

General Certificate of Education  
June 2006  
Advanced Level Examination



**MATHEMATICS**  
**Unit Further Pure 4**

**MFP4**

Monday 12 June 2006 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 Two planes,  $\Pi_1$  and  $\Pi_2$ , have equations  $\mathbf{r} \cdot \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix} = 0$  and  $\mathbf{r} \cdot \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} = 0$  respectively.

(a) Determine the cosine of the acute angle between  $\Pi_1$  and  $\Pi_2$ . *(4 marks)*

(b) (i) Find  $\begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix} \times \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ . *(2 marks)*

(ii) Find a vector equation for the line of intersection of  $\Pi_1$  and  $\Pi_2$ . *(2 marks)*

2 A transformation is represented by the matrix  $\mathbf{A} = \begin{bmatrix} 0.28 & -0.96 & 0 \\ 0.96 & 0.28 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .

(a) Evaluate  $\det \mathbf{A}$ . *(1 mark)*

(b) State the invariant line of the transformation. *(1 mark)*

(c) Give a full geometrical description of this transformation. *(3 marks)*

3 Express the determinant  $\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$  as the product of four linear factors. *(6 marks)*

4 The plane transformation T maps points  $(x, y)$  to points  $(x', y')$  such that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{where } \mathbf{A} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

(a) (i) State the line of invariant points of T. *(1 mark)*

(ii) Give a full geometrical description of T. *(2 marks)*

(b) Find  $\mathbf{A}^2$ , and hence give a full geometrical description of the single plane transformation given by the matrix  $\mathbf{A}^2$ . *(3 marks)*

5 A set of three planes is given by the system of equations

$$\begin{aligned}x + 3y - z &= 10 \\2x + ky + z &= -4 \\3x + 5y + (k-2)z &= k+4\end{aligned}$$

where  $k$  is a constant.

(a) Show that  $\begin{vmatrix} 1 & 3 & -1 \\ 2 & k & 1 \\ 3 & 5 & k-2 \end{vmatrix} = k^2 - 5k + 6.$  (2 marks)

(b) In each of the following cases, determine the **number** of solutions of the given system of equations.

(i)  $k = 1.$

(ii)  $k = 2.$

(iii)  $k = 3.$  (7 marks)

(c) Give a geometrical interpretation of the significance of each of the three results in part (b) in relation to the three planes. (3 marks)

6 The matrices  $\mathbf{P}$  and  $\mathbf{Q}$  are given by

$$\mathbf{P} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & t & -2 \\ 3 & 2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} 1 & 1 & 1 \\ -7 & -1 & 5 \\ 11 & -1 & -7 \end{bmatrix}$$

where  $t$  is a real constant.

(a) Find the value of  $t$  for which  $\mathbf{P}$  is singular. (2 marks)

(b) (i) Determine the matrix  $\mathbf{R} = \mathbf{PQ}$ , giving its elements in terms of  $t$  where appropriate. (3 marks)

(ii) Find the value of  $t$  for which  $\mathbf{R} = k\mathbf{I}$ , for some integer  $k$ . (2 marks)

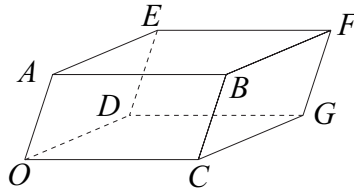
(iii) Hence find the matrix  $\mathbf{Q}^{-1}$ . (1 mark)

(c) In the case when  $t = -3$ , describe the geometrical transformation with matrix  $\mathbf{R}$ . (2 marks)

**Turn over for the next question**

**Turn over ►**

7 The diagram shows the parallelepiped  $OABCDEFG$ .



Points  $A$ ,  $B$ ,  $C$  and  $D$  have position vectors

$$\mathbf{a} = \begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 6 \\ 1 \\ 6 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

respectively, relative to the origin  $O$ .

- (a) Show that  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are linearly dependent. (1 mark)
- (b) Determine the volume of the parallelepiped. (3 marks)
- (c) Determine a vector equation for the plane  $ABDG$ :
- (i) in the form  $\mathbf{r} = \mathbf{u} + \lambda\mathbf{v} + \mu\mathbf{w}$ ; (2 marks)
- (ii) in the form  $\mathbf{r} \cdot \mathbf{n} = d$ . (4 marks)
- (d) Find cartesian equations for the line  $OF$ , and hence find the direction cosines of this line. (4 marks)

8 For real numbers  $a$  and  $b$ , with  $b \neq 0$  and  $b \neq \pm a$ , the matrix

$$\mathbf{M} = \begin{bmatrix} a & b+a \\ b-a & -a \end{bmatrix}$$

- (a) (i) Show that the eigenvalues of  $\mathbf{M}$  are  $b$  and  $-b$ . (3 marks)
- (ii) Show that  $\begin{bmatrix} b+a \\ b-a \end{bmatrix}$  is an eigenvector of  $\mathbf{M}$  with eigenvalue  $b$ . (2 marks)
- (iii) Find an eigenvector of  $\mathbf{M}$  corresponding to the eigenvalue  $-b$ . (2 marks)
- (b) By writing  $\mathbf{M}$  in the form  $\mathbf{UDU}^{-1}$ , for some suitably chosen diagonal matrix  $\mathbf{D}$  and corresponding matrix  $\mathbf{U}$ , show that

$$\mathbf{M}^{11} = b^{10}\mathbf{M} \quad \text{(7 marks)}$$

**END OF QUESTIONS**