student is doing.
- The unit of θ is the radian, rad. Seeing the word ‘power’ encourages many students to give it the unit watt, W.

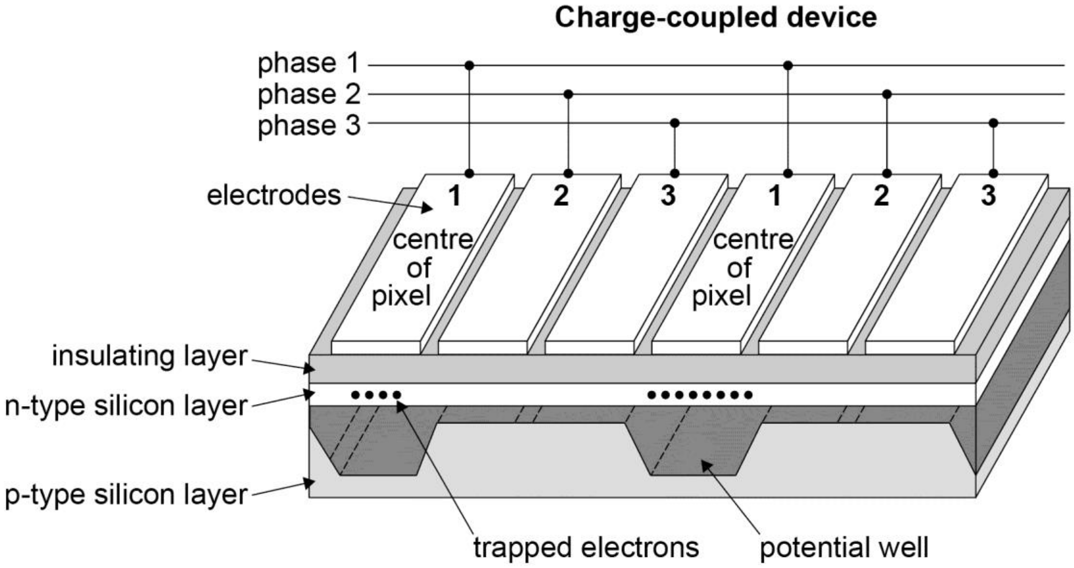
The wavelength and diameter have to have consistent units. Aperture diameters can be given in metres, millimetres and even kilometres for some large baseline interferometry devices. Wavelengths are often in nanometers, but can also be in micrometers, millimetres or metres. Problems with indices are very common. Students should be encouraged to practise calculating the resolving power of a wide range of telescopes which use many different parts of the electromagnetic spectrum.

The angle θ is a theoretical minimum. In reality other effects can make the calculated angle irrelevant when considering the resolving power of a telescope, particularly at shorter wavelengths. These effects include the refraction of light as it passes through the atmosphere. It has also already been stated that the spider holding the secondary mirror in a Cassegrain arrangement causes diffraction problems.

In the examination if the resolving power is asked for, it is this angle which is needed.

The detection system

Although the main emphasis of this section is about the advantages of large diameter telescopes, it is also worth considering the impact the detection system has on both the amount of information collected and the resolution.



It is important to consider the role of the Charge-Coupled Device (CCD) in modern astronomy because of the huge impact it has had. Students may also be familiar with their use in cameras and mobile phones.

The diagram looks fairly complicated, and the structure and operation of a CCD is beyond the requirements of the specification. Essentially, the structure of the CCD is fairly straight forward. It can be thought of as a piece of silicon, which can be several centimetres along each side. This silicon chip is made up of perhaps 16 million picture elements or pixels, formed into a two-dimensional array. Electrodes connected to the device help form potential wells which allow electrons to be first trapped, and then moved in order for the information to be processed.

A telescope is used to form an image on the CCD. Photons striking the silicon liberate electrons which are then trapped in the potential wells of the pixels. Exposure continues until sufficient electrons have been trapped to allow the required image to be obtained.

When exposure is complete, the electrodes are used to 'shuffle' the electrons along the array so that the contents of each well can be measured and the charge measurement used to create the image.

CCDs have been used to detect several regions of the electromagnetic spectrum, not just visible light. An important part of their behaviour is their linear response so that very faint parts of a relatively bright image can be obtained.

In examinations on the previous specification, questions were asked about the operation and structure of the CCD. The emphasis in this specification is a comparison with the eye.

Quantum efficiency

The number of electrons trapped in each well of a CCD is proportional to the number of photons hitting the pixel, ie to the intensity of the light falling on the pixel. This means that the pattern of electrons in the array is the same as the image pattern formed by the photons. The quantum efficiency, which is the percentage of incident photons which cause an electron to be liberated, can be 70% to 80%. The response of the eye to light is fairly complicated. Mechanisms exist to increase the eye's sensitivity in low light conditions. One very fast effect is the increase in aperture size, ie the pupil gets larger. True light adaptation can take up to half an hour, however. At low light levels, the quantum efficiency of the eye is approximately 1% at $550\ \mathrm{nm}$. Another problem with eyesight at low light levels is the loss of colour vision, as the colour sensitive cones no longer work, and the light detection is taken over by the rods.

Resolution

The resolution of a CCD itself is related to the size of each pixel – smaller pixels allow for more detail to be observed as each pixel integrates the light falling on it. Typical pixel size can be as small as $4\ \mu\mathrm{m} \times 4\ \mu\mathrm{m}$. However, it is more common to quote the number of pixels on the CCD wafer. In this case a $1\ \mathrm{cm}^2$ wafer would contain over 6 Mega pixels. Comparing this with the human eye, the pixel size (ie the size of a light sensitive cell) is approximately $2\ \mu\mathrm{m}$ and the most sensitive part of the eye, the fovea, has a diameter of about $1\ \mathrm{mm}$. This suggests that the human eye and CCD are roughly similar in their resolution. However, the function of the eye is fairly complex, and this is only a very simple analysis.

Convenience of use

Although it could be said that nothing could be easier than just looking through a telescope to see what is there, it could also be argued that CCDs are even more convenient. They allow for remote viewing (for example see the Bradford Robotic Telescope, http://www.telescope.org/), direct computer analysis, long exposure times and the detection of wavelengths beyond the visible spectrum.

Section 2 Classification of stars

a) Classification by luminosity

A brief look at the night sky would show students that some stars look brighter than others.

In about 120 BC the Greek astronomer Hipparchus produced a catalogue of over 1000 stars and their relative brightness. He used a 6 point scale, with 1 being the brightest and 6 the dimmest.

The modern version of this is called the apparent magnitude scale. In simple qualitative terms this is the brightness of the star as seen from Earth. This is to avoid problems with the non-visible parts of the spectrum which should not be included in this simpler definition. So, terms like 'luminosity' or 'intensity' should not be used. Luminosity is the total power radiated by a star, and the intensity is the power per unit area at the observer.

In this sense, apparent magnitude is simply a scale of brightness (as judged by an observer) which decreases as brightness increases, and which has a value of about 6 for the faintest stars which can be seen with the naked eye on a good night. Students can be introduced to apparent magnitude in this way, and therefore treat it initially as a qualitative scale. You could imagine students being asked to look at the stars in, say, The Plough, and to write them down in order of brightness and use some sort of scale to compare them. It should be stressed that what the students are using is the 'visible luminosity'. As a star's luminosity is its total power output at all wavelengths, it is often referred to as the 'bolometric luminosity'. It would not be helpful to catalogue a star's bolometric luminosity for observers to use with optical telescopes.

The fact that some stars appear brighter than others can be due to two reasons; it could be closer, or it could be emitting more power at visible wavelengths. The brightest star in The Plough, for example, happens to be the furthest away.

With modern measuring techniques (eg photography, CCD cameras), a more quantitative approach to apparent magnitude can be made. The devices allow the intensity of the light from a star to be measured. The apparent magnitude scale now takes on a more precise meaning.

The branch of astronomy that deals with this is called photometry, and towards the end of the 1700s, William Herschel devised one simple (though inaccurate) method to measure the brightness of stars. One key point that arose from his work was the fact that a first magnitude star delivers about 100 times as much light to Earth as that of a sixth magnitude star.

In 1856, following the development of a more precise method of photometry, Norman Pogson produced a quantitative scale of apparent magnitudes. Like Herschel he suggested that observers receive 100 times more light from a first magnitude star than from a sixth magnitude star; so that, with this difference of five magnitudes, there is a ratio of 100 in the light intensity received.

Because of the way light is perceived by an observer, equal intervals in brightness are actually equal ratios of light intensity received – the scale is logarithmic. Pogson therefore proposed that an increase of one on the apparent magnitude scale corresponded to a change in the intensity received by a factor of 2.51, ie the fifth root of 100. Thus a fourth magnitude star is 2.51 times brighter than a fifth magnitude star, but 6.31 times brighter than a sixth magnitude star.

With the faintest stars having an apparent magnitude of 6, this scale allows the brightest stars to have negative apparent magnitudes, the brightest star visible from Earth (the Sun) having an apparent magnitude of –26.

Pogson's scale allows the intensity, I, measured to be converted to a numerical apparent magnitude

$m = -2.51 \log_{10} (I/2.56 \times 10^{-6})$

Where 2.56×10^{-6} lux is the intensity for a magnitude 0 star.

Questions will not be set on this equation, although its important features should be taught: in particular, the negative relationship between intensity and apparent magnitude and the 2.51 ratio for a difference in apparent magnitude of 1, and, therefore, the log scale used for apparent magnitudes. Tables such as the one below are easy to obtain using sources on the internet.

object	apparent magnitude, ***m***
the Sun	–26
the full Moon	–19
Venus	–4
Sirius	–1.5
Vega	0.0
Altair	0.8
Polaris	2.0

b) Absolute magnitude, M

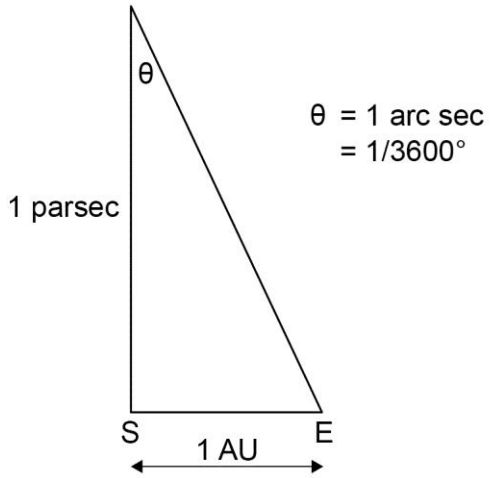
Units of distance

It is clear that apparent magnitude tells you very little about the properties of the star itself, unless you know how far away it is. Astronomical distances are extremely big – and several different units of distance exist to make the numbers smaller and more manageable.

The light year is the distance travelled by light in a vacuum in one year. It can be easily converted into metres by multiplying the speed of light in a vacuum (3×10^{8} m s^{-1}) by the number of seconds in a year ($365 \times 24 \times 3600$) to get 9.46×10^{15} m.

The AU (astronomical unit) is the mean distance from the Sun to the Earth and has a value of 1.5×10^{11} m.

The unit which causes students the biggest problem is the parsec: the distance from which 1 AU subtends an angle of 1 arc second (1/3600th of a degree). This is most easily defined from a diagram such as this:



The parsec is an important unit because of the way distances to nearby stars can be determined – trigonometrical parallax. This involves measuring how the apparent position of a star, against the much more distant background stars, changes as the Earth goes around the Sun.

1 pc is approximately 3.26 light year.

Definition of absolute magnitude, M

The absolute magnitude of a star is the apparent magnitude it would have at a distance of 10 pc from an observer. It is a measure of a star's inherent brightness. This means that absolute and apparent magnitudes are measured on the same logarithmic scale.

As light intensity reduces in proportion to the inverse square of the distance, the relationship between apparent and absolute magnitude can be related to distance, d, by the equation:

$$m - M = 5\log\frac{d}{10}$$

'd' is measured in parsec, and the log is to base 10. A quick glance shows that:

- Stars which are closer than 10 pc (about 33 light years) have a brighter (more negative) apparent magnitude than absolute magnitude. $M – M < 0$
- Stars further than 10 pc have a dimmer (more positive) apparent magnitude than absolute magnitude. $m – M > 0$
- If $m = M$, the star must be 10 pc away.

star	apparent magnitude m	absolute magnitude M	$m–M$	distance / light year
Sirius	–1.46	1.43	–2.89	9
Pollux	1.1	1.1	0	33
Castor	1.58	0.59	0.99	52

Care must be taken comparing magnitudes – does a bigger magnitude mean brighter (greater intensity) or dimmer (bigger number). It is safer to talk about brighter or dimmer magnitudes rather than larger or smaller.

Students should also be able to manipulate this equation in order to work out distances. As it involves logs, students unfamiliar with this aspect of maths are likely to find this difficult.

b) Classification by temperature, black body radiation

All the information we get about stars comes from the electromagnetic radiation we receive.

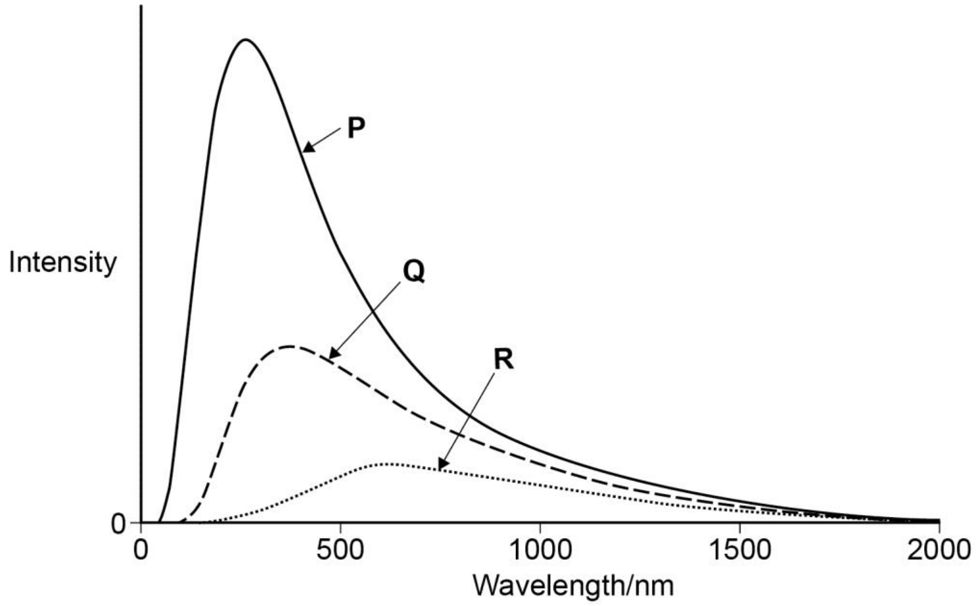
For example, an analysis of the intensity of the light at different wavelengths allows astronomers to measure the 'black body' temperature of a star. Using this and an estimation of the power output of a star allows its diameter to be determined.

Black body radiation and Wien's displacement law

We are used to the terms 'red hot' and 'white hot' when applied to sources of heat. We are also aware that hot objects emit radiation at infrared wavelengths. This behaviour of hot objects and the electromagnetic radiation they produce was investigated at the end of the 19th century and contributed to the development of quantum theory by Max Planck.

A black body is one that absorbs all the em radiation that falls on it. (In practical terms, it is the hole in the wall of an oven, painted black on the inside). Analysis of the intensity of the em radiation emitted from a black body at different wavelengths produces the following result.

The three lines are for three different temperatures.



The graph shows that the hottest object (P) has a peak at the shortest wavelength. In fact, the relationship between the wavelength of the peak, λ_{max}, and the temperature, T, is called Wien's displacement law:

$$\lambda_{\max} T = \text{const} = 2.9 \times 10^{-3} \text{ m K}$$

λ_{max} is measured in metres and T is the absolute temperature and is therefore measured in kelvin, K.

Hotter stars will therefore produce more of their light at the blue/violet end of the spectrum and will appear white or blue-white. Cooler stars look red as they produce more of their light at longer wavelengths. When analysing the light from stars in this way it is assumed that the star acts as a black body, and that no light is absorbed or scattered by material between the star and observer.

Stefan's Law

Stefan's law relates the total power output, P, of a star to its black body temperature, T, and surface area, A.

$$P = \sigma A T^4$$

where σ is called Stefan's constant and has the value 5.67×10^{-8} W m^{-2} K^{-4}. The law is sometimes called the Stefan-Boltzmann Law as it was derived by Stefan experimentally and independently by Ludwig Boltzmann from theory.

Qualitative comparison of two stars

The law tells us that, if two stars have the same black body temperature (are the same spectral class), the star with the brighter absolute magnitude has the larger diameter. This fact is very important when interpreting the Hertzsprung-Russell diagram, which is considered shortly.

It is quite common for Stefan's Law to be used qualitatively when comparing the size of stars. For example, Antares and Proxima Centauri are both M class stars. Although they have approximately the same black-body temperature, Antares is one of the brightest stars, with an absolute magnitude of -5.3, and Proxima Centauri is one of the dimmest, with an absolute magnitude of 15. It is not surprising to work out, therefore, that the diameter of Antares is approximately 5500 times that of Proxima Centauri.

This YouTube clip is one of several which shows a comparison of stars [Star Size Comparison HD](#) .

Quantitative use to determine the diameter of a star

Consider two stars of black-body temperatures T_1 and T_2. If the ratio of their power output (P_1/P_2) is known, Stefan's law can be used to calculate the ratio of their diameters, d_1/d_2

$$\frac{d_1}{d_2} = \frac{T_2^2}{T_1^2}\sqrt{\frac{P_1}{P_2}}$$

This equation is also useful when considering how the size of a star must change as it goes through its life cycle.

The inverse-square law

There are several inverse-square laws in Physics. Essentially any property which reduces to a quarter of its original value when the distance is doubled follows an inverse-square law.

$$I = \frac{I_0}{d^2}$$

Derivation of $m - M = 5\log(\frac{d}{10})$

As has already been stated, when the inverse-square law equation is combined with Pogson's equation for apparent magnitude

$$m = -2.5\log(\frac{I}{2.56\times10^{-6}})$$

the distance equation can be obtained. The specification does not require this derivation, but it is worth demonstrating at this point.

I_0 is the intensity of a light source at 1 pc, and I is the intensity at distance, d. Therefore, when we substitute in the inverse-square law, we get:

$$m = -2.5\log(\frac{I_0}{2.56\times10^{-6}d^2})$$

When d = 10 pc, m = M, so:

$$M = -2.5\log(\frac{I_0}{2.56\times10^{-6}10^2})$$

Subtracting:

$$m - M = -2.5\log(\frac{I_0}{2.56\times10^{-6}d^2}) + 2.5\log(\frac{I_0}{2.56\times10^{-6}10^2})$$

Tidying:

$$m - M = -2.5\log(\frac{1}{d^2}) + 2.5\log(\frac{1}{10^2})$$

And therefore:

$$m - M = 5\log(d) - 5\log(10)$$

So:

$$m - M = 5\log(\frac{d}{10})$$

Other uses of the inverse-square law

Using this equation, the intensity of a light source can be determined at different distances.

For example, if the Sun is used to provide the energy for solar cells on a space probe, the equation can be used to determine what happens as the probe gets further from the Sun.

The inverse-square law can also be used to estimate the power output of different objects.

For example, Doppler shift information suggests that some quasars are billions of light years away. Using the inverse-square law, it can be shown that the power output of a quasar must be equivalent to that of a whole galaxy.

Assumptions in its application

Use of the inverse-square law assumes that no light is absorbed or scattered between the source and the observer and that the source can be treated as a point.

d) Principles of the use of stellar spectral classes

When the light created within a star passes through its ‘atmosphere’, absorption of particular wavelengths takes place. This produces gaps in the spectrum of the light from the star, resulting in an absorption spectrum. The wavelengths are related to frequency ($c = f\lambda$) and therefore to particular energies ($\Delta E = hf$). Electrons in the atoms and molecules of the star’s atmosphere are absorbing the light, and therefore ‘jumping’ to higher energy levels. The difference in these energy levels are discrete (have particular values) and therefore the frequencies of the absorbed light are discrete.

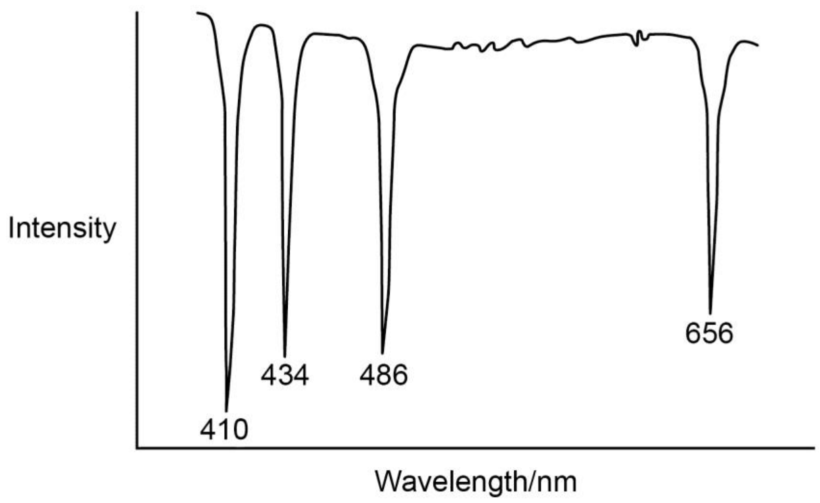
Some early attempts to classify stars according to their spectra used alphabetic labels.

When the processes were better understood, it transpired that a more logical order, related to temperature, was adopted. The original alphabetical order was then lost.

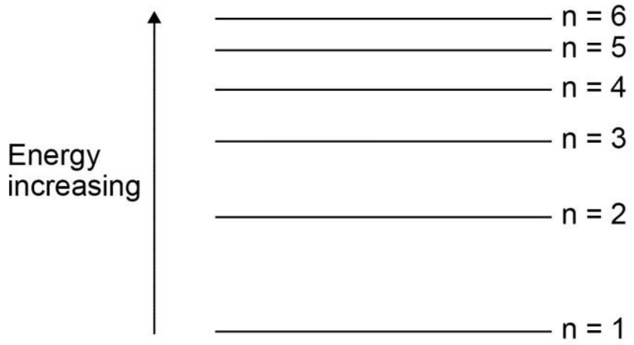
Spectral class	Intrinsic colour	Temperature / K	Prominent absorption lines
O	blue	25 000–50 000	He+, He, H
B	blue	11 000–25 000	He, H
A	blue-white	7500–11 000	H (strongest) ionised metals
F	white	6000–7500	ionised metals
G	yellow-white	5000–6000	ionised and neutral metals
K	orange	3500–5000	neutral metals
M	red	< 3500	neutral atoms, TiO

Probably the most famous mnemonic for remembering the order is 'Oh be a fine girl, kiss me!'

The relationship between temperature and spectra is due to the effect of the energy on the state of the atoms or molecules. At low temperatures there may not be enough energy to excite atoms, or break molecular bonds, resulting in the TiO and neutral atoms in the M class spectra. At higher temperatures atoms have too much energy to form molecules and ionisation can take place (as seen in classes F and G). The abundance of hydrogen and helium in the atmosphere of the hottest stars means that their spectral lines start to dominate. The Hydrogen Balmer series is a useful indicator here.



The spectrum above shows absorption lines at particular wavelengths in the visible spectrum. They are due to the absorption of light by excited hydrogen. The diagram below shows the electron energy levels of a hydrogen atom.



The electrons start in the *n=2* state. To achieve this with a significant proportion of the hydrogen, the atmosphere of the star must be fairly hot. Any hotter than about 10 000 K, however, and the hydrogen starts to be ionised. Absorbing the correct amount of energy excites the electrons even further, raising them to *n=3, 4* etc. When the electrons de-excite, the light is emitted in all directions, and the electrons may do it in several steps or miss out *n=2* and jump down to *n=1*. This results in the gap, or reduction in intensity in the direction of the observer. The name given to the spectrum produced when transitions from *n=2* are made is the Hydrogen Balmer Absorption Spectrum. Chemists will be familiar with other series such as Lyman and Paschen.

In terms of the Balmer series, the following points can be noted:

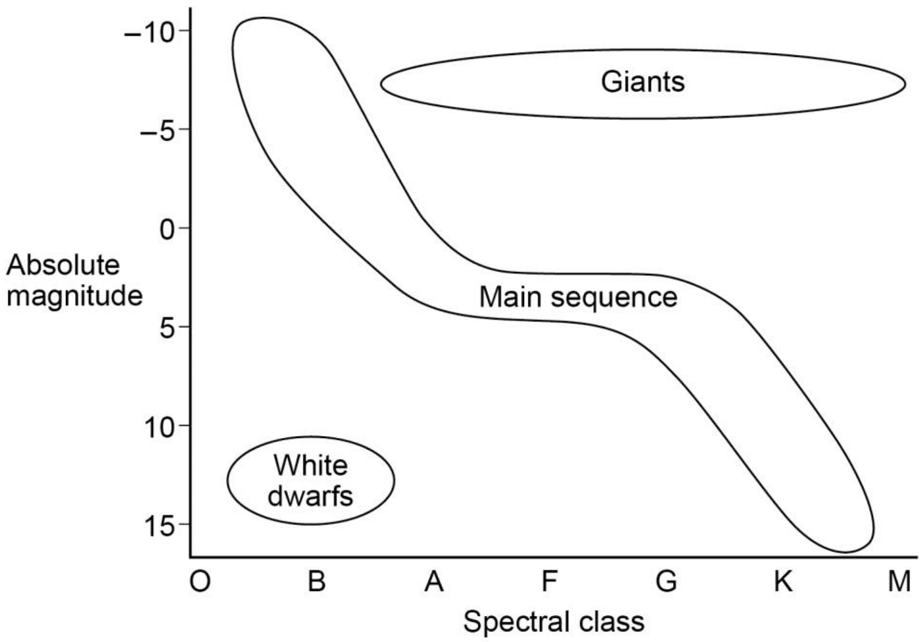
Spectral class	Prominence of Balmer lines	Explanation
O	weak	star's atmosphere too hot hydrogen likely to be ionised
B	slightly stronger	
A	strongest	high abundance of hydrogen in *n*=2 state
F	weak	too cool, hydrogen unlikely to be excited
G	very weak/none	too little atomic hydrogen, far too cool to be excited
K		
M		

When searching for the properties of stars, it will be clear that the spectral classes have been subdivided, so that each spectral class has a 10 point scale. The Sun, for example is a class G2 star.

Evidence from stellar absorption spectra supports temperature calculations based on black-body radiation. As has been seen, temperature and absolute magnitude can give some indication of the size of a star. It is this relationship that is dealt with in the next section.

e) The Hertzsprung-Russell (H-R) diagram

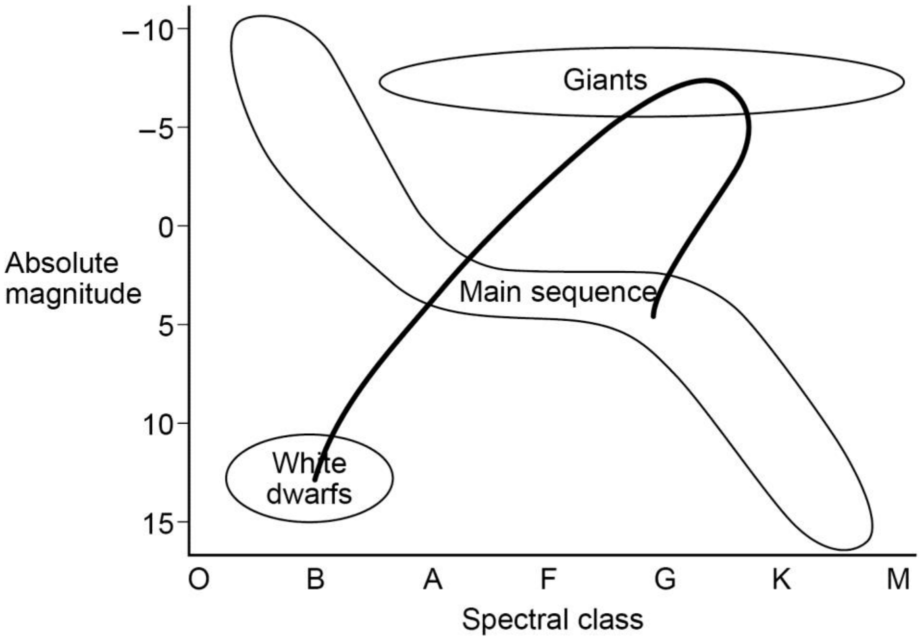
There are many versions of the H-R diagram, but the one required on this specification is shown below. It is an x-y scattergraph, with stars as the points on the graph. The y-axis is absolute magnitude, from $+15$ to -10, whilst the x-axis can be spectral class, as shown, or temperature (from $50\ 000\ \mathrm{K}$ to $2500\ \mathrm{K}$). For simplicity, the diagram is reduced to just three regions, although there are others, such as the supergiants.



Stefan's law can be used to explain why some stars are referred to as giants or dwarfs.

For example, a dim (absolute magnitude 12) star in spectral class B must be much smaller than a much brighter star (absolute magnitude -10) in the same spectral class (ie at the same temperature).

It is also useful to be able to follow a star, such as the Sun, as it evolves on the H-R diagram.



The Sun is currently a G class star with an absolute magnitude of 5. This puts it on the main sequence. As it uses up the hydrogen fuel in its core, it will start to fuse the hydrogen in its outer layers and expand. At this point it will leave the main sequence and become a giant star. Its diameter is likely to extend beyond the orbit of the Earth. As it expands, the outer surface of the Sun will cool. As it uses up this fuel it will collapse and the increase in temperature and pressure in the core will initiate helium fusion. When this fuel is used up, helium shell fusion can then occur and the Sun will again expand. Eventually the outer material of the Sun will be pushed away to form a planetary nebula, leaving the extremely hot but small core as a white dwarf. Fusion has finished, and the white dwarf cools eventually to a black dwarf.

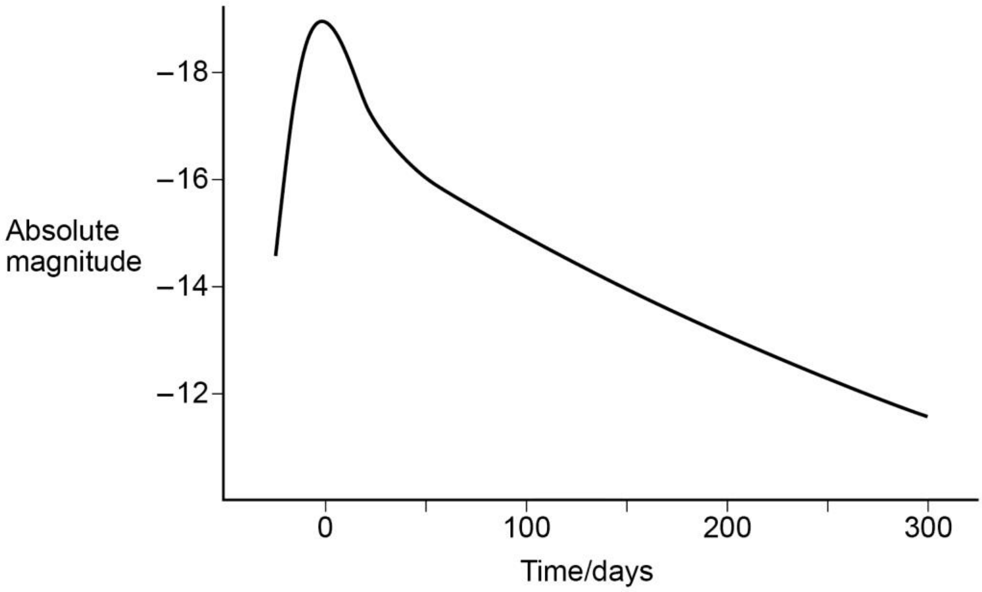
f) Supernovae, neutron stars and black holes

The evolution of a star is related to its mass. The Sun is relatively small, and its demise will be relatively unspectacular. Larger stars are more likely to have a catastrophic ending.

Supernovae

Supernovae are objects which exhibit a rapid and enormous increase in absolute magnitude. Type 1a supernovae are useful because they reach the same peak value of absolute magnitude, making them standard candles, ie the absolute magnitude is known, the apparent magnitude can be measured, so the distance can be determined.

The diagram shows the variation of absolute magnitude with time for a typical type 1a supernova. Graphs of this kind are called light curves.



The peak value of absolute magnitude is -19.3, and occurs after about 20 days from the start of the increase in brightness. Conventionally, time is measured from the peak.

Type 1a supernovae are the result of an exploding white dwarf star, which is part of a binary system. The white dwarf increases in mass as it attracts material from its companion. Eventually the white dwarf reaches a size which allows fusion to start again, (the fusion of carbon which requires very high pressures and temperatures), which in some cases causes the star to explode. This occurs when the star reaches a critical mass, and produces a very consistent light curve. The other advantage of using Type 1a supernovae as standard candles is that they are very bright. This means they can be used to measure the distance to the furthest galaxies.

Recent measurements of this kind have led to the idea that the expansion of the Universe may be accelerating, and that the controversial 'dark energy' may be driving this expansion. This is dealt with later.

Other supernovae also exist. As the core of giant stars stop generating energy by fusion, gravitational collapse can take place. The core may form a neutron star or black hole, and the gravitational energy released can cause the stars outer layers to heat up and be expelled.

Neutron stars

When a giant star has exploded its outer layer, the core may be too massive to become a white dwarf and further collapse can take place. A core with a mass of about twice that of the Sun will form a neutron star. As its name suggests, it is believed to largely consists of neutrons, although some theories suggest it may have a 'crust' made of atomic nuclei and electron fluid, with protons and neutrons making up the interior. Due to the complex nature of its structure, other properties are considered to be its defining features.

A neutron star is believed to have the density of nuclear matter and be relatively small (about $12\ \mathrm{km}$ in diameter). Due to conservation of angular momentum as the core collapses, they tend to be spinning very rapidly, although they do lose energy and slow down with time. They can be very powerful radio sources due to their extremely strong magnetic fields, and this, combined with their spinning, produces pulsars – which behave like radio 'light houses'. An example can be found in the Crab Nebula.

Black holes

More massive cores (more than about 3 solar masses) can continue to collapse and form black holes. The defining feature of a black hole is that its escape velocity is greater than the speed of light. The boundary at which the escape velocity is equal to the speed of light is called the Event Horizon. Anything within the Event Horizon of a black hole cannot escape, not even light.

The radius of the Event Horizon is referred to as the Schwarzschild radius, R_S. The equation is quite easily obtained from Newtonian gravitational theory (although a more correct analysis uses general relativity). The escape velocity of an object of mass M is found by considering the minimum KE needed to move from the surface of the object to infinity, where the GPE is zero.

KE lost = GPE gained

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R}$$

rearranging and cancelling for the mass of the object escaping, m:

$$R = \frac{2GM}{v^2}$$

For a black hole, with an escape velocity equal to the speed of light, c,

$$R_s = \frac{2GM}{c^2}$$

Although many believe a black hole to be a singularity, this distance gives a more useful boundary distance. It is important to note that, unlike a neutron star, the density of the black hole within this boundary is not necessarily very large. It is believed that supermassive black holes, whose mass may be several million times the mass of the Sun, exist at the centre of most galaxies. The density of these black holes may be similar to that of water. The radius of a black hole is proportional to its mass, which means that the volume is proportional to mass^3.

As

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

the density of a black hole must be proportional to the inverse square of its mass, ie the more massive the black hole, the less dense it is.

It is believed that a quasar is produced as a supermassive black hole at the centre of an active galaxy consumes nearby stars.

Section 3 Cosmology

a) Doppler Effect

Most people are familiar with the change in pitch that occurs as an ambulance moves past with its siren going. This is due to the Doppler Effect. As the source of the sound waves approaches, the sound waves are effectively ‘bunched together’, reducing the wavelength and therefore increasing the frequency of the sound (higher pitch). As the source recedes, the waves are ‘stretched out’, increasing the wavelength and reducing the frequency (lower pitch).

The same effect occurs with light. However, as the size of the effect depends on the ratio of the speed of the object to the speed of the wave, it is less noticeable with light unless the source is moving extremely quickly.

As with sound, if the source of light is moving away from the observer the wavelength is increased. This is often referred to as ‘redshift’ as red is at the longer wavelength end of the visible spectrum. The red shift is given the symbol z and can be calculated from the equation:

$$z = \frac{\Delta f}{f} = \frac{v}{c}$$

where v is the speed of the source and Δf is the change in frequency.

In terms of wavelength

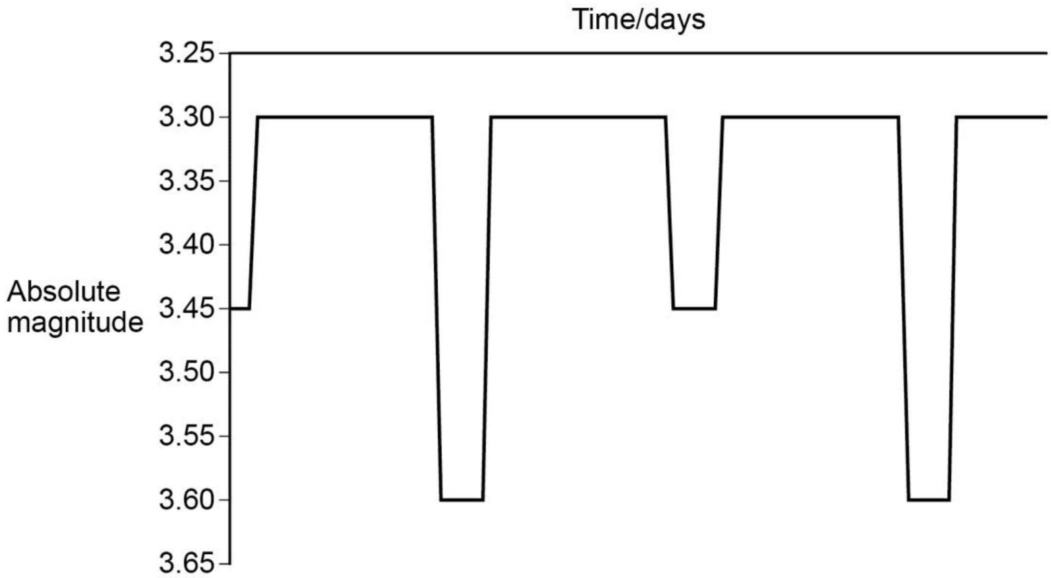
$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

Notice the negative sign. This reminds us that the wavelength decreases if the source is approaching.

The sign of z can cause some confusion. As quoted, z would be positive for an increase in frequency, ie for an approaching object. However, as z is called the red shift (rather than just the Doppler shift) it is treated as positive for receding objects. In effect, it can be treated as a modulus – the information about the direction of the source is in its name rather than sign.

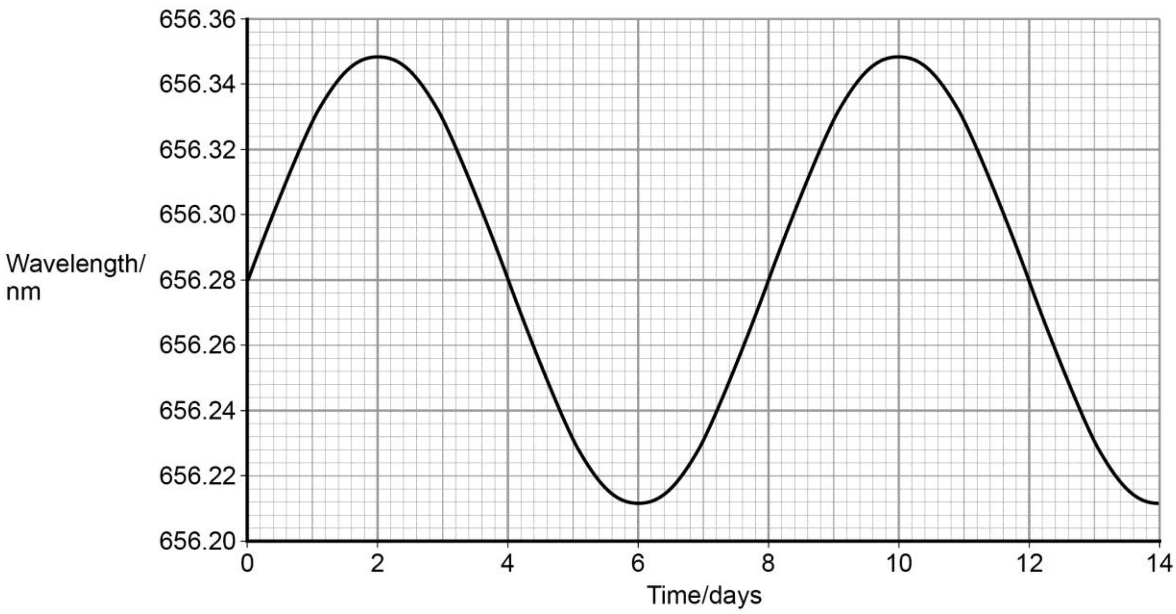
Binary star systems

A typical application of the Doppler Effect in cosmology is the study of eclipsing binary star systems, ie two stars rotating about a common centre of mass. In our line of sight, one star passes in front of the other, causing the eclipse. A simplified light curve from an eclipsing binary system is shown below.



Notice that the apparent magnitude scale increases going upwards, ie the dips in the graph correspond to the light dimming. In this example, the two stars have different surface temperatures (ie they emit different amounts of light per unit surface area – see Stefan's Law). When the apparent magnitude is 3.30, the system is at its brightest as both stars can be seen. The deeper dips are caused by the cooler star passing in front of the hotter star – the apparent magnitude is 3.60. The shallower dips, to 3.45, occur when the hotter star passes in front of the cooler.

If the light from one star is analysed in more detail, the shift in wavelength of one particular spectral line can be measured as the star follows its circular orbit.



The peak in the graph (at 656.35 nm) occurs when the star is receding from our point of view, at its maximum velocity, ie the two stars are next to each other. The eclipses occur when the star is moving at right angles to our line of sight and the wavelength is 656.28 nm.

The orbital period is 8 days.

The Doppler equation can be applied to calculate the maximum recessional velocity, which is equal to the orbital speed of the star.

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

rearranging and substituting

$$\frac{(656.35 - 656.28) \times 3 \times 10^8}{656.28} = 3.2 \times 10^4 \text{ m s}^{-1}$$

With the orbital speed and time period, the diameter of the orbit can be calculated using the circular motion equation.

circumference of orbit = orbital speed × orbital period

$= 3.2 \times 10^4 \times 8 \times 24 \times 3600$

$= 2.21 \times 10^{10}$ m

diameter = $2.21 \times 10^{10}/3.14 = 7.04 \times 10^9$ m

b) Hubble's Law

Observation of distant galaxies showed significant red shifts. Further analysis suggested that the further away the galaxy is, the greater the red shift (see section A, Doppler Effect).

The Doppler equation was used to calculate the recessive velocity of galaxies. This led to the development of Hubble's Law. The law deals with the relationship between recessive velocity, v, and distance, d, for galaxies

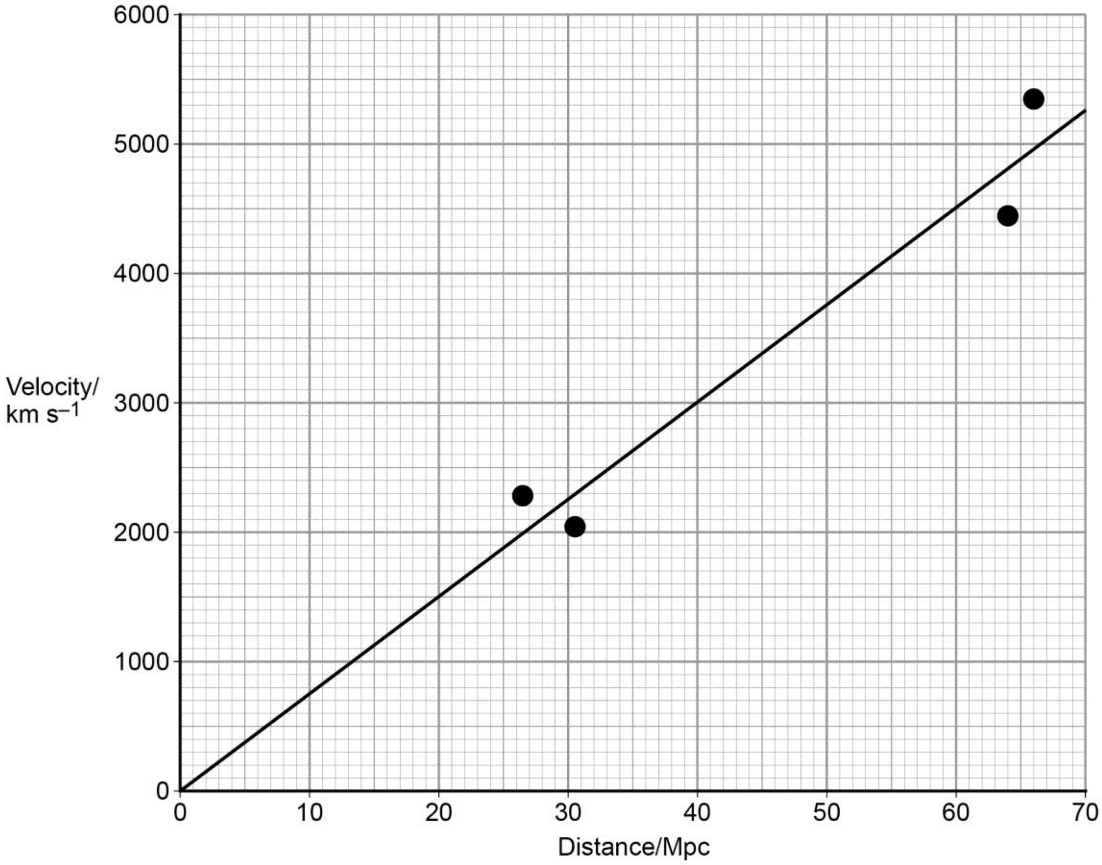
$$v = Hd$$

where H is Hubble's constant, the currently accepted value of which is $65\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$. Notice that the unit of velocity is therefore $\mathrm{km\ s^{-1}}$, and the unit of distance is the Mpc (mega parsec).

The data from some typical galaxies is given in the table.

galaxy	velocity / $\mathrm{km\ s^{-1}}$	distance / **Mpc**
NGC 1357	2094	28
NGC 1832	2000	31
NGC 5548	5270	67
NGC 7469	4470	65

A graph of velocity against distance using this data can be used to determine a value for Hubble's constant. Simply calculate the gradient of a line of best fit for these points which passes through the origin. The scatter in the points does give some indication of the approximate nature of the relationship.



The data suggest that the Universe is expanding. It is like being an average runner in a race – those in front of you are faster than you and moving away from you. You are faster than those behind and you are moving away from them. From your point of view, everyone appears to be moving away from you. Just like a race, Hubble's Law suggests that there was once one common starting point. As the distance between the galaxies was zero at the start, the time since the beginning of the expansion (the age of the Universe) can be found by dividing the distance to a galaxy by its recessional velocity, ie the age of the universe = d/v = $1/H$. This gives an age for the Universe of approximately 15 billion years.

The equation assumes that the expansion rate is constant. Recent evidence suggests, however, that the rate of expansion may be increasing. This relates to the controversial 'dark energy'.

Dark energy

At the end of the last century, astronomers measured the distance to several very distant Type 1a Supernovae, and measured the red shift of their host galaxies. They expected to find that the rate of expansion was slowing down and that the supernovae would appear brighter than suggested by the extent of their red shift and Hubble's Law. What they found was that the supernovae were dimmer than expected, suggesting that the rate of expansion of the Universe is increasing. Further measurements since then have supported this idea, and in order to explain this increasing expansion some astronomers have used the controversial idea of 'dark energy'.

At the moment there is no known mechanism for this expansion, or the Dark energy that drives it. Some astronomers believe it is linked to the cosmological constant introduced and then discarded by Einstein during his work on gravity. Some believe it is due to a 'negative vacuum pressure' or a quantum field effect. The important issue is that there is evidence for its existence, and that, as no one knows what it is, it is controversial.

The Big Bang

Dark energy is just the latest cosmological controversy in a long line. Ten years ago it was quasars, and fifty years ago it was the Big Bang.

The theory suggests that, over the past 15 billion years or so, the Universe has expanded from an extremely hot and dense point, and is still expanding.

Evidence clearly comes from the Hubble relationship and observations of the red shift of distant galaxies. Further supporting evidence came from the discovery of the cosmological microwave background radiation. This is a 'glow' from all parts of the Universe, the spectrum of which follows a black-body radiation curve with a peak in the microwave region. This peak corresponds to a temperature of $2.7\ \mathrm{K}$. It can be interpreted as the left over 'heat' from the big bang, the photons having been stretched to longer wavelengths and lower energies (more correctly perhaps, it is the radiation released when the Universe cooled sufficiently for matter and radiation to 'decouple', with the combination of protons and electrons to form neutral atoms).

The final piece of evidence comes from the formation of nuclear matter. The Big Bang theory suggests that a very brief period of fusion occurred when the Universe was very young, resulting in the production of helium from fusing hydrogen. The Universe then expanded and cooled too rapidly for the creation of larger nuclei, resulting in a relative abundance of Hydrogen and Helium (in the ratio of 3:1) spread uniformly throughout the Universe, and a lack of larger elements. This is consistent with observation.

c) Quasars

Red shift measurements suggest that quasars are some of the most distant and most powerful observable objects. Using the inverse-square law, calculations suggest that they can produce the same power output of several galaxies. Variations in their output, however, suggest that they may be only the size of a solar system. They appear in the centre of young (ie distant) galaxies, often obscuring the host galaxy because they are so bright.

Whilst they were once controversial, they are now believed to come from supermassive black holes at the centre of young active galaxies. The typical power output of a quasar may be approximately 10^{42} W.

Quasars were first discovered as very powerful radio sources, and, due to the poor resolving power of radio telescopes, it was very difficult to associate a quasar with its optical counterpart. It was only with very large baseline interferometry that quasars could be pinpointed and we now know that quasars emit em radiation from the whole spectrum.

d) Detection of exoplanets

A recent development in astrophysics has been the successful search for planets outside our solar system, known as exoplanets. Direct observations of exoplanets are difficult because the light from them tends to be obscured by the much brighter star they orbit, and the planet and star tend to be too close together for the optics to resolve them. Planets far enough away from the star to be resolved, are likely to be very dim as they will not reflect much light.

Despite this several exoplanets have been directly imaged. They tend to be very large and in large orbits. They are also likely to be hot – so that they can be found from the infrared they emit rather than from the reflected light from their star. It also helps if the planet orbits a brown dwarf or other dim star. An example is 2M1207b, a planet, discovered by the VLT in Chile, orbiting a brown dwarf in Centaurus. It is believed to have a mass several times that of Jupiter, and orbit at about 40 AU, roughly the orbital distance of Pluto around the Sun. It was discovered by observations of the I-R it emits. Technology has advanced, and various masking techniques have been developed to block out the light from stars so that the planets orbiting might be directly imaged.

Fortunately there are many alternative methods that can be applied to the search for exoplanets. The two investigated in this specification are the radial velocity method, that uses Doppler shift measurements, and the transit method.

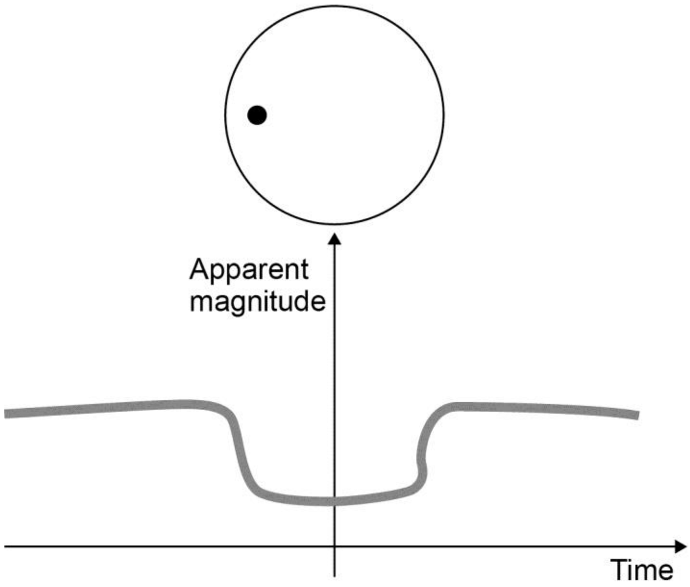
Radial velocity method

When a planet orbits a star, the gravitational pull causes the star to "wobble". This can cause a Doppler shift in the light received from the star. The amount and direction of shift is related to the component of the star's velocity towards and away from the Earth – ie radially from the Earth's point of view. The analysis is similar to that of binary systems discussed earlier. One of the first exoplanets discovered was a Jupiter-sized object orbiting 51 Pegasi, a main sequence star similar to our Sun. The time period was found to be 4.2 days, and the orbital speed $136\ \text{km s}^{-1}$. Using this data it is quite easy to show that the planet is incredibly close to its star, with an orbital radius of about 0.05 AU. (To give this some sense of scale, the diameter of our Sun is about 0.01 AU)

For detailed information about the discovery of the planet orbiting the star 51 Pegasi see pa.msu.edu/courses/2011spring/AST208/mayorQueloz.pdf

Transit method

When a planet passes in front of a star there is very small dimming of its apparent magnitude. Again, the analysis is similar to that of eclipsing binaries discussed earlier.



The graph above is a typical light curve. The apparent magnitude dims (increases in numerical terms) as the planet passes in front of the star. The graph of apparent magnitude against time is called a light curve. Many other processes can cause this shape of light curve, such as sun spots or variations in the output of the star itself. Repeated transits are observed to confirm the existence of a planet. The NASA Kepler website contains a lot of information about the detection of exoplanets using the transit method.

An important aspect of the search for exoplanets is the need for corroboration. Much care is taken to exclude other possible causes of Doppler shifts or periodic apparent magnitude variations before the existence of an exoplanet is announced.

So far, most of the planets discovered have been large, so-called “hot Jupiters” or “super Earths”. The search continues for an Earth sized planet in the so called “Goldilocks” zone around a star where liquid water could exist on the surface. Kepler 186f may be the first. nasa.gov/ames/kepler/nasas-kepler-discovers-first-earth-size-planet-in-the-habitable-zone-of-another-star/ provides more information.