

General Certificate of Education  
January 2007  
Advanced Level Examination



**MATHEMATICS**  
**Unit Pure Core 3**

**MPC3**

Thursday 18 January 2007 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 Use the mid-ordinate rule with four strips of equal width to find an estimate for

$$\int_1^5 \frac{1}{1 + \ln x} dx, \text{ giving your answer to three significant figures.} \quad (4 \text{ marks})$$

- 2 Describe a sequence of **two** geometrical transformations that maps the graph of  $y = \sec x$  onto the graph of  $y = 1 + \sec 3x$ . (4 marks)

- 3 The functions  $f$  and  $g$  are defined with their respective domains by

$$f(x) = 3 - x^2, \text{ for all real values of } x$$

$$g(x) = \frac{2}{x+1}, \text{ for real values of } x, x \neq -1$$

- (a) Find the range of  $f$ . (2 marks)

- (b) The inverse of  $g$  is  $g^{-1}$ .

(i) Find  $g^{-1}(x)$ . (3 marks)

(ii) State the range of  $g^{-1}$ . (1 mark)

- (c) The composite function  $gf$  is denoted by  $h$ .

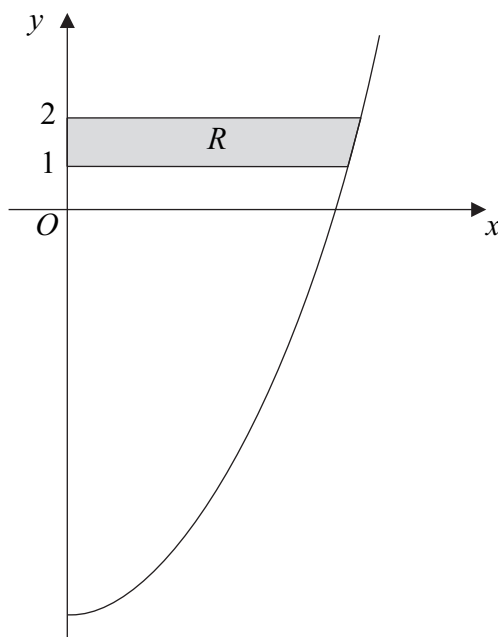
(i) Find  $h(x)$ , simplifying your answer. (2 marks)

(ii) State the greatest possible domain of  $h$ . (1 mark)

4 (a) Use integration by parts to find  $\int x \sin x \, dx$ . (4 marks)

(b) Using the substitution  $u = x^2 + 5$ , or otherwise, find  $\int x\sqrt{x^2 + 5} \, dx$ . (4 marks)

(c) The diagram shows the curve  $y = x^2 - 9$  for  $x \geq 0$ .



The shaded region  $R$  is bounded by the curve, the lines  $y = 1$  and  $y = 2$ , and the  $y$ -axis.

Find the exact value of the volume of the solid generated when the region  $R$  is rotated through  $360^\circ$  about the  $y$ -axis. (4 marks)

5 (a) (i) Show that the equation

$$2 \cot^2 x + 5 \operatorname{cosec} x = 10$$

can be written in the form  $2 \operatorname{cosec}^2 x + 5 \operatorname{cosec} x - 12 = 0$ . (2 marks)

(ii) Hence show that  $\sin x = -\frac{1}{4}$  or  $\sin x = \frac{2}{3}$ . (3 marks)

(b) Hence, or otherwise, solve the equation

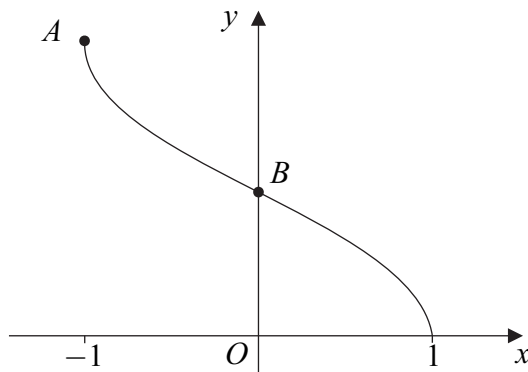
$$2 \cot^2(\theta - 0.1) + 5 \operatorname{cosec}(\theta - 0.1) = 10$$

giving all values of  $\theta$  in radians to two decimal places in the interval  $-\pi < \theta < \pi$ .

(3 marks)

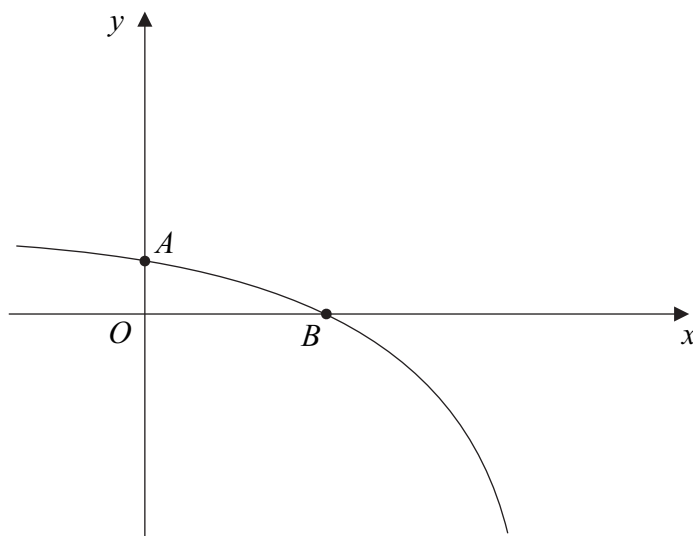
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- 6 (a) Find  $\frac{dy}{dx}$  when:
- (i)  $y = (4x^2 + 3x + 2)^{10}$ ; (2 marks)
- (ii)  $y = x^2 \tan x$ . (2 marks)
- (b) (i) Find  $\frac{dx}{dy}$  when  $x = 2y^3 + \ln y$ . (1 mark)
- (ii) Hence find an equation of the tangent to the curve  $x = 2y^3 + \ln y$  at the point (2,1). (3 marks)
- 7 (a) Sketch the graph of  $y = |2x|$ . (1 mark)
- (b) On a separate diagram, sketch the graph of  $y = 4 - |2x|$ , indicating the coordinates of the points where the graph crosses the coordinate axes. (3 marks)
- (c) Solve  $4 - |2x| = x$ . (3 marks)
- (d) Hence, or otherwise, solve the inequality  $4 - |2x| > x$ . (2 marks)
- 8 The diagram shows the curve  $y = \cos^{-1} x$  for  $-1 \leq x \leq 1$ .



- (a) Write down the exact coordinates of the points  $A$  and  $B$ . (2 marks)
- (b) The equation  $\cos^{-1} x = 3x + 1$  has only one root. Given that the root of this equation is  $\alpha$ , show that  $0.1 \leq \alpha \leq 0.2$ . (2 marks)
- (c) Use the iteration  $x_{n+1} = \frac{1}{3}(\cos^{-1} x_n - 1)$  with  $x_1 = 0.1$  to find the values of  $x_2$ ,  $x_3$  and  $x_4$ , giving your answers to three decimal places. (3 marks)

- 9 The sketch shows the graph of  $y = 4 - e^{2x}$ . The curve crosses the  $y$ -axis at the point  $A$  and the  $x$ -axis at the point  $B$ .



- (a) (i) Find  $\int (4 - e^{2x}) dx$ . (2 marks)
- (ii) Hence show that  $\int_0^{\ln 2} (4 - e^{2x}) dx = 4 \ln 2 - \frac{3}{2}$ . (2 marks)
- (b) (i) Write down the  $y$ -coordinate of  $A$ . (1 mark)
- (ii) Show that  $x = \ln 2$  at  $B$ . (2 marks)
- (c) Find the equation of the normal to the curve  $y = 4 - e^{2x}$  at the point  $B$ . (4 marks)
- (d) Find the area of the region enclosed by the curve  $y = 4 - e^{2x}$ , the normal to the curve at  $B$  and the  $y$ -axis. (3 marks)

**END OF QUESTIONS**

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