



## **General Certificate of Education**

# **Mathematics 6360**

**MD02      Decision 2**

## **Mark Scheme**

*2007 examination - January series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A <sub>2,1</sub>	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MD02**

Q	Solution	Marks	Total	Comments
<p><b>1(a)</b></p>	<p>Network attempted up to 2 slips (boxes or arrows) correct network</p> <p>Earliest start time    Duration    Latest finish time</p>	<p>M1 A1 A1</p>	<p>3</p>	<p>SCA</p>
<p><b>(b)</b></p>	<p>Forward pass correct</p>	<p>M1 A1</p>	<p>2</p>	
<p><b>(c)</b></p>	<p>Backward pass correct</p>	<p>M1 A1</p>	<p>2</p>	
<p><b>(d)</b></p>	<p>Minimum completion time: 13 weeks Critical paths: <i>ACGIJ</i> <i>BEGIJ</i> <i>BEHJ</i></p>	<p>B1 B1 B1 B1</p>	<p>4</p>	
	<p style="text-align: right;"><b>Total</b></p>		<p><b>11</b></p>	

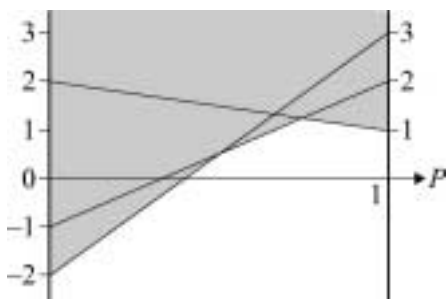
**MD02 (cont)**

Q	Solution	Marks	Total	Comments
2(a)	Hungarian algorithm <b>minimises</b>	E1	2	idea of high becoming low, etc.  array giving $15 - x$  reduce rows (or columns then rows)  reduced array correct  <div style="text-align: center;"> <math display="block">\begin{array}{cccccc} &amp;   &amp; &amp;   &amp; &amp;   \\ \hline 2 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; \\ \hline \end{array}</math>                     augmented array                 </div> <div style="text-align: center;"> <math display="block">\begin{array}{ccccc} 0 &amp; 1 &amp; 3 &amp; 3 &amp; \triangle 0 \\ \triangle 0 &amp; 2 &amp; 2 &amp; 4 &amp; 3 \\ 2 &amp; 3 &amp; 5 &amp; \triangle 0 &amp; 0 \\ 3 &amp; 1 &amp; \triangle 0 &amp; 0 &amp; 4 \\ 3 &amp; \triangle 0 &amp; 0 &amp; 2 &amp; 1 \end{array}</math> </div>
	15 - x gives measure of criteria NOT met which need minimising in order to maximise scores	E1		
	(b)	$\begin{array}{ccccc} 2 & 4 & 6 & 5 & 2 \\ 0 & 3 & 3 & 4 & 3 \\ 3 & 5 & 7 & 1 & 1 \\ 4 & 3 & 2 & 1 & 5 \\ 3 & 1 & 1 & 2 & 1 \\ \hline 0 & 2 & 4 & 3 & 0 \\ 0 & 3 & 3 & 4 & 3 \\ 2 & 4 & 6 & 0 & 0 \\ 3 & 2 & 1 & 0 & 4 \\ 2 & 0 & 0 & 1 & 0 \end{array}$	B1	
		Zeros can be covered with only <b>4 lines</b> so adjustment needed	E1	
		Reduction by subtracting 1 from each uncovered element and adding 1 to each element at intersection of two lines	M1	
		Matching on particular zeros	M1	
		Alex ↔ (5)    Don ↔ (3)	A1	
		Bill ↔ (1)    Ed ↔ (2)		
		Cath ↔ (4)		
		If adjustment <b>not</b> done correctly and matching made, award B1 for 3 correct and B1 for rest correct		
(c)	Deleting row 2 and column 4 either in final matrix or reworking	M1	3	<div style="text-align: center;"> <math display="block">\begin{array}{ccccc} \square 0 &amp; 1 &amp; 3 &amp; 3 &amp; 0 \\ 0 &amp; 2 &amp; 2 &amp; 4 &amp; 3 \\ 2 &amp; 3 &amp; 5 &amp; 0 &amp; \square 0 \\ 3 &amp; 1 &amp; \square 0 &amp; 0 &amp; 4 \\ 3 &amp; \square 0 &amp; 0 &amp; 2 &amp; 1 \end{array}</math> </div>
	Final solution:	A1		
	A ↔ (1)    C ↔ (5)	A1		
D ↔ (3)    E ↔ (2)				
If no method award B2 if matching is all correct				
<b>Total</b>			<b>13</b>	

**MD02 (cont)**

Q	Solution	Marks	Total	Comments
<b>3(a)</b>	P $x$ $y$ $z$ $s$ $t$ Value			
	1    -5    -8    -7    0    0    0	M1		SCA
	0    3    2    1    1    0    12	A2	3	- 1 EE
	0    2 <b>4</b> 5    0    1    16			
<b>(b)(i)</b>	$\frac{12}{2} = 6; \frac{16}{4} = 4$ and $4 < 6$	E1	1	
<b>(ii)</b>	1    -1    0    3    0    2    32	M1		using 4 as pivot and possibly dividing third row by 4
	0 <b>2</b> 0 $-1\frac{1}{2}$ 1 $-\frac{1}{2}$ 4	A1		top row correct
	0 $\frac{1}{2}$ 1 $1\frac{1}{4}$ 0 $\frac{1}{4}$ 4	A1		second row correct; may have 0    2    4    5    0    1    16
	choice of pivot from $x$ -column	M1		pivot = <b>2</b> identified and used
	1    0    0 $2\frac{1}{4}$ $\frac{1}{2}$ $1\frac{3}{4}$ 34			
	0    1    0 $-\frac{3}{4}$ $\frac{1}{2}$ $-\frac{1}{4}$ 2	m1		row operations
	0    0    1 $1\frac{5}{8}$ $-\frac{1}{4}$ $\frac{3}{8}$ 3	A1	6	correct or scaled up 0    0    4 $6\frac{1}{2}$ -1 $1\frac{1}{2}$ 12
<b>(iii)</b>	Max $P = 34$ $x = 2, y = 3, z = 0$ }	B1✓ B1	2	all correct
<b>(iv)</b>	Yes - no negative values in first row	E1✓	1	no – if negatives in top row
	<b>Total</b>		<b>13</b>	

**MD02 (cont)**

Q	Solution	Marks	Total	Comments
4(a)(i)	Row min -4 -2 -1	M1		Attempt at row minimum and column maximum
	Col max 5 -1 3 min (col max) = max (row min) ⇒ stable solution	A1 E1	3	all figures correct
(ii)	Ros plays III and Col plays Y value of game = -1	B1 B1	2	
(b)(i)	Ros plays $R_1$ with probability $p$ and $R_2$ with probability $1 - p$  Expected gains when Col plays: $C_1 : 3p - 2(1 - p) = 5p - 2$ $C_2 : 2p - (1 - p) = 3p - 1$ $C_3 : p + 2(1 - p) = 2 - p$	M1 A1		attempt at least 2 correct unsimplified
	Plot expected gains against $p$ for $0 \leq p \leq 1$	M1		
		A1		correct (must see 0 or 1 on $P$ axis, or implied by their numbers) A0 if not possible to see highest point of region being correct
	Choose highest point of region below lines ⇒ $3p - 1 = 2 - p$	M1		must be this pair of lines or their highest point
	leading to $p = \frac{3}{4}$	A1		
	Therefore Ros plays $R_1$ with prob $\frac{3}{4}$ and plays $R_2$ with prob $\frac{1}{4}$	B1✓	7	fit their $p$ from any lines
(ii)	Value of game = $3 \times \frac{3}{4} - 1$ or $\left(2 - \frac{3}{4}\right) = 1\frac{1}{4}$	B1	1	
<b>Total</b>			<b>13</b>	

**MD02 (cont)**

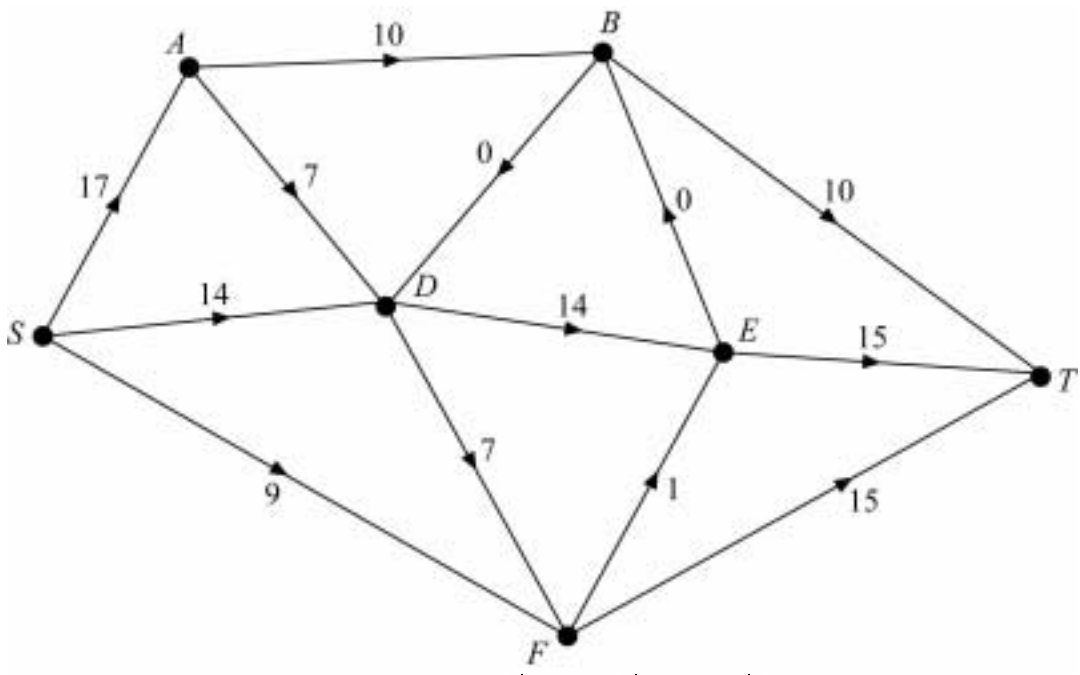
Q	Solution				Marks	Total	Comments
<b>5(a)</b>	<i>SAET</i> has least day's sunshine of 5 hours whereas for <i>SACT</i> least value is only 4 hours				M1 A1	2	Reasonable understanding Mention of 4 and 5 hours and clear idea that minimum is larger in <i>SAET</i>
<b>(b)</b>	Stage	Initial State	Action	Value	M1		General idea of stage and state
	1	<i>C</i>	<i>CT</i>	7*	A1		First stage correct (may be reversed)
		<i>D</i>	<i>DT</i>	9*			
		<i>E</i>	<i>ET</i>	5*			
	2	<i>A</i>	<i>AC</i>	$\min(4, 7) = 4$	M1		Finding least value from 2 legs
			<i>AD</i>	$\min(4, 9) = 4$			
			<i>AE</i>	$\min(5, 5) = 5^*$	m1		Finding max of minima (star values)
		<i>B</i>	<i>BC</i>	$\min(6, 7) = 6^*$	A1		All values in second stage correct
			<i>BD</i>	$\min(5, 9) = 5$			
			<i>BE</i>	$\min(7, 5) = 5$			
	3	<i>S</i>	<i>SA</i>	$\min(9, 5) = 5$	A1		All values in third stage correct
			<i>SB</i>	$\min(8, 6) = 6^*$	A1		All values correct (inc max of min all correct) <b>and</b> minimum comparison clearly shown at each stage, particularly (9, 5) and (8, 6) in third stage
	Maximin route is <i>SBCT</i>				B1	8	Award B1 even without dynamic programming
<b>Total</b>						<b>10</b>	



**MD02 (cont)**

Q	Solution	Marks	Total	Comments												
6(a)(i)	$15 + 0 + 14 + 7 + 9 = 45$	B1	1													
(ii)	Maximum flow $\leq 45$	M1 A1	2	$\leq$ their value or $< 45$ correct												
(b)	<i>SABT</i> flow 10 <i>SDET</i> flow 14 <i>SFT</i> flow 9  (may appear in table below)	B1  B1	  2	one correct  two more correct												
(c)(i)	<p>Additional route with correct flow one more correct route and flow table complete correct use of potential and used flows values correctly updated</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Route</th> <th>Flow</th> </tr> </thead> <tbody> <tr> <td><i>SABT</i></td> <td>10</td> </tr> <tr> <td><i>SDET</i></td> <td>14</td> </tr> <tr> <td><i>SFT</i></td> <td>9</td> </tr> <tr> <td><i>SADFT</i></td> <td>6</td> </tr> <tr> <td><i>SADFET</i></td> <td>1</td> </tr> </tbody> </table>	Route	Flow	<i>SABT</i>	10	<i>SDET</i>	14	<i>SFT</i>	9	<i>SADFT</i>	6	<i>SADFET</i>	1	M1 A1 A1 A1 M1 A1	6	correct total flow of 40 on network (may use double edges) <b>strict</b>  several possibilities
Route	Flow															
<i>SABT</i>	10															
<i>SDET</i>	14															
<i>SFT</i>	9															
<i>SADFT</i>	6															
<i>SADFET</i>	1															

**MD02 (cont)**

Q	Solution	Marks	Total	Comments
<p>6 (cont) (c)(ii)</p>	 <p>Maximum flow = 40 Network showing flow of 40</p>	<p>B1 B1</p>	<p>2</p>	
<p>(iii)</p>	<p>Cut through saturated arcs <i>AB, BD, DE, DF, SF</i> Minimum cut shown to be 40 with statement linking to maximum flow</p>	<p>M1 A1</p>	<p>2</p>	
	<b>Total</b>		<b>15</b>	
	<b>TOTAL</b>		<b>75</b>	