General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Friday 22 June 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 1, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

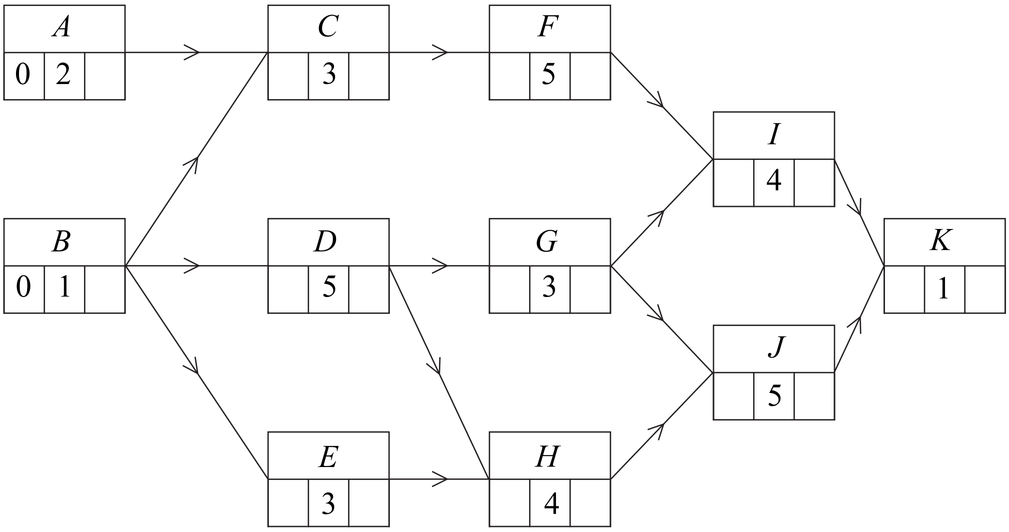
Instructions
- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

The following diagram shows an activity diagram for a building project. The time needed for each activity is given in days.



(a) Complete the precedence table for the project on **Figure 1**. *(2 marks)*

(b) Find the earliest start times and latest finish times for each activity and insert their values on **Figure 2**. *(4 marks)*

(c) Find the critical path and state the minimum time for completion of the project. *(2 marks)*

(d) Find the activity with the greatest float time and state the value of its float time. *(2 marks)*

2 The daily costs, in pounds, for five managers A, B, C, D and E to travel to five different centres are recorded in the table below.

	A	**B**	**C**	**D**	**E**
Centre 1	10	11	8	12	5
Centre 2	11	5	11	6	7
Centre 3	12	8	7	11	4
Centre 4	10	9	14	10	6
Centre 5	9	9	7	8	9

Using the Hungarian algorithm, each of the five managers is to be allocated to a different centre so that the overall total travel cost is minimised.

(a) By reducing the **rows first** and then the columns, show that the new table of values is

3	6	3	6	0
4	0	6	0	2
6	4	3	6	0
2	3	8	3	0
0	2	0	0	2

(3 marks)

(b) Show that the zeros in the table in part (a) can be covered with three lines and use adjustments to produce a table where five lines are required to cover the zeros. *(5 marks)*

(c) Hence find the two possible ways of allocating the five managers to the five centres with the least possible total travel cost. *(3 marks)*

(d) Find the value of this minimum daily total travel cost. *(1 mark)*

3 Two people, Rose and Callum, play a zero-sum game. The game is represented by the following pay-off matrix for Rose.

Callum

		$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
	$\mathbf{R_1}$	5	2	−1
Rose	$\mathbf{R_2}$	−3	−1	5
	$\mathbf{R_3}$	4	1	−2

(a) (i) State the play-safe strategy for Rose and give a reason for your answer. *(2 marks)*

(ii) Show that there is no stable solution for this game. *(2 marks)*

(b) Explain why Rose should never play strategy $\mathbf{R_3}$. *(1 mark)*

(c) Rose adopts a mixed strategy, choosing $\mathbf{R_1}$ with probability p and $\mathbf{R_2}$ with probability $1 - p$.

(i) Find expressions for the expected gain for Rose when Callum chooses each of his three possible strategies. Simplify your expressions. *(3 marks)*

(ii) Illustrate graphically these expected gains for $0 \leqslant p \leqslant 1$. *(2 marks)*

(iii) Hence determine the optimal mixed strategy for Rose. *(3 marks)*

(iv) Find the value of the game. *(1 mark)*

4 A linear programming problem involving variables x and y is to be solved. The objective function to be maximised is $P = 3x + 5y$. The initial Simplex tableau is given below.

P	***x***	***y***	***s***	***t***	***u***	***value***
1	−3	−5	0	0	0	0
0	1	2	1	0	0	36
0	1	1	0	1	0	20
0	4	1	0	0	1	39

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, write down **three** inequalities involving x and y for this problem. *(2 marks)*

(b) (i) By choosing the first pivot from the ***y*-column**, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Explain how you know that the optimal value has not been reached. *(1 mark)*

(c) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and state the values of the slack variables. *(3 marks)*

Turn over for the next question

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A maker of exclusive furniture is planning to build three cabinets A, B and C at the rate of one per month. The order in which they are built is a matter of choice, but the costs will vary because of the materials available and suppliers' costs. The expected costs, in pounds, are given in the table.

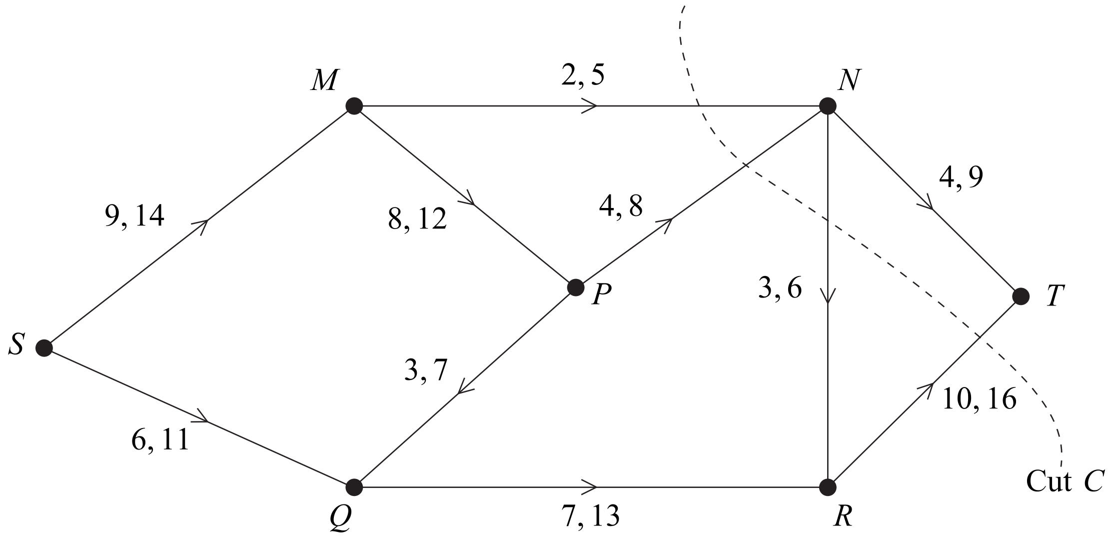
Month	**Already built**	**Cost**		
		A	***B***	***C***
1	–	500	440	475
2	***A*** ***B*** ***C***	– 510 520	440 – 490	490 500 –
3	***A* and *B*** ***A* and *C*** ***B* and *C***	– – 510	– 500 –	520 – –

(a) Use dynamic programming, working **backwards** from month 3, to determine the order of manufacture that **minimises** the total cost. You may wish to use **Figure 3** for your working. *(6 marks)*

(b) It is discovered that the figures given were actually the profits, not the costs, for each item. Modify your solution to find the order of manufacture that **maximises** the total profit. You may wish to use the final column of **Figure 3** for your working. *(4 marks)*

6 [**Figures 4, 5 and 6**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) (i) Find the value of the cut C. *(1 mark)*

(ii) State what can be deduced about the maximum flow from S to T. *(1 mark)*

(b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 20 litres per second from S to T. Indicate, on **Figure 4**, the flows along the edges MP, PN, QR and NR. *(4 marks)*

(c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. *(2 marks)*

(ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(iii) Illustrate the maximum flow on **Figure 6**. *(2 marks)*

END OF QUESTIONS

There are no questions printed on this page

Surname		Other Names								
Centre Number						Candidate Number				
Candidate Signature										

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Insert

Insert for use in **Questions 1, 5 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Figure 1 (for use in Question 1)

Activity	Immediate Predecessors
A	–
B	–
C	
D	
E	
F	
G	
H	
I	
J	
K	

Figure 2 (for use in Question 1)

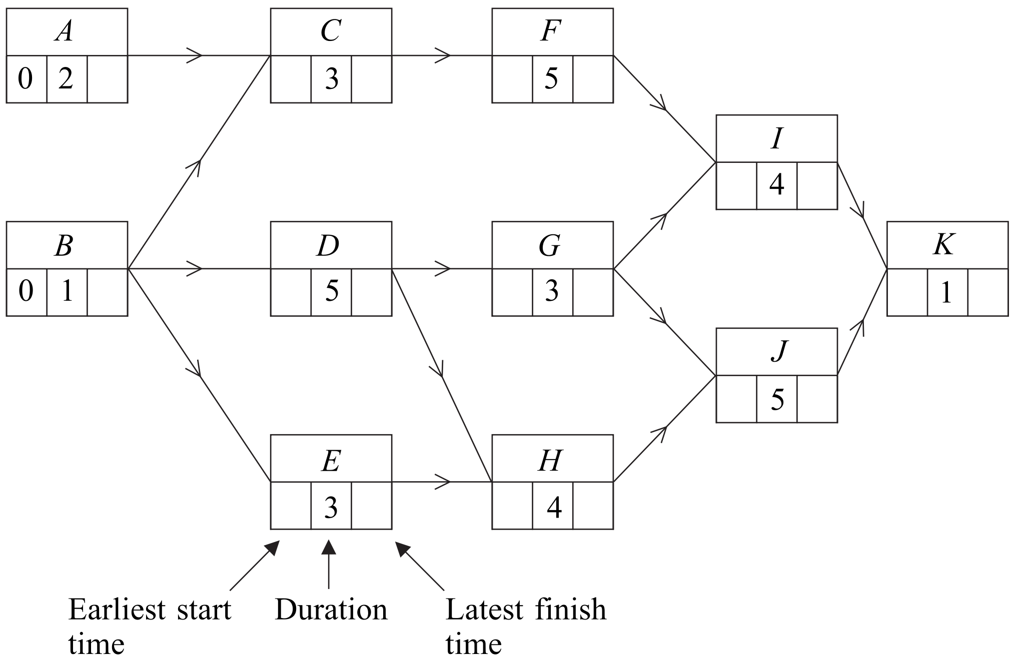


Figure 3 (for use in Question 5)

Month	**Already built**	**Machine built**	For use in part (a)		For use in part (b)
3	*A* and *B*	*C*			
	A and *C*	*B*			
	B and *C*	*A*			
2	*A*	*B*			
		C			
	B	*A*			
		C			
	C	*A*			
		B			

Turn over ▶

Figure 4 (for use in Question 6)

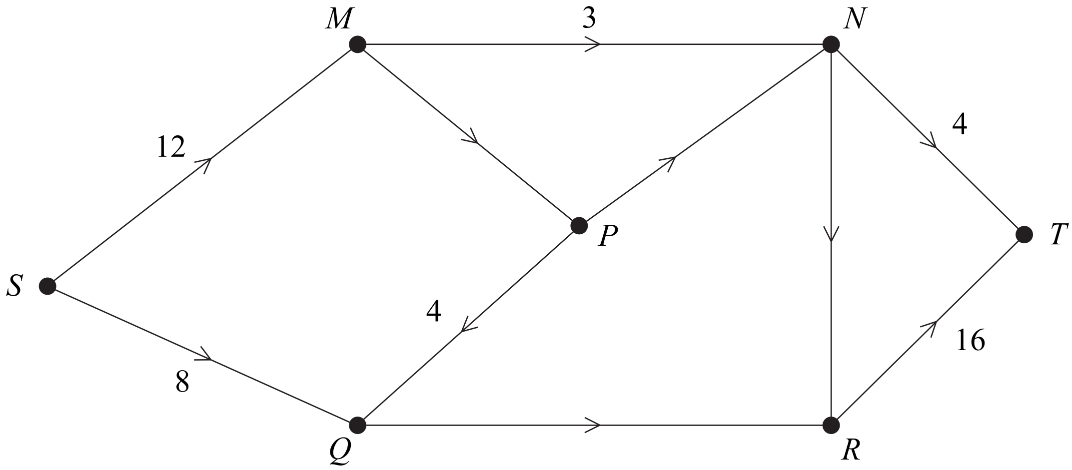
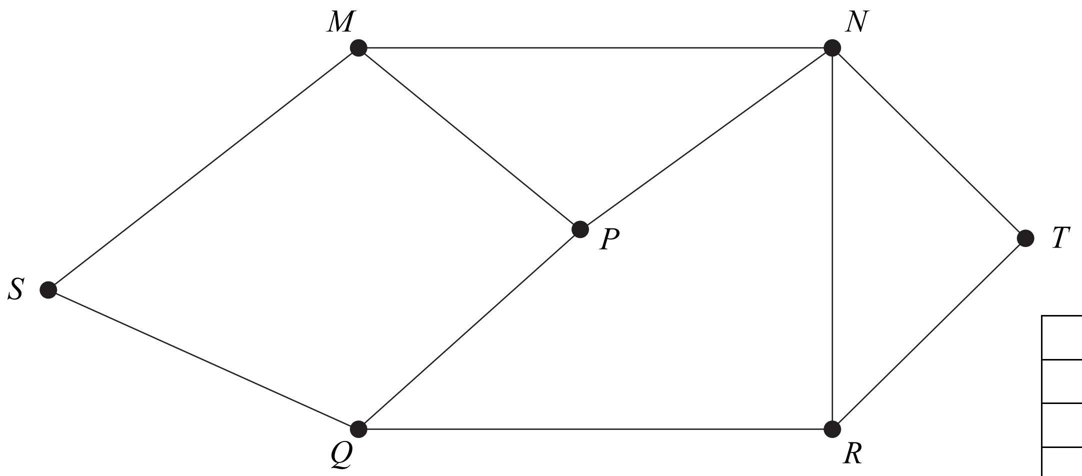


Figure 5 (for use in Question 6)



Path	Flow

Figure 6 (for use in Question 6)