

General Certificate of Education  
June 2007  
Advanced Level Examination



**MATHEMATICS**  
**Unit Further Pure 2**

**MFP2**

Tuesday 26 June 2007 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 (a) Given that  $f(r) = (r - 1)r^2$ , show that

$$f(r + 1) - f(r) = r(3r + 1) \quad (3 \text{ marks})$$

- (b) Use the method of differences to find the value of

$$\sum_{r=50}^{99} r(3r + 1) \quad (4 \text{ marks})$$

- 2 The cubic equation

$$z^3 + pz^2 + 6z + q = 0$$

has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

- (a) Write down the value of  $\alpha\beta + \beta\gamma + \gamma\alpha$ . (1 mark)

- (b) Given that  $p$  and  $q$  are real and that  $\alpha^2 + \beta^2 + \gamma^2 = -12$ :

(i) explain why the cubic equation has two non-real roots and one real root; (2 marks)

(ii) find the value of  $p$ . (4 marks)

- (c) One root of the cubic equation is  $-1 + 3i$ .

Find:

(i) the other two roots; (3 marks)

(ii) the value of  $q$ . (2 marks)

- 3 Use De Moivre's Theorem to find the smallest positive angle  $\theta$  for which

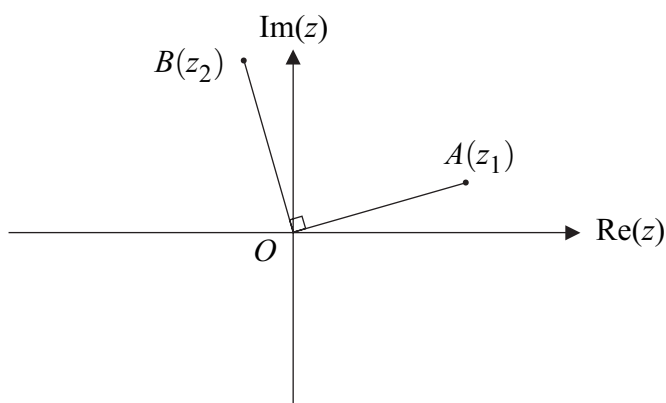
$$(\cos \theta + i \sin \theta)^{15} = -i \quad (5 \text{ marks})$$

4 (a) Differentiate  $x \tan^{-1} x$  with respect to  $x$ . (2 marks)

(b) Show that

$$\int_0^1 \tan^{-1} x \, dx = \frac{\pi}{4} - \ln \sqrt{2} \quad (5 \text{ marks})$$

5 The sketch shows an Argand diagram. The points  $A$  and  $B$  represent the complex numbers  $z_1$  and  $z_2$  respectively. The angle  $AOB = 90^\circ$  and  $OA = OB$ .



(a) Explain why  $z_2 = iz_1$ . (2 marks)

(b) On a **single** copy of the diagram, draw:

(i) the locus  $L_1$  of points satisfying  $|z - z_2| = |z - z_1|$ ; (2 marks)

(ii) the locus  $L_2$  of points satisfying  $\arg(z - z_2) = \arg z_1$ . (3 marks)

(c) Find, in terms of  $z_1$ , the complex number representing the point of intersection of  $L_1$  and  $L_2$ . (2 marks)

6 (a) Show that

$$\left(1 - \frac{1}{(k+1)^2}\right) \times \frac{k+1}{2k} = \frac{k+2}{2(k+1)} \quad (3 \text{ marks})$$

(b) Prove by induction that for all integers  $n \geq 2$

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \dots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n} \quad (4 \text{ marks})$$

**Turn over for the next question**

**Turn over ►**

7 A curve has equation  $y = 4\sqrt{x}$ .

- (a) Show that the length of arc  $s$  of the curve between the points where  $x = 0$  and  $x = 1$  is given by

$$s = \int_0^1 \sqrt{\frac{x+4}{x}} \, dx \quad (4 \text{ marks})$$

- (b) (i) Use the substitution  $x = 4 \sinh^2 \theta$  to show that

$$\int \sqrt{\frac{x+4}{x}} \, dx = \int 8 \cosh^2 \theta \, d\theta \quad (5 \text{ marks})$$

- (ii) Hence show that

$$s = 4 \sinh^{-1} 0.5 + \sqrt{5} \quad (6 \text{ marks})$$

- 8 (a) (i) Given that  $z^6 - 4z^3 + 8 = 0$ , show that  $z^3 = 2 \pm 2i$ . (2 marks)

- (ii) Hence solve the equation

$$z^6 - 4z^3 + 8 = 0$$

giving your answers in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta \leq \pi$ . (6 marks)

- (b) Show that, for any real values of  $k$  and  $\theta$ ,

$$(z - ke^{i\theta})(z - ke^{-i\theta}) = z^2 - 2kz \cos \theta + k^2 \quad (2 \text{ marks})$$

- (c) Express  $z^6 - 4z^3 + 8$  as the product of three quadratic factors with real coefficients. (3 marks)

**END OF QUESTIONS**