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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Statistics 6380

SS04 Statistics 4

Mark Scheme

2008 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

SS04

Q	Solution	Marks	Total	Comments
1(a)	$H_0 : \mu = 7$ $H_1 : \mu > 7$	B1		Both
	Test statistic $= \dfrac{7.5 - 7}{1.15/\sqrt{12}}$	M1 m1		Correct form of ts SD divided by $\sqrt{12}$
	$= 1.51$	A1		AWRT
	$\nu = 11$	B1		
	Critical value of $t = 1.796$	B1√		ft on ν
	$1.51 < 1.796$	E1		Must compare ts with cv or use diagram
	Not enough evidence at the 5% significance level to claim that daffodils last longer than 7 days on average	A1√	8	dep on M1 and m1; ft on ts and cv
(b)	eg Lifetimes of daffodils normally distributed Random sample of (lifetimes of) daffodils Daffodil lifetimes independent	B2	2	B1 each; any two
	Total		**10**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\hat{p} = \frac{10}{90} = \frac{1}{9}$	B1		
	Distribution of proportion is approximately normal			
	$\text{SD} = \sqrt{\frac{\frac{1}{9} \times \frac{8}{9}}{90}} \quad (= 0.0331)$	M1 A1		All values correct
	$z = 2.5758$	B1		
	99% confidence limits are:			
	$\frac{1}{9} \pm 2.5758 \times \text{SD}$	M1		
	giving (0.0258, 0.196)	A1	6	(0.0256 to 0.026, 0.196 to 0.20) A0 for wrong z
(b)	$\bar{x} = 62.3$	B1		
	$s = 7.79$	B1		
	$\mu = 10 - 1 = 9$	B1		
	$t = \pm 3.250$	B1		
	99% confidence limits are:			
	$62.3 \pm 3.250 \times \frac{7.79}{\sqrt{10}}$	M1 m1		SD divided by $\sqrt{10}$
	giving (54.3, 70.3)	A1√	7	AWRT; ft on s and t, but not z
(c)	99% confidence limits are:			
	$16.6 \pm 2.5758 \times \frac{3.02}{\sqrt{80}}$	M1 m1		Accept 2.58; t_{79} or t_{80} = 2.64 $3.02/\sqrt{80}$
	giving (15.7, 17.5)	A1√	3	AWRT; ft on incorrect z from (a)
(d)	From (a), upper bound of CI < 25%	E1		
	From (b) and (c), lower bound of CI for mean time for cheques > 3 × upper for cash payments ⇒ it is highly probable that cheques take more than three times as long as cash on average	E2		
	99% confidence level so can be very confident of results	E1		Must evaluate level of confidence
	Andy should make the change	B1	5	
	Total		**21**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	H_0 : Mean number for departure = 4.5			Accept λ or μ for mean
	H_1 : Mean number for departure > 4.5	B1		Both (here or in part (ii))
	Under H_0, $X \sim$ Po(4.5)			
	$P(X \geq 9)$	M1		Attempt at $P(X \geq 9)$ or $P(X > 9)$
	$= 1 - P(X \leq 8)$	m1		
	$= 1 - 0.9597$			
	$= 0.0403$	A1		CAO
	$0.0403 < 5\%$ so reject H_0	E1		Allow if using $P(X \geq 9)$ or $P(X > 9)$ found from exact Poisson distribution
	Significant evidence at 5% level that mean number of cars for departure is greater than 4.5	A1√	6	dep on M1 and m1
(ii)	H_0 : Mean number for departure = 4.5			
	H_1 : Mean number for departure > 4.5			
	Under H_0, $Y \sim Po(8 \times 4.5) = Po(36)$	B1		
	$\approx N(36, 36)$	B1√		ft on Poisson mean
	Test statistic $= \frac{42.5 - 36}{\sqrt{36}} = 1.083$	M1		N(4.5, 4.5) with $X = 5.375$
		A1		
	or $\frac{43 - 36}{\sqrt{36}} = 1.167$			
	Critical value of z is 1.6449; ts < cv	B1		z value or comparison of probability with 0.05
				or $P(z > 1.083) = 0.139 > 5\%$, so cannot reject H_0
	Not enough evidence at 5% level to claim that the mean number of cars for departure is greater than 4.5	A1√	6	ft on cv and ts but not if N(4.5, 4.5) used
(b)	First survey supports the view that average demand has increased, but later, larger survey does not	B1√		Valid comment based on results of hypothesis tests
	Possible that TV programme had short term effect which soon tailed off			
	Early increase may be nothing to do with programme - eg weather or school holiday	B1	2	Sensible comment in context of question
	Total		**14**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$X+Y \sim \text{N}(11.8, 5.2)$	B2		Mean B1; variance B1
	$\text{P}(X+Y<15) = \Phi\left(\dfrac{15-11.8}{\sqrt{5.2}}\right)$	M1		
	$= \Phi(1.403) = 0.919$ to 0.920	A1√	4	ft on numerical errors in mean and variance
(b)(i)	$\text{E}(X-2Y) = \text{E}(X) - 2\text{E}(Y)$	M1		
	$\text{Var}(X-2Y) = \text{Var}(X) + 2^2\text{Var}(Y)$	M1		
	$X-2Y \sim \text{N}(-0.5, 6.28)$	A1		
(ii)	$\text{P}(X>2Y) = \text{P}(X-2Y>0)$	M1		
	$= 1-\Phi\left(\dfrac{0-(-0.5)}{\sqrt{6.28}}\right)$	m1		
	$= 1-\Phi(0.200)$			
	$= 0.421$	A1	6	CAO; 0.42 to 0.422
(c)	$\text{P}(Y_1 > Y_2) = 0.5$	B1	1	
(d)(i)	A longer consultation is likely to mean more information to add to record so longer update	E1	1	
(ii)	Unlikely to be independent. If first consultation is long it will increase the waiting times for the next few patients	E1	1	
	Total		**13**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$A \sim \mathrm{B}(n, 0.038)$	B2	2	B1 for binomial B1 for parameters; allow if B(85, 0.038) shown later
(ii)	All applicants independent in likelihood of colour blindness (eg no identical twins)	B1	1	
(b)(i)	$\mathrm{B}(85, 0.038) \approx \mathrm{Po}(3.23)$	B2		B1 Poisson; B1 mean
	$\mathrm{P}(X = 5) = \mathrm{e}^{-3.23} \times \dfrac{(3.23)^5}{5!}$	M1 m1		Attempt at Poisson formula Correct values entered M1 only if Po(3.2) and tables used
	$= 0.116$	A1	5	AWRT (Exact binomial max B1) (Normal approximation max B1 mean and variance; M1 m1 finding probability)
(ii)	Binomial with large n and very small p	B1 B1	2	Award if normal used
(c)	$\mathrm{H}_0 : p = 0.75$ $\mathrm{H}_1 : p \neq 0.75$	B1		Both
	Under H_0, $Y \sim \mathrm{B}(96, 0.75)$ $\approx \mathrm{N}(72, 18)$	B1		Normal approximation
		M1		Attempt at parameters
	Test statistic $= \dfrac{67.5 - 72}{\sqrt{18}} = -1.06$	M1 A1		Accept positive value
	or ts $= \dfrac{67 - 72}{\sqrt{18}} = -1.18$			Using proportions: sd $= \sqrt{\dfrac{0.75 \times 0.25}{96}}$ M1A1
	Critical value of $z = \pm 1.96$; ts is not in critical region	B1		ts $= \dfrac{\frac{67}{96} - 0.75}{\text{sd}} = -1.18$ M1A1
	Or P($z \leq -1.06$) = 0.144 > 2.5%			
	Not enough evidence at the 5% significance level to claim that proportion who accept places is not 75%	A1√	7	ft on ts and cv
	Total		**17**	
	TOTAL		**75**	