Teaching guide: Turning points in physics

This teaching guide aims to provide background material for teachers preparing students for the Turning Points in physics option of our A-level Physics specification (7408). It gives teachers more detail on specification topics they may not be familiar with and should be used alongside the specification. This guide is not designed to be used as a comprehensive set of teaching notes.

Contents

Introduction

Turning Points in physics is intended to enable key developments in physics to be studied in depth so that students can appreciate, from a historical viewpoint, the significance of major conceptual shifts in the subject, both in terms of the understanding of the subject and in terms of its experimental basis.

Each of the three sections within the course represents a different approach to the subject. The discovery of the electron and the determination of e/m and e took the subject forward through important experimental work by Thomson and Millikan. Further work on electron beams led to support for relativity and the discovery of electron diffraction. Recent 'Millikan' experiments have unsuccessfully sought evidence for fractional charge.

The section on wave particle duality follows the changes in the theory of light arising from experimental discoveries over two centuries from Newton to Einstein, and links up to the prediction and discovery of matter waves. Recent important developments in electron microscopy illustrate the unpredicted benefits of fundamental research.

Einstein's theory of special relativity overthrew the concept of absolute motion. The null result of the Michelson-Morley experiment presented a challenge to accepted ideas of space and time which was not solved until Einstein put forward his revolutionary theory of special relativity.

Each section offers opportunities to consider the relationship between experimental and theoretical physics. The nature of scientific proof can be developed in each section in line with the philosophy of science; no amount of experimentation can ever prove a theory fully, yet one experiment can be sufficient to overthrow it.

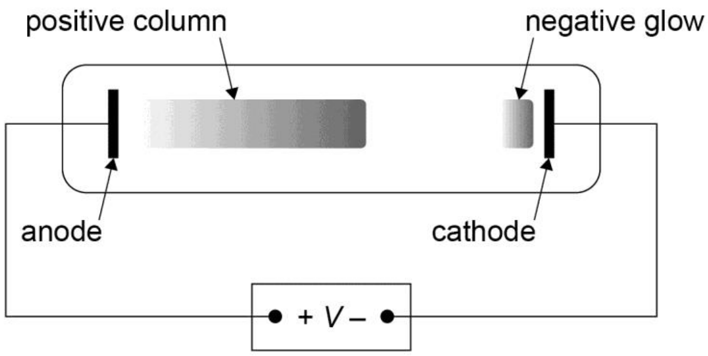
Section 1 The discovery of the electron

a) Cathode rays

Students should know what cathode rays are and how they are produced in a discharge tube. They should know how a beam of electrons is produced by thermionic emission and they should be able to relate the speed of an electron in such a beam to the pd used to accelerate it.

William Crookes in the 1870s discovered that gases at sufficiently low pressure in sealed glass tubes conduct electricity and glow with light of a characteristic colour (**Figure 1**). Investigations into the cause of the glow showed that the glowing gas near the anode, the positive column, was easily distorted when a magnet was brought near to the tube. This observation showed charged particles move through the gas when it conducts electricity. In a specially-devised tube, radiation from the direction of the cathode forced 'a paddle wheel' placed in the tube to rotate. This radiation was referred to as 'cathode rays'. Placing a magnet near the tube stopped the paddle wheel rotating by deflecting the cathode rays away from it.

Figure 1 A discharge tube



Further tests showed the cathode rays to be negatively charged particles. Exactly what cathode rays are puzzled scientists for over 20 years until Joseph J Thomson carried out a series of experiments and proved that the negative particles are the same, regardless of which gas is used in the tube. The cathode ray particles became known as electrons.

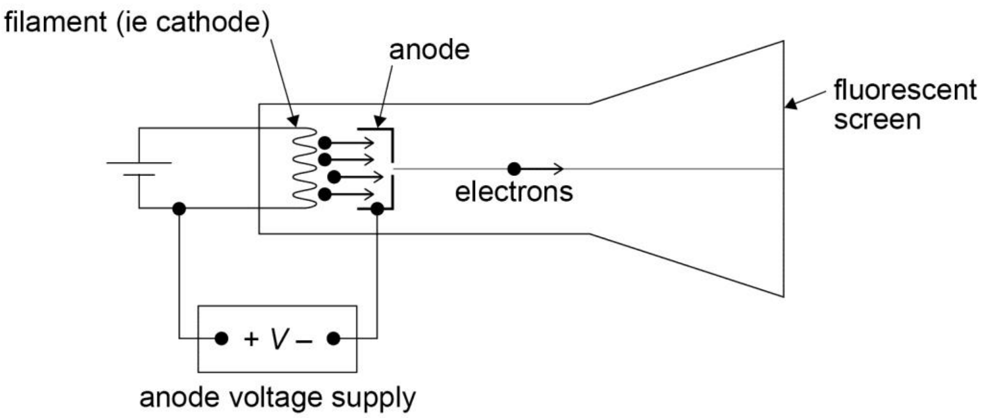
The emission of light from a discharge tube happens because the voltage applied to the tube is so high that some of the gas atoms in the tube are ionised because the electric field pulls electrons out of them. Positive ions created near the cathode are attracted onto the cathode surface, causing free electrons from the cathode surface to be emitted. These electrons are accelerated towards the anode and collide with gas atoms causing them to be ionised. The glowing gas near the cathode, the 'negative glow', is due to photons emitted when some of the positive ions and electrons produced by ionisation recombine.

Some of the electrons pulled out of the gas atoms do not recombine and are attracted to the anode and therefore move away from the cathode – hence the term 'cathode rays' was used to describe them. These electrons move towards the anode and cause excitation by collision of gas atoms in the tube. The positive column of glowing gas is due to de-excitation of these excited gas atoms. The processes of recombination and de-excitation result in the emission of photons of visible and ultraviolet light.

Thermionic emission

Thermionic emission is a much simpler way of producing an electron beam than using a discharge tube. When a metal is heated, some of the electrons that move about freely inside the metal (referred to as 'free' or 'conduction' electrons) gain sufficient kinetic energy to leave the metal at its surface. In practice, the metal is a wire filament which is heated by passing an electric current through it. The filament or 'cathode' is at one end of an evacuated glass tube with a metal plate or 'anode' nearby, as shown in **Figure 2**.

Figure 2 Thermionic emission



The electrons emitted from the filament are attracted to the anode by connecting a high voltage power supply between the anode and the cathode, with the anode positive relative to the filament. Because there are no gas molecules in the tube to stop the electrons, the electrons are accelerated to the anode where some of them pass through a small hole to form a narrow beam.

The work done by the potential difference (pd) V between the anode and the cathode on each electron = eV, where e is the charge of the electron.

The kinetic energy of each electron passing through the hole = $\frac{1}{2}mv^2$ where v is the speed of each electron at this position.

Since the work done on each electron increases its kinetic energy from a negligible value at the cathode, then the speed, v, of each electron leaving the anode is given by

$$\frac{1}{2}mv^2 = eV$$

For the above equation to apply, the speed of the electrons must be much less than the speed of light in free space, c. Students will not be expected in this section to use the relativistic expression for kinetic energy.

b) The specific charge of the electron

Students should know what is meant by the specific charge of the electron and should be able to understand the principles underlying its determination through the use of the relevant equations in the Data and Formulae Booklet, namely:

$$F = \frac{eV}{d}$$

$$F = Bev$$

$$r = \frac{mv}{Be}$$

$$\frac{1}{2}mv^2 = eV$$

As outlined below, given a diagram and description of suitable apparatus and sufficient relevant data, they should be able to use one of more of the above equations to calculate the speed of the electrons in a beam and/or determine the specific charge of the electron from the given data. They should also be able to describe one method, including the data to be collected, to determine the specific charge of the electron. They should appreciate why electron tubes in general need to be evacuated and why the tube in **Figure 3** needs to contain a gas at low pressure.

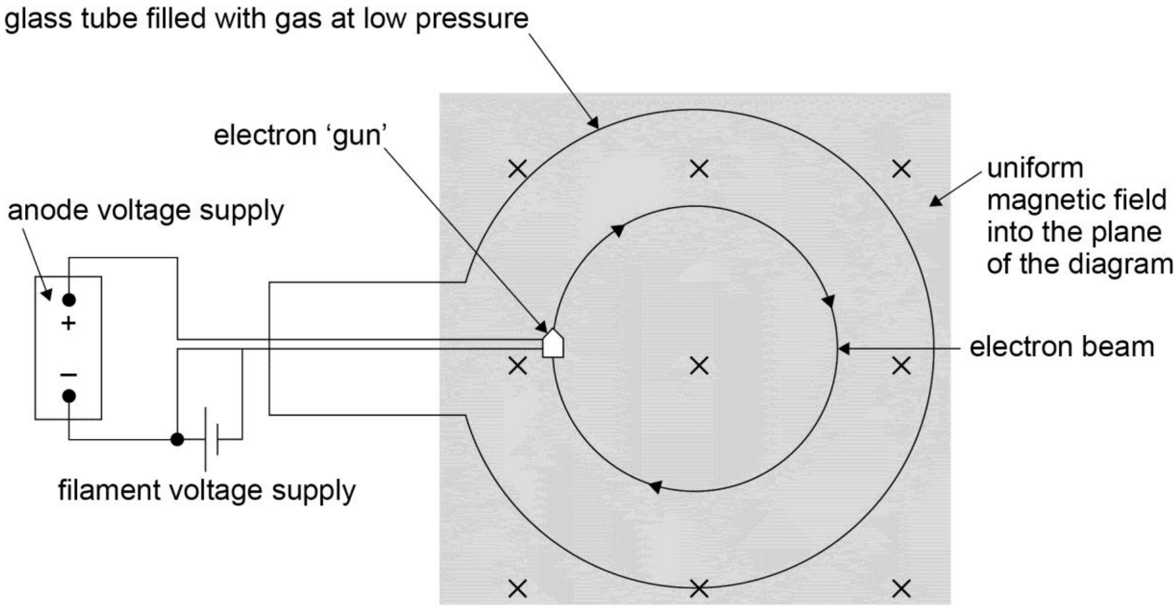
They should also appreciate that the specific charge of the electron was found by Thomson to be much larger than the previous largest specific charge, namely that of the hydrogen ion.

(i) Using a magnetic field to deflect the beam

(a) The radius of curvature r of the beam in a uniform magnetic field of flux density B may be measured using the arrangement shown in **Figure 3** or using a 'Teltron tube' arrangement in which a straight beam enters the field and is deflected on a circular arc by a magnetic field.

Details of how to measure the radius of curvature in each case are **not** required.

Figure 3 Deflection by a magnetic field



Because the magnetic force on each electron ($=Bev$) provides the centripetal force

($= mv2/r$), then $Bev = mv^2/r$ which gives $r = mv/Be$.

To determine e/m, the anode pd V_A must also be measured.

By combining $r = mv/Be$ and $\frac{1}{2}mv^2 = eV_A$ (where V_A is the anode pd), the following equation for e/m is obtained

$$\frac{e}{m} = \frac{2V_A}{B^2 r^2}$$

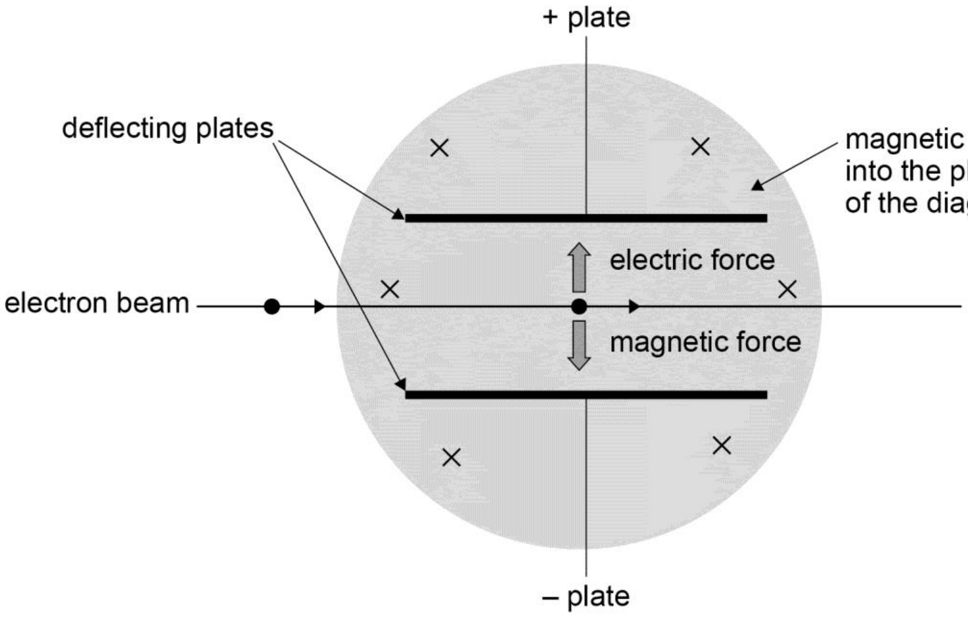
Note students should be able to explain why no work is done on the electrons by the magnetic field.

(ii) Using a magnetic field to deflect the beam and an electric field to balance the deflection

a) The radius of curvature r of the beam in a uniform magnetic field of flux density B is measured. As before, $r = mv/Be$ from $Bev = mv^2/r$.

b) The speed, v, of the electrons is measured directly using an electric field E that is perpendicular to the beam and to the magnetic field B. This arrangement of the fields is referred to as ‘crossed fields’. When the beam is straightened out (ie undeflected), the forces due to the crossed fields are balanced.

Figure 4 Balanced forces due to crossed E and B fields



As the magnetic force (Bev) on each electron is equal and opposite to the electric force (eE), the speed of the electrons passing through undeflected is given by the equation $v = E/B$ from $eE = Bev$.

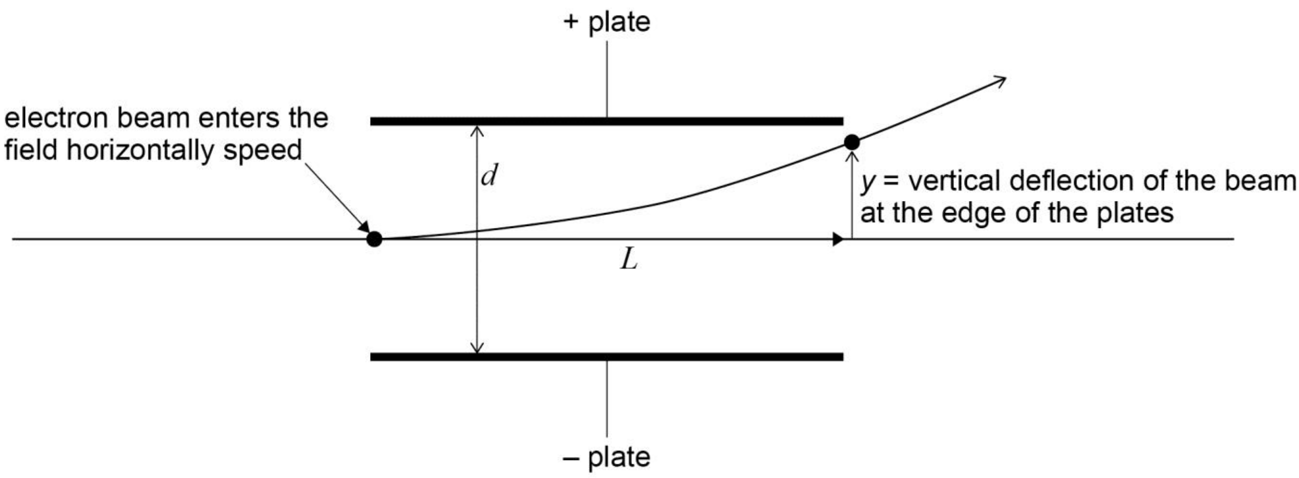
Note that $E = \dfrac{\text{pd between the deflecting plates}}{\text{plate separation}}$

The measured values of v, B and r can then be substituted into the equation

$r = mv/Be$ and the value of e/m calculated.

(iii) Using an electric field to deflect the beam and a magnetic field to balance the deflection

Figure 5 Deflection by a uniform electric field



a) The deflection, y, of the beam at the edge of the plates is measured for a measured plate pd, V_P.

The time taken by each electron to pass between the plates,

$$t = \frac{\text{plate length } L}{\text{electron speed } v}$$

The measurement of the speed of the electrons is explained below.

The acceleration, a, of each electron towards the positive plate can then be calculated from the measured deflection y using the equation $y = \frac{1}{2} a t^2$.

The value of e/m can then be determined as the acceleration, a, of each electron towards the positive plate, $a = F/m = e V/md$ where V is the plate pd and d is the plate separation. Hence $e/m = ad/V$.

b) The speed, v, of the electrons is measured directly using a magnetic field B perpendicular to the beam and to the electric field to straighten the beam out. As explained before, the speed of the electrons passing through undeflected is given by the equation $v = E/B$ (from $eE = Bev$ where E = pd between the deflecting plates/plate separation). Hence the speed can be calculated if E and B are measured when the beam is undeflected.

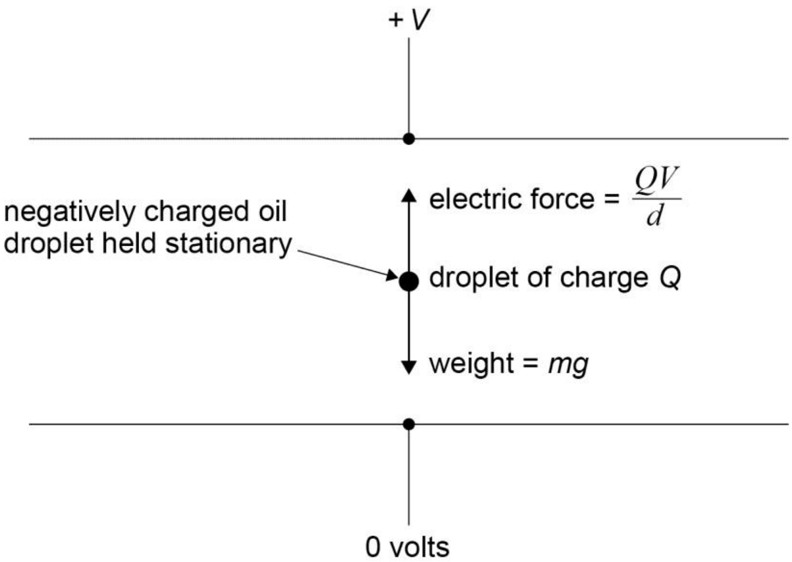
c) Millikan's determination of the charge of the electron

Students are expected to know how a vertical electric field affects the motion of a charged oil droplet. They should have sufficient awareness of the methods used by Millikan to describe how a droplet can be made to be stationary or to move up or down. They should be able to describe the forces on a droplet when it is stationary and when it is moving and be able to explain why a moving droplet reaches a terminal speed. They should know how to find the mass of a droplet from suitable measurements given appropriate data and how to determine the droplet charge from its mass and the electric field strength or plate pd needed to hold it stationary.

They should also appreciate the significance of Millikan's results, in particular the concept of quantisation of charge, and know that the charge of a droplet is an integral multiple of the charge of an electron and that the charge of the electron was considered to be the quantum of charge. Note that students are **not** expected to include the upthrust on the droplet in their considerations.

(i) A charged droplet held stationary by a vertical electric field

Figure 6 A stationary oil droplet

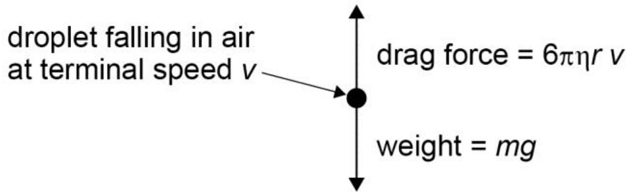


When the droplet is stationary, the electric force (= $Q\, V/d$) on it acting vertically upwards is equal and opposite to the force of gravity on it (ie its weight mg).

Therefore $\frac{QV}{d} = m\,g$ for a droplet of charge Q and mass m held stationary between plates at separation d with a pd V between them.

(ii) A droplet falling vertically with no electric field present

Figure 7 An oil droplet falling at terminal speed



The droplet falls at constant speed because the drag force on it acts vertically upwards and is equal and opposite to its weight. Using Stokes' Law for the drag force $F_D = 6\pi\eta rv$ therefore gives $6\pi\eta rv = mg$.

Assuming the droplet is spherical, its volume = $4\pi r^3/3$ hence its mass m = its density $\rho \times$ its volume = $4\pi\rho r^3/3$.

Hence $6\pi\eta rv = 4\pi\rho r^3/3$ which gives $r^2 = \dfrac{9\eta v}{2\rho g}$.

Thus the radius can be calculated if the values of the speed v, the oil density ρ and the viscosity of air η are known.

Section 2 Wave particle duality

a) Theories of light

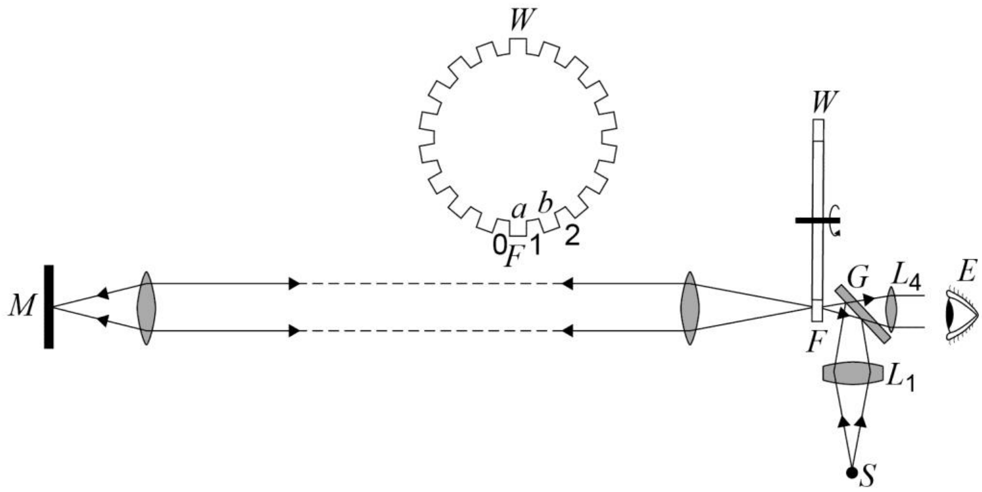
Students should be able to use Newton's corpuscular theory to explain reflection and refraction in terms of the velocity or momentum components of the corpuscles parallel and perpendicular to the reflecting surface or the refractive boundary. They should be able to explain reflection and refraction using wave theory in outline. Proof of Snell's law or the law of reflection is not expected. Newton's ideas about refraction may be demonstrated by rolling a marble down and across an inclined board which has a horizontal boundary where the incline becomes steeper. They should know why Newton's theory was preferred to Huygens' theory.

They should also be able to describe and explain Young's fringes using Huygens' wave theory and recognise that interference cannot be explained using Newton's theory of light as it predicts the formation of two fringes corresponding to the two slits. In addition, they should know that Huygens explained refraction by assuming that light travels slower in a transparent substance than in air, in contrast with Newton's assumption that its speed is faster in a transparent substance. They should appreciate that Newton's theory of light was only rejected in favour of wave theory long after Young's discovery of interference when the speed of light in water was measured and found to be less than the speed in air

Measuring the speed of light.

Early scientists thought that light travelled at an infinite speed. However, the earliest reliable experiments involving astronomical observations (Römer, 1676) suggested that light had a finite speed. In 1849, Fizeau obtained a value that differed by only 5% from that now accepted using a terrestrial method. The arrangement used is shown in **Figure 8**.

Figure 8 Fizeau's experiment for determining speed of light



The principle was to determine the time taken for a beam of light to travel to a mirror and back to the observer. The time was measured using a toothed wheel that was rotated at high speed. Pulses of light were transmitted through the gaps in the wheel. At low speeds of rotation, light from the source passed through a gap and then passed through the same gap on its return so the observer could see the light. As the speed of rotation increased there came a time when the returning beam found its path blocked by the adjacent tooth. As this was the same for all the gaps the observer did not now see any reflected beam. For example, no light would have been seen when light that passes through gap 0 finds its path blocked by tooth a on return, light passing through gap 1 would be blocked by tooth b and so on. When the speed was doubled, the light passing through 0 could pass though gap 1 on return so the reflected beam was once again observable.

If there are n teeth and n gaps then a tooth replaces a gap after $\frac{1}{2n}$ of a revolution. If the frequency is f then the time for 1 revolution is $\frac{1}{f}$ and a tooth replaces a gap after $\frac{1}{2nf}$ seconds. If the distance from F to M is d the speed of light is given by $\frac{2d}{\frac{1}{2nf}} = 4dnf$

In Fizeau's arrangement, the distance from the point of origin of the light F to the mirror M was 8.6 km.

Fizeau's wheel had 720 teeth and 720 gaps. He found that the first time that the light disappeared was at a speed of 12.6 revolutions per second.

This gave the speed of light to be 3.13×10^8 m s^{-1}.

Students are expected to understand the physics principles that underlie the Fizeau experiment, including how the equation for the speed of light, $c = 4dnf$, is derived.

Further experiments on the speed of light

Fizeau and other contemporary scientists refined the experiments and showed the speed of light in water to be lower than that in air, which disproved Newton's corpuscular theory. They also conducted experiments using interference effects to measure the speed in moving water and found that light travelled quicker when emitted in the direction of flow than against the flow. This was known as the 'ether(aether) drift'. However, they found that the speed was lower than the expected simple sum of the speed of light in water plus the speed of the fluid which they could not explain. Their values were later found to be consistent with those predicted by the addition of speeds using Einstein's theory of relativity and was supporting evidence for the theory.

Maxwell and Hertz

In considering Maxwell's theory of electromagnetic waves, students should recognise that Maxwell predicted electromagnetic waves in terms of oscillating electric and magnetic fields before there was any experimental evidence for electromagnetic waves. In addition to being able to describe the nature of an electromagnetic wave, students should know that Maxwell derived the equation $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ for their speed and used it to show that the speed of an electromagnetic wave in a vacuum is the same as the speed of light in free space. In this way, he showed that light is an electromagnetic wave and infra-red and ultraviolet radiations beyond the visible spectrum are also electromagnetic waves. The subsequent later separate discoveries of X-rays and of radio waves confirmed the correctness of Maxwell's predictions. Maxwell's theory demonstrates the power of a theory that predicts observable effects that agree with experimental observations. Students should be aware that ε_0 is a constant that relates the strength of an electric field in free space to the charge that creates it and that μ_0 is a constant that relates the magnetic flux density of a magnetic field to the current that creates it.

Hertz's discovery of radio waves and their properties draw on previous knowledge of polarisation and stationary waves. Hertz showed that radio waves are produced when high voltage sparks jump across an air gap and he showed they could be detected using:

- either a wire loop with a small gap in it,
- or a 'dipole' detector consisting of two metal rods aligned with each other at the centre of curvature of a concave reflector.

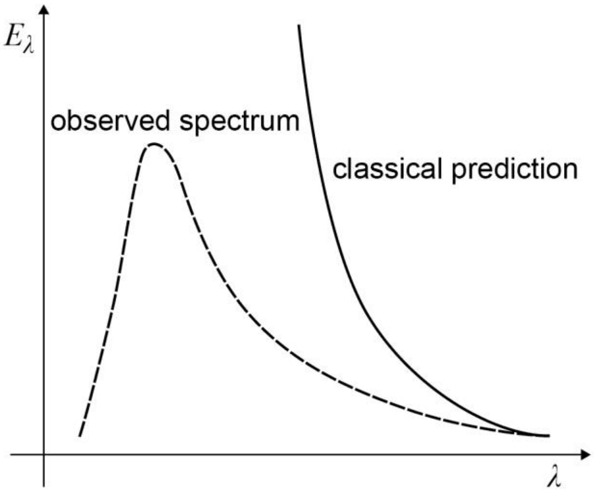
Students should be aware that Hertz also found that the radio waves are reflected by a metal sheet and discovered that a concave metal sheet placed behind the transmitter made the detector sparks stronger. Hertz also produced stationary radio waves by using a flat metal sheet to reflect the waves back towards the transmitter. Students should be able to explain why stationary waves are produced in this way and should be able to calculate the speed of the radio waves from their frequency and the distance between adjacent nodes. Hertz also discovered that insulators do not stop radio waves and he showed that the radio waves he produced are polarised. Students should be able to explain why the detector signal changes in strength when the detector is rotated about the line between the transmitter and the detector in a plane perpendicular to this line.

The ultraviolet catastrophe

The ultraviolet catastrophe refers to the disagreement between practical measurements of the energy intensity at different wavelengths from a black body at a given temperature and the theoretical predictions using classical physics.

A black body is one that emits all wavelengths of radiation that are possible for that temperature. The dotted lines in the graph in **Figure 9** show the practical variation of energy intensity E_λ emitted at different wavelengths with wavelength λ for a particular temperature.

Figure 9 Black body radiation curve



The solid line shows the prediction made using classical physics. This predicts that most of the energy would be emitted at short (ultraviolet) wavelengths and would become infinite at very short wavelengths. Measurements indicate however that there is a peak as shown by the solid line. This peak occurs at a shorter wavelength as the temperature increases (Wein's Law).

The problem was resolved when Planck suggested that:

- radiation is emitted in quanta (which we now call photons)
- the quantum is related to a single frequency
- an energy quantum is hf so high frequency radiation is emitted in larger 'chunks' of energy.

Using the idea of energy 'quanta' as packets of energy Planck was able to develop a theory to explain the shape of the observed spectrum. The proposition by Planck also laid the foundations to solve other areas of physics where classical physics was failing to explain what was being observed.

Photoelectricity was one such phenomenon and the photon theory became firmly established when Einstein's used the photon theory of light to explain the observations.

Photoelectricity

Note that a study of the photoelectric effect is in the Physics core (section 3.2.2.1) so this knowledge will be assumed.

It is included here to complete the study on the nature of light.

Just as students are expected to be aware of the main differences between wave theory and corpuscular theory, students should know why wave theory was rejected in favour of the photon theory and they should be aware of the differences between the photon theory and corpuscular theory. They should know that metals emit electrons when supplied with sufficient energy and that thermionic emission involves supplying the required energy by heating the metal, whereas photoelectric emission involves supplying energy by illuminating the metal with light above a certain frequency.

Photoelectric emission was first discovered by Hertz when he was investigating radio waves using a spark gap detector. He observed that the sparks were much stronger when ultraviolet radiation was directed at the spark gap contacts. Investigations showed that for any given metal:

- photoelectric emission does not occur if the frequency of the incident light is below a threshold frequency
- photoelectric emission occurs at the instant that light of a suitably high frequency is incident on the metal surface
- the photoelectrons have a range of kinetic energies from zero up to a maximum value that depends on the type of metal and the frequency of the incident light.

The number of photoelectrons emitted from the metal surface per second is proportional to the intensity of the incident radiation (ie the light energy per second incident on the surface).

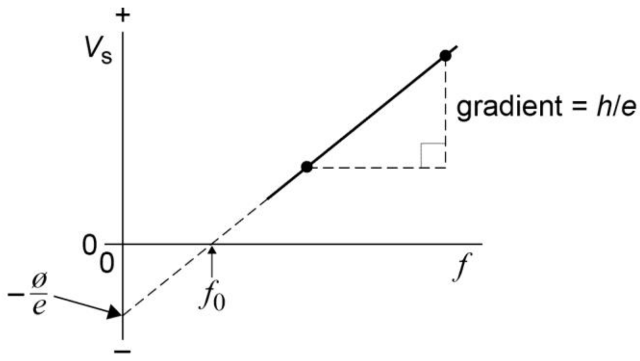
Students should know why the wave theory of light fails to explain the threshold frequency and the instantaneous emissions and they should be able to use the photon theory to explain these observations. According to wave theory, light of any frequency should cause photoelectric emission. Wave theory predicted that the lower the frequency of the light, the longer the time taken by electrons in the metal to gain sufficient kinetic energy to escape from the metal. So the wave theory could not account for the existence of the threshold frequency and it could not explain the instant emission of photoelectrons or their maximum kinetic energy.

To explain the existence of a threshold frequency of light for each metal, students should know that in order for a conduction electron to escape, it needs to absorb a single photon and thereby gain energy hf and that the electron uses energy equal to the work function ϕ of the metal to escape. They should be able to explain and use the equation $E_K = hf - \phi$ and they should be able to explain why the threshold frequency of the incident radiation, $f_0 = \phi/h$. They should also be aware that the mean kinetic energy of a conduction electron in a metal at room temperature is negligible compared with the work function of the metal so that the electron can only escape if the energy it gains from a photon is greater than or equal to the work function of the metal.

Students should know that photoelectrons need to do extra work to move away from the metal surface if it is positively charged (relative to a collecting electrode) and that the number of photoelectrons emitted per second decreases as the potential of the metal is made increasingly positive. They should know that at a certain potential, referred to as the stopping potential, V_S, photoelectric emission is stopped because the maximum kinetic energy has been reduced to zero and they should be able to recall and explain:

- why the stopping potential is given by $eV_S = hf - \phi$
- why the graph of V_S against f is a straight line with a gradient and intercepts as shown in Figure 10.

Figure 10 Stopping potential v frequency



They should appreciate how the stopping potential may be measured using a potential divider and a photocell and how the measurements may be plotted to enable the value of h and the value of ϕ to be determined.

The first measurements were obtained by RA Millikan and gave results and a graph as shown in **Figure 10** above, that confirmed the correctness of Einstein's explanation and thus confirmed Einstein's photon theory of light. Einstein thus showed that light consists of photons which are wavepackets of electromagnetic radiation, each carrying energy hf, where f is the frequency of the radiation. Einstein was awarded the 1921 Nobel Prize for physics for the photon theory of light which he put forward in 1905, although it was not confirmed experimentally until 10 years later.

Students should know that the photon is the least quantity or 'quantum' of electromagnetic radiation and may be considered as a massless particle. It has a dual 'wave particle' nature in that its particle-like nature is observed in the photoelectric effect and its wave-like nature is observed in diffraction and interference experiments such as Young's double slits experiment.

b) Matter waves

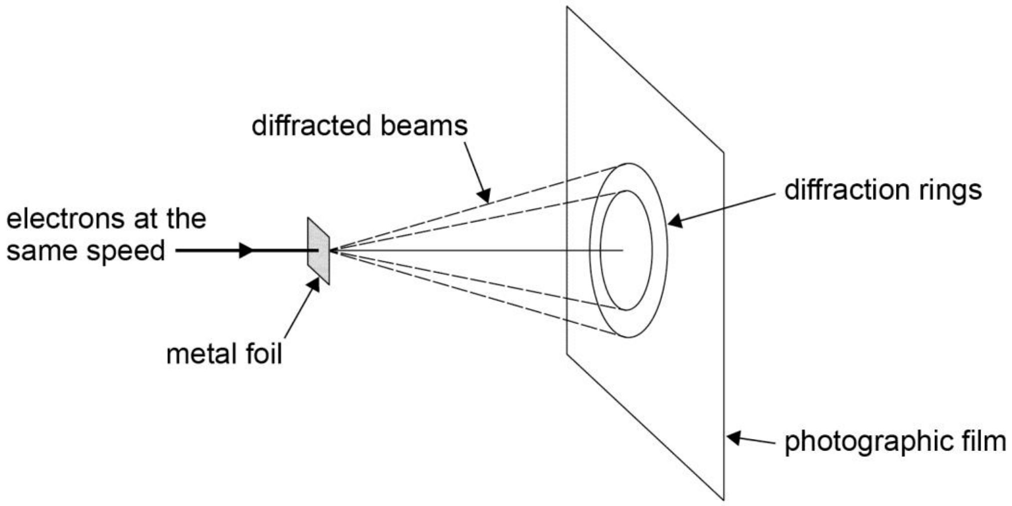
Students should know from their AS course that de Broglie put forward the hypothesis that all matter particles have a wave-like nature as well as a particle-like nature and that the particle momentum mv is linked to its wavelength by the equation:

$m\,v \times \lambda = h$ where h is the Planck constant.

De Broglie arrived at this equation after successfully explaining one of the laws of thermal radiation by using the idea of photons as 'atoms of light'. Although photons are massless, in his explanation he supposed a photon of energy hf to have an equivalent mass m given by $mc^2 = hf$ and therefore a momentum $mc = hf/c = h/\lambda$ where λ is its wavelength.

De Broglie's theory of matter waves and equation 'momentum × wavelength = h' remained a hypothesis for several years until the experimental discovery that electrons in a beam were diffracted when they pass through a very thin metal foil. **Figure 11** shows an arrangement.

Figure 11 Diffraction of electrons



Photographs of the diffraction pattern showed concentric rings, similar to those obtained using X-rays. Since X-ray diffraction was already a well-established experimental technique for investigating crystal structures, it was realised that similar observations with electrons instead of X-rays meant that electrons can also be diffracted and therefore they have a wave-like nature. So de Broglie's hypothesis was thus confirmed by experiment. Matter particles do have a wave-like nature.

The correctness of de Broglie's equation was also confirmed as the angles of diffraction were observed to increase (or decrease) when the speed of the electrons was decreased (or increased).

Students should be able to outline suitable experimental evidence such as above in support of de Broglie's hypothesis and should be able to state and explain the effect of an increase (or decrease) in the speed of the electrons on the de Broglie wavelength and hence on the observations (eg the angle of diffraction above). They should know that the electrons are diffracted more (or less) if their wavelength is increased (or decreased), but they are not expected to explain the diffraction pattern (eg the formation of concentric rings above).

In addition, students should be able to:

- relate the de Broglie wavelength for a beam of charged particles (eg electrons) to the pd through which they have been accelerated from rest
- calculate the de Broglie wavelength for a given pd.

For example, for electrons produced by thermionic emission, the speed v of the electrons depends on the anode potential V in accordance with the equation:

$\frac{1}{2}mv^2 = eV$ assuming $v << c$, the speed of light in free space.

Rearranging this equation gives $mv = (2\,m\,e\,V)^{1/2}$. Hence, the de Broglie wavelength λ given in terms of the anode potential V_A is given by

$$\lambda = \frac{h}{\sqrt{(2\,m\,e\,V)}}$$

c) Electron microscopes

Students should be able to describe the main features of the transmission electron microscope (TEM) and the scanning tunneling microscope (STM) and be able to explain how each type of microscope gives an image. In addition, they should know the significance of the wave nature of the electron in relation to image formation in both microscopes and in relation to image resolution in the TEM. In addition, they should know why the image resolution of the TEM is limited.

i) The transmission electron microscope

The transmission electron microscope was invented in 1931 by Max Knoll and Ernst Ruska who realised that electron waves in an 'electron microscope' would produce much higher resolving power than light waves in an optical microscope.

Figure 12 Transmission electron microscope

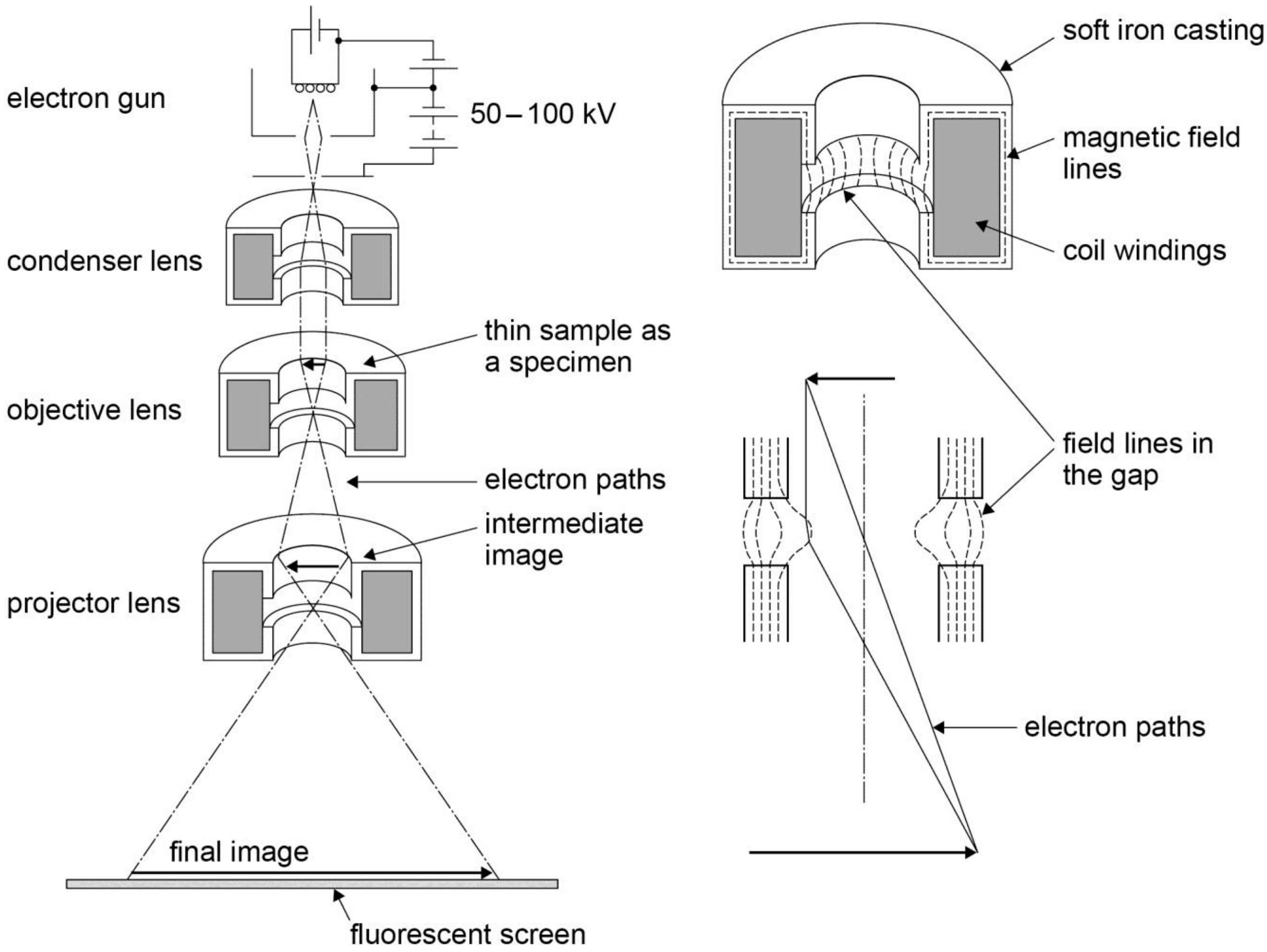


Figure 12 shows an outline of the transmission electron microscope in which the beam of electrons passes through the sample to form an image on a fluorescent screen. The electrons are produced by thermionic emission from a filament wire and are accelerated through a pd of between 50 to $100\ \mathrm{kV}$. The system of magnetic lenses in the latest TEMs is capable of l07 times magnification at most, enabling images no smaller than about $0.1\ \mathrm{nm}$ to be seen.

Students should know the effect of the magnetic lenses on the electrons transmitted through the object as shown in **Figure 12**. Electrons passing near the gap in the soft iron shield are deflected towards the axis of the microscope; electrons passing through the centre are undeflected. Hence a magnetic lens can be compared with an optical convex lens which deflects light rays passing near the edge towards the centre and allows light rays through the centre without change of direction. The condenser lens deflects the electrons into a wide parallel beam incident uniformly on the sample. The objective lens then forms an image of the sample. The projector lens then casts a second image onto the fluorescent screen.

They should know that the amount of detail in the image is determined by the resolving power which increases as the wavelength of the electrons decreases. They should also know that the wavelength becomes smaller at higher electron speeds so that raising the anode potential in the microscope gives a more detailed image. In addition, they should know that the amount of detail possible is limited by lens aberrations (because the lenses are unable to focus electrons from each point on the sample to a point on the screen since some electrons are moving slightly faster than others) and sample thickness (because the passage of electrons through the sample causes a slight loss of speed of the electrons which means that their wavelength is slightly increased, thus reducing the detail of the image).

For a given anode potential, as outlined earlier, students should be able to calculate the de Broglie wavelength of the electrons using the de Broglie equation $\lambda = \frac{h}{\sqrt{(2meV)}}$; for example, the electrons in the beam of a TEM operating at $80\ \mathrm{kV}$ would have a de Broglie wavelength of about $0.004\ \mathrm{nm}$. In addition, students should appreciate that:

- in theory, electrons of such a small wavelength ought to be able to resolve atoms less than $0.1\ \mathrm{nm}$ in diameter
- in practice in most TEMs, electrons of such a small wavelength do not resolve such small objects for the reasons outlined above.

Note, the TEAM 0.5 electron microscope at the US Lawrence Berkeley Laboratory is the most powerful electron microscope in the world. Aberration correctors developed at the laboratory are fitted in the $80\ \mathrm{kV}$ microscope, enabling individual atoms to be seen.

ii) The scanning tunneling microscope

The scanning tunneling microscope (STM), invented in 1981, gives images of individual rows of atoms. Students should know that the STM is based on a fine-tipped probe that scans across a small area of a surface and that the probe's scanning movement is controlled to within $0.001\ \mathrm{nm}$ by piezoelectric transducers. They should be aware that the probe is at a small constant potential, with the tip held at a fixed height of no more than $1\ \mathrm{nm}$ above the surface so that electrons 'tunnel' across the gap. They should also know that if the probe tip moves near a raised atom or across a dip in the surface, the tunneling current increases or decreases respectively due to a respective decrease or increase of the gap width. They should also know that:

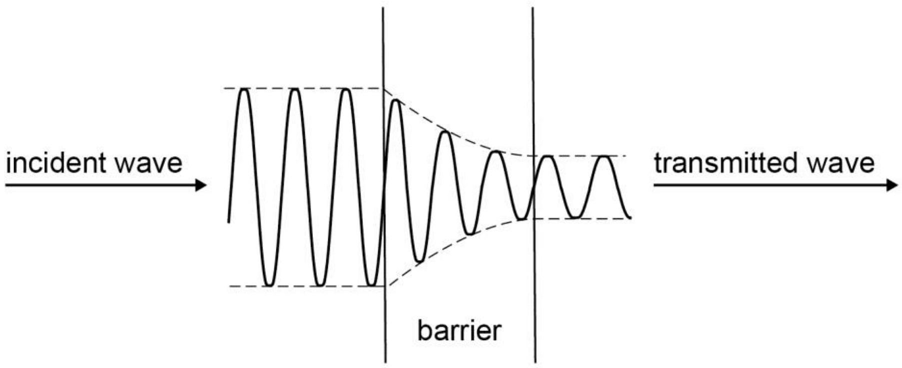
- in constant height mode, the change of current is used to generate an image of the surface provided the probe’s vertical position is unchanged

- in constant current mode, the change of current is used to move the probe vertically upwards or downwards respectively until the gap width and the current is the same as before.

The vertical resolution is of the order of $0.001\ \mathrm{nm}$, much smaller than the size of the smallest atom.

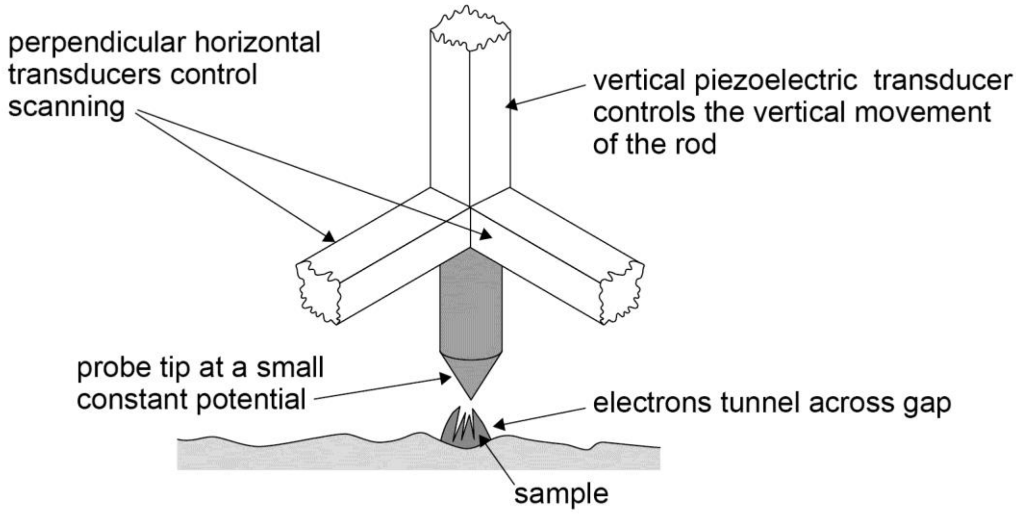
The principle of tunneling is based on the wave nature of particles. Light can be seen through a thin metal film because the amplitude of the light waves is not reduced to zero by the passage of the light in the film. In the same way, the amplitude of matter waves in a barrier does not become zero if the barrier is sufficiently narrow. This process is referred to as 'tunneling'.

Figure 13A Quantum tunnelling

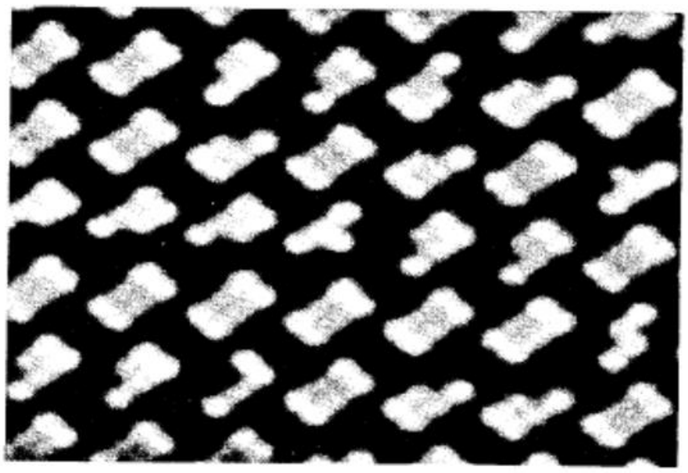


The 'de Broglie wavelength' of an electron depends on its momentum, and equals about $1\ \mathrm{nm}$ for electrons in a metal at room temperature. Hence tunneling is possible for gaps of the order of $1\ \mathrm{nm}$ or so and the tunneling current is sensitive to changes of the gap width as little as $0.001\ \mathrm{nm}$.

Figure 13B Scanning tunneling microscope



An example of an atomic resolution STM image is shown below where the dimensions of this image are 10.5×7.1 nm. The image is of one particular reconstruction of a 111-V semiconductor surface, namely the (100) surface of indium antimonide (InSb), and shows pairs of Sb atoms (dimers) arranged in groups of three in a brickwall-like structure. This image is an example of the best spatial resolution achievable from this class of materials. Note that, although some of these groups are incomplete, each bright dot represents the position of a single Sb atom.



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Section 3 Special relativity

The notes below are intended to indicate the depth of study expected of students. Proofs for the formulae in the specification are **not** required and will **not** be examined.

a) Frames of reference

Einstein's theory of special relativity is the result of analysing the consequences of the absence of any universal frame of reference. Special relativity concentrates on frames of reference which are moving at constant velocity relative to each other. These frames are known as inertial frames of reference.

Special relativity is based on two postulates:

- the laws of physics, expressed in equations, have the same form in all inertial frames
- the speed of light in free space is invariant (ie the same for all observers regardless of their state of motion and of the speed of the light source).

Einstein realised that these postulates were a consequence of Maxwell's equations and that if the first postulate is to hold, then the speed of light in free space has to be independent of the observer.

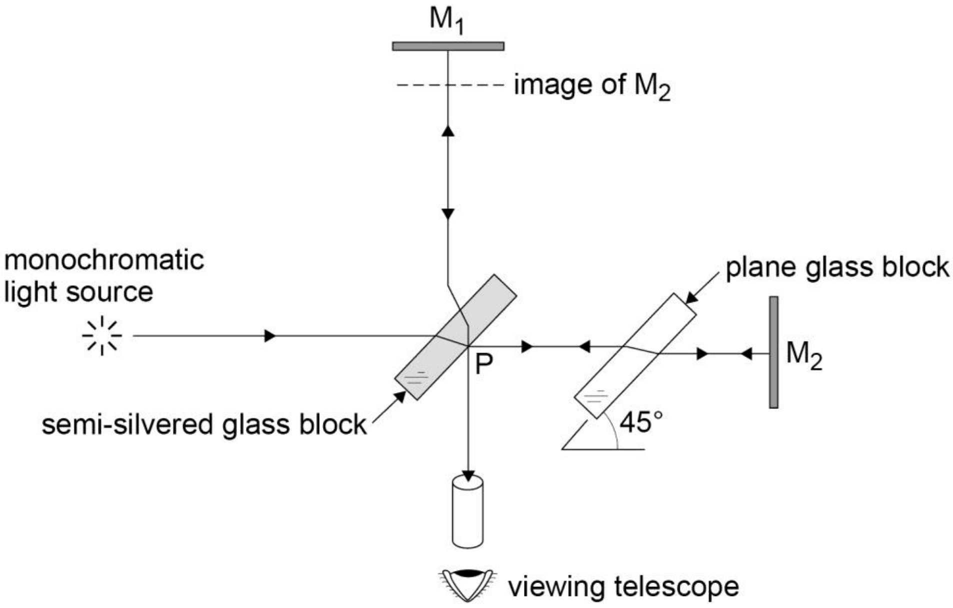
b) The Michelson-Morley experiment

This experiment was devised to try to determine the absolute speed of the Earth through the 'ether' (or 'aether'). The ether was thought to be a substance that filled space and that was essential for the propagation of light.

The proposition was that, if an ether existed, then as the Earth moved though the ether, it should cause light to travel at different speeds when travelling in the same direction as the Earth than in the opposite direction (recall that Fizeau had discovered this variation in his experiments with light through moving water). The experiment was first carried out in 1881 to find out if the speed of light in the direction of the Earth's motion through space (through the ether) differed from the speed of light in a perpendicular direction (where there would be no ether drag).

This was investigated using the Michelson interferometer, as outlined in **Figure 14.**

Figure 14 Michelson-Morley experiment



The semi-silvered glass block splits the beam of monochromatic light into two beams at P which is on the semi-silvered side of the block. The plane block is necessary to ensure that both beams pass through the same thickness of glass and air. Plane mirror M_1 and the image of plane mirror M_2 are effectively a parallel sided film of air; hence, with an extended monochromatic source, concentric fringes of equal separation w are seen. An analogy of the experiment can be demonstrated using a suitable microwave transmitter and receiver on a board on a rotating turntable with hardboard squares in place of the glass blocks and metal plates in place of the mirrors.

Michelson and Morley calculated that, if the speed of light in free space depends on the Earth's velocity, the interference pattern should shift by about $0.4\,w$ of a fringe when the apparatus is rotated through $90°$.

Suppose the two paths PM_1 and PM_2 are of the same length and PM_1 lies in the direction of the Earth's velocity. The time taken for light to travel from P to M_1 and back to P would be greater than the time taken from P to M_2 and back to P if the speed of light depends on the Earth's velocity. Rotating the apparatus through $90°$ would cause the time difference to reverse, making the interference pattern shift.

Although the apparatus was capable of detecting shifts of 0.05 fringe, no shift was detected then or in later experiments, thus failing to detect absolute motion.

The experiment showed that:

- either the ether did not exist so light travels without the need for a material medium, unlike all other known mechanical waves
- or the Earth was dragging the ether with it.

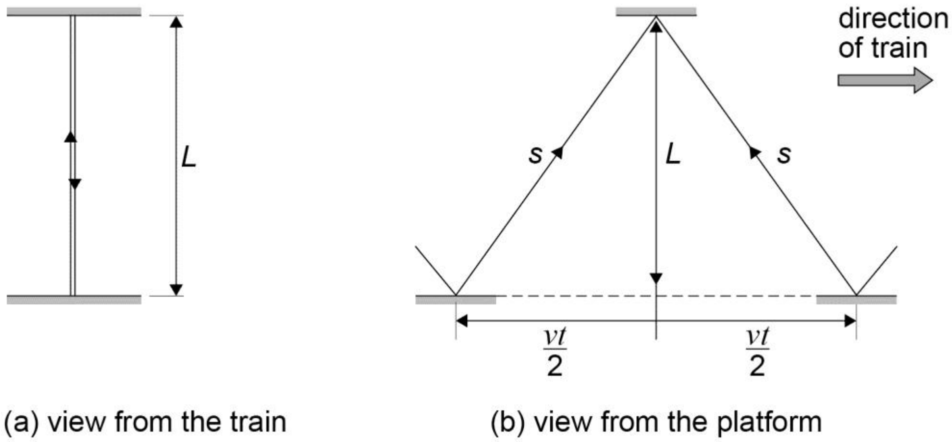
Another conclusion from the null result is that the speed of light was not affected by the Earth's motion, (ie the speed of light is invariant in free space). This was not explained until 1905 when Einstein put forward the theory of Special Relativity.

Students are expected to understand the physics of the interferometer and outline how it is used as a means of attempting to detect absolute motion. In addition, students should be able to explain the significance of the null result in terms of the invariance of the speed of light and Einstein's theory of Special Relativity.

c) Time dilation and proper time

A consequence of the invariance of the speed of light in free space is that 'moving clocks run slow' or time runs slower when you are moving. Consider an observer sitting in a moving train with a clock which is used to time a light pulse reflected between two horizontal mirrors in the carriage, one directly above the other at distance L apart, as shown in Figure 15. The train is travelling along a track parallel to a platform with a second observer watching.

Figure 15 Time dilation



The moving observer times how long a light pulse takes to travel from one mirror to the other mirror, and back again. Since the distance travelled by the light pulse is $2L$, the time taken is $2L/c$ where c is the speed of light in free space. This is the proper time t_0.

The platform observer sees the light pulse travel a distance 2 s where

$$s = \sqrt{\left(L^2 + \frac{v^2t^2}{4}\right)}.$$

According to this observer, the light pulse takes time t which is greater than t_0 because the pulse travels a greater distance.

Distance travelled, s, is given by

$$s = ct = 2\sqrt{\left(L^2 + \frac{v^2t^2}{4}\right)}$$

Rearranging this equation gives

$$t = \frac{2L}{(c^2 - v^2)^{\frac{1}{2}}} = t_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

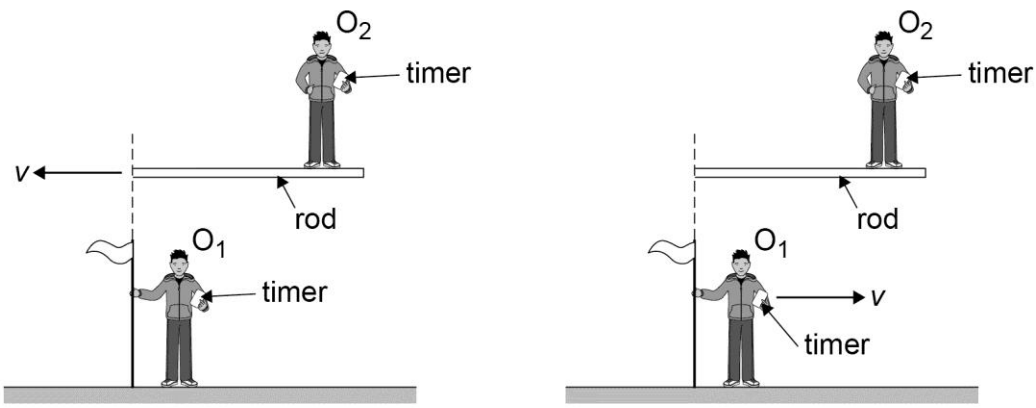
The observer in the train measures the proper time, t_0, since they are stationary with respect to the light clock. The time interval, t, according to the platform observer is greater than the proper time t_0. Time runs more slowly for the moving observer.

d) Length contraction and proper length

Having seen that moving clocks run slow, another consequence of special relativity is that the length of a moving rod is found to be different when measured by an observer moving parallel to the rod and an observer at rest relative to the rod.

The proper length, L_0, of the rod is its length as measured by an observer at rest relative to the rod. To understand why the length measured by a moving observer is less, consider the situation as seen by each observer in turn.

Figure 16 Length contraction



- Observer O_1 measures the time taken, t_0, for the rod to pass by when it is moving at velocity, v. Hence the length of the rod according to this observer is $L = vt_0$.
- The time taken, t, for observer O_1, moving at velocity, v, to pass the rod as measured by observer O_2 is equal to

$$t = \frac{t_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

since O_1 is moving at velocity v relative to O_2. This is a consequence of time dilation.

Hence, the length of the rod according to O_2, its proper length, L_0 is equal to vt.

$$\text{Therefore } L_0 = v \times \frac{t_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}} = \frac{L}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

$$\text{which gives } l = l_0 \left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

e) Evidence for time dilation and length contraction

When cosmic rays reach the ionosphere above the Earth's surface, unstable sub-atomic particles called muons are created, travelling at 0.996 c. These particles can also be produced in the laboratory. The half-life of low-energy muons produced in the laboratory is 1.5 μs.

Measurements by Bruno Rossi showed that about 80% of the muons produced by cosmic radiation on a mountain top reached an observatory 2 km below. In one half-life of 1.5 μs, muons moving at 0.996 c would travel no further than 450 m (= 0.996 c × 1.5 μs). Thus a height difference of 2 km ought to reduce the intensity of cosmic muons

by $\left(\frac{1}{2}\right)^{\frac{2000}{450}}$ which is less than 10% of the initial intensity.

The discrepancy can be explained in terms of time dilation or length contraction;

The time dilation formula gives the half-life of muons moving at 0.996 c as 17 μs = 1.5 μs × (1 – 0.9962)–0.5.

Muons at this speed cover a distance of 2 km in 6.69 μs (= 2 km / 0.996 c). Since 6.69 μs is 0.40 of the half-life of 17 μs, the intensity, I, of cosmic muons after travelling a distance of 2 km should be given by

$I = \frac{I_0}{2^{0.40}}$ = 76% of I_0 which agrees with observation.

The length contraction formula can be applied by considering the relativistic contraction of the distance of 2 km as 'seen' by the muons moving at 0.996. This distance contracts to

177 m (= 2000 × $\sqrt{(1-0.966^2)}$)

in the muon frame of reference.

Muons travel a distance of 450 m in one half-life of 1.5 μs. Since 177 m / 450 m equals 0.40, the intensity I of cosmic muons after travelling this distance should be given by

$I = \frac{I_0}{2^{0.40}}$ = 76% of I_0 which agrees with observation.

Relativistic energy

By considering the law of conservation of momentum in different inertial frames of reference, Einstein showed that the mass m of an object depends on its speed v in accordance with the equation

$$m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

where m_o is the rest mass or 'proper mass' of the object (ie its mass at zero speed).

The equation above shows that:

- the mass increases with increasing speed
- as the speed increases towards the speed of light, the mass becomes ever larger.

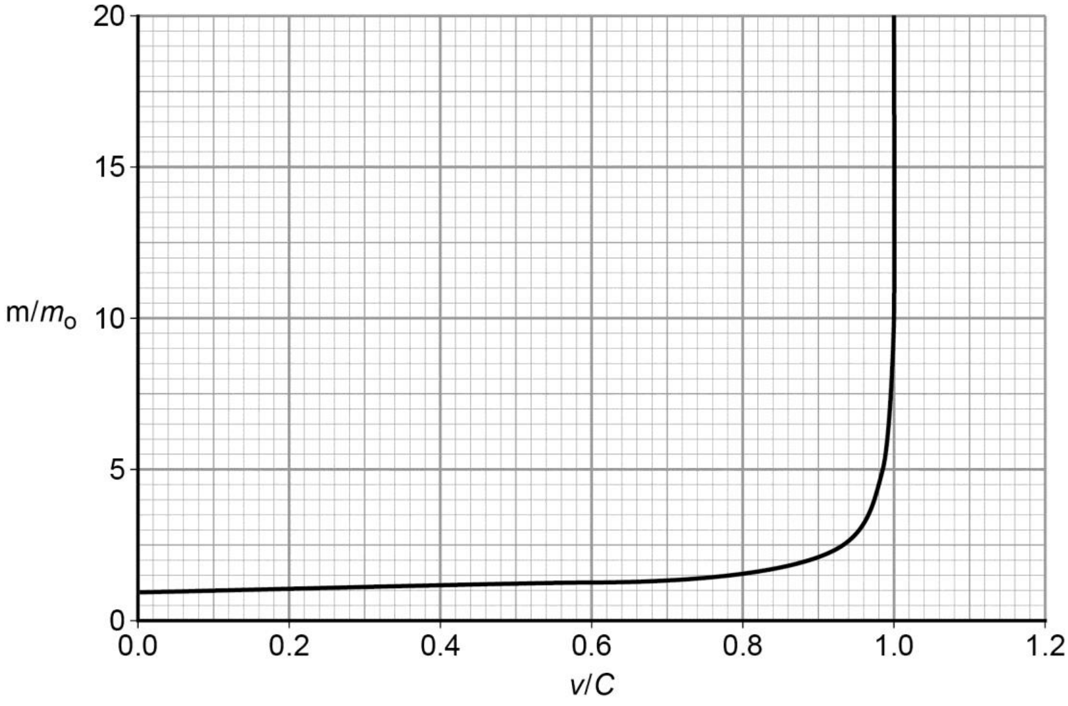
Experimental evidence for relativistic mass was obtained from electron beam experiments soon after Einstein published the theory of special relativity. The specific charge of the electron, e/m, was measured for electrons moving at different speeds. For example, at a speed of 0.69 c, e/m was measured as 1.28×10^{11} C kg^{-1} which can be compared with the accepted value of 1.76×10^{11} C kg^{-1} for the 'rest' value, e/m_o. Using the relativistic mass formula, the value of e/m at 0.69 c is 1.28×10^{11} C kg^{-1}.

Figure 17 shows how the mass of an object varies with its speed in accordance with the relativistic mass equation. The graph shows that:

- at speed $v << c$, $m = m_0$
- as speed $v \to c$, m increases gradually to about $2\ m_0$ at $v \approx 0.9$ c and then increases sharply and tends to infinity as v approaches c.

Einstein's relativistic mass formula means that no material object can ever reach the speed of light as its mass would become infinite and therefore no material object can ever travel faster than light.

Figure 17 Relativistic mass v speed



f) Mass and energy

In his theory of special relativity, Einstein proved that transferring energy in any form:

- to an object increases its mass.
- from an object decreases its mass.

He showed that energy E and mass m are equivalent (ie interchangeable) on a scale given by the equation

$E = m\,c^2$

Since the value of $c = 3.0 \times 10^8$ m s^{-1}, then 1 kg of mass is equivalent to 9.0×10^{16} J (= $1 \times (3.0 \times 10^8)^2$).

In terms of the rest mass m_0 of an object, the above equation may be written as

$$E = \frac{m_0 c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

At zero speed, $v = 0$, $E = m_0\,c^2$ which represents the rest energy E_0 of the object.

At speed v, the difference between its total energy E and its rest energy E_0 represents its energy due to its speed (ie its kinetic energy). Therefore,

its kinetic energy $E_k = m\,c^2 - m_0\,c^2$

For example, if an object is travelling at a speed $v = 0.99$ c, the relativistic mass formula gives its

mass $m = \dfrac{m_0}{\sqrt{(1-0.99^2)}}$

$= 7.1\ m_0$ so its kinetic energy $E_k = 7.1\ m_0\, c^2 - m_0\, c^2 = 6.1\ m_0\, c^2$.

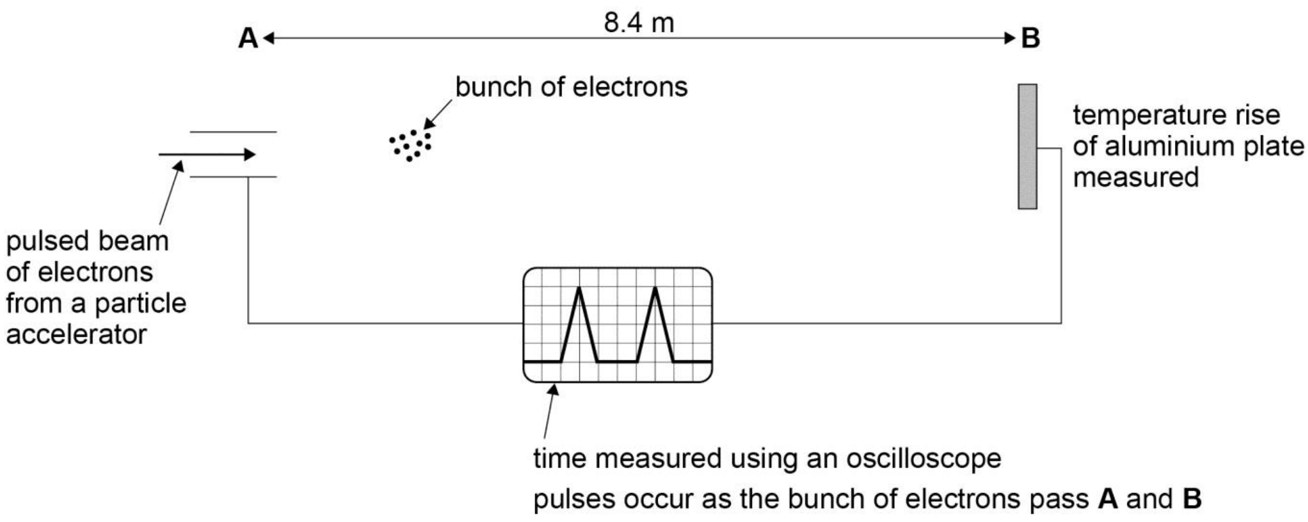
Bertozzi's experiment

This experiment set out to determine the variation of the kinetic energy of an electron with velocity based on direct measurements.

When a charged particle of charge Q is accelerated from rest through a potential difference V to a certain speed, the work done on it is $W = Q\,V$. Its kinetic energy after being accelerated is therefore equal to $Q\,V$ and its total energy $E = m\,c^2 = \mathrm{m_0}\,\mathrm{c}^2 + QV$. In previous experiments the kinetic energy had been determined indirectly using $E_K = VQ$.

Bertozzi (1962) used the apparatus shown schematically in **Figure 18**.

Figure 18 Bertozzi's experiment



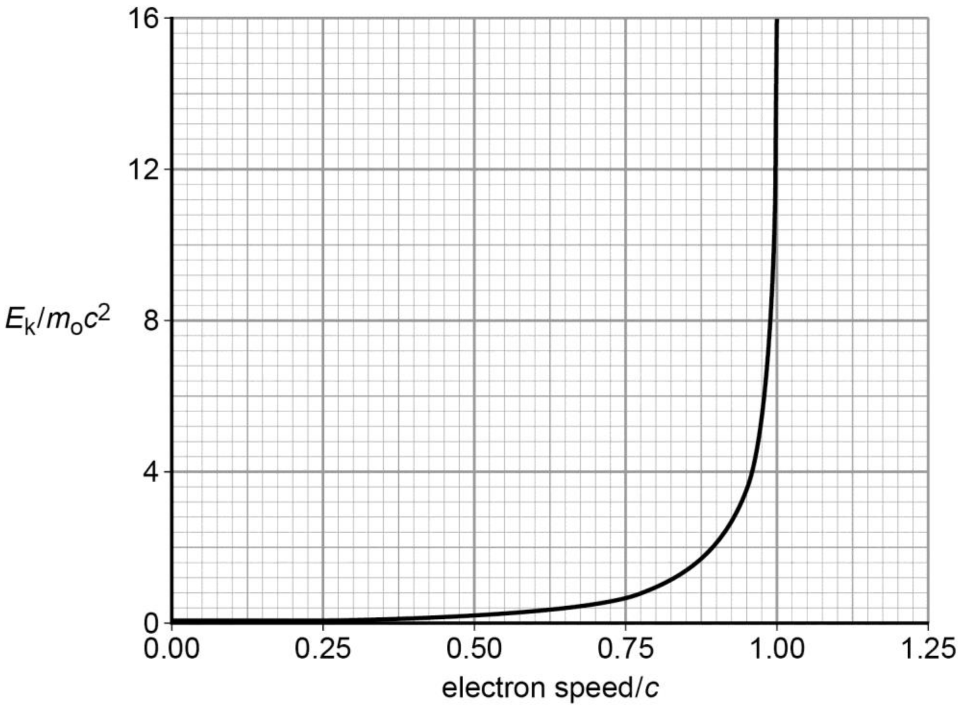
The speeds of bunches of electrons that had been accelerated in a particle accelerator were measured directly for five different accelerating voltages. The speeds were measured over a distance of 8.4 m using a 'time of flight' method and agreed with those expected from the principle of relativity.

For each speed, the kinetic energy was measured directly at the end of its flight by a calorimetric method. The electrons gave up their energy to an aluminium plate of mass m and specific heat capacity c. The temperature rise $\Delta\theta$ of the plate was measured for a known number n of electrons hitting the plate. The kinetic energy E_K of an electron was then $mc\Delta\theta/n$ where m was the mass of the aluminium plate.

The graph in **Figure 19** shows how the expected variation of kinetic energy varies with the velocity of the electron beam special relativity,

$$\frac{m_0 c^2}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} - m_o c^2.$$

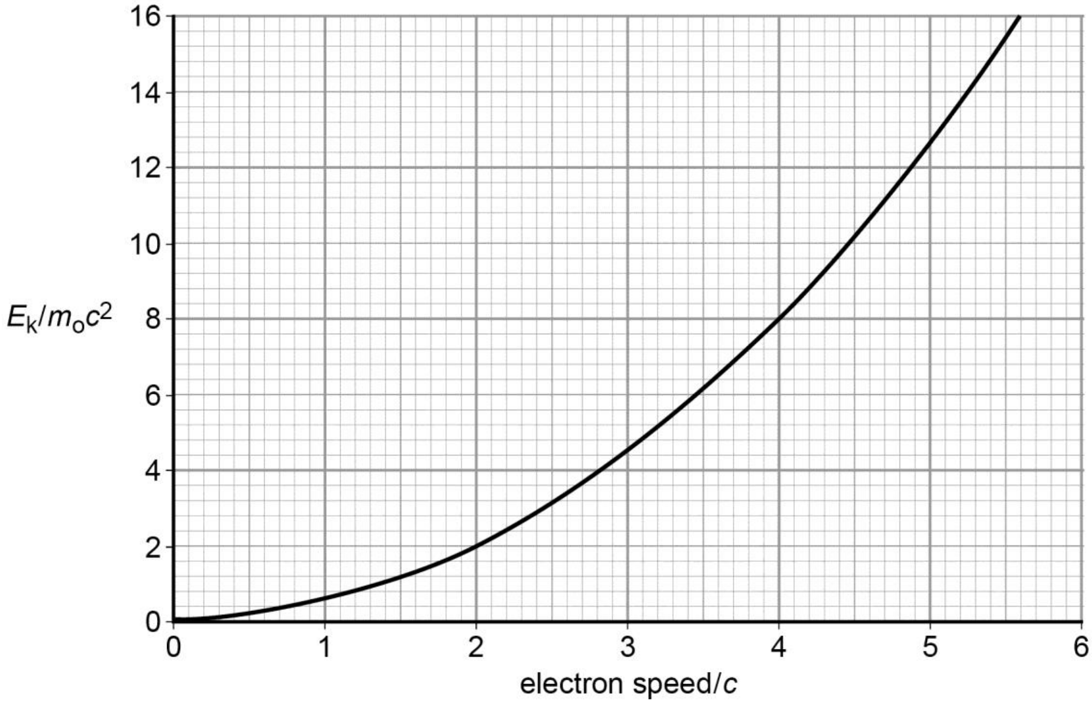
Figure 19 Variation of kinetic energy with velocity using relativity



For each electron, kinetic energy used by Bertozzi, the velocity of the electron beam was found to be within 10% of the values expected by the special relativity formula. The results confirmed that although the kinetic energy continues to increase, the speed of the electrons approaches a limiting value.

Using classical physics $E = mv^2/2$ for the same range of kinetic energies, the electron speeds would vary as shown in **Figure 20**. However, this only agrees with the practical results for the variation of E_K with v for very low speeds ($v<<c$).

Figure 20 Variation of kinetic energy with velocity using *$mv^2/2$*



The graphs show that to reach a kinetic energy equal to $16 \times m_oc^2$ using classical Newtonian physics would need the speed to reach about $5.5\ c$ whereas Bertozzi's results show this energy to be achieved without the speed exceeding c.

Note: At speeds $v << c$, the above kinetic energy formula $E_k = mc^2 - m_0 c^2 \rightarrow \frac{1}{2}mv^2$ as $(1 - (v^2/c^2))^{-1/2} \rightarrow (1 + v^2/2c^2)$ for $v << c$. Students do **not** need to know this but it **may be** helpful for teachers to know how the formula $E_k = \frac{1}{2}mv^2$ fits in.

Appendix A Suggested experiments and demonstrations

1. Discharge demonstration using a (demountable) discharge tube
2. Determination of e/m
3. Measurement of e
4. Estimate of h using a vacuum photocell or different coloured LEDs
5. Measurement of the speed of light, c
6. Microwave analogue of the Michelson-Morley experiment
7. Investigation of the variation of the wavelength of electrons with kinetic energy using an electron diffraction tube
8. Determination of the speed of β–particles using magnetic deflection

Experiments 2, 3 and 5 are essentially demonstrations that support the option specification closely and may be used where appropriate to obtain values of e/m, e and c. Experiment 4 may be carried out as a demonstration or as a student experiment. The speed of light measurement, the microwave analogue and the electron diffraction experiment can be carried out as demonstrations to provide awareness of key experiments in the relevant topics. The provision of adequate instructions and close supervision is vital where a student is conducting an experiment using evacuated tubes, high voltage equipment or radioactive sources.