General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

Monday 11 June 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The magnitude of the gravitational force, F, between two planets of masses m_1 and m_2 with centres at a distance x apart is given by

$$F = \frac{Gm_1m_2}{x^2}$$

where G is a constant.

(a) By using dimensional analysis, find the dimensions of G. *(3 marks)*

(b) The lifetime, t, of a planet is thought to depend on its mass, m, its initial radius, R, the constant G and a dimensionless constant, k, so that

$$t = km^{\alpha}R^{\beta}G^{\gamma}$$

where α, β and γ are constants.

Find the values of α, β and γ. *(5 marks)*

2 The unit vectors **i**, **j** and **k** are directed due east, due north and vertically upwards respectively.

Two helicopters, A and B, are flying with constant velocities of $(20\mathbf{i} - 10\mathbf{j} + 20\mathbf{k})\,\mathrm{m\,s^{-1}}$ and $(30\mathbf{i} + 10\mathbf{j} + 10\mathbf{k})\,\mathrm{m\,s^{-1}}$ respectively. At noon, the position vectors of A and B relative to a fixed origin, O, are $(8000\mathbf{i} + 1500\mathbf{j} + 3000\mathbf{k})\,\mathrm{m}$ and $(2000\mathbf{i} + 500\mathbf{j} + 1000\mathbf{k})\,\mathrm{m}$ respectively.

(a) Write down the velocity of A relative to B. *(2 marks)*

(b) Find the position vector of A relative to B at time t seconds after noon. *(3 marks)*

(c) Find the value of t when A and B are closest together. *(5 marks)*

3 A particle P, of mass 2 kg, is initially at rest at a point O on a smooth horizontal surface. The particle moves along a straight line, OA, under the action of a horizontal force. When the force has been acting for t seconds, it has magnitude $(4t + 5)$ N.

(a) Find the magnitude of the impulse exerted by the force on P between the times $t = 0$ and $t = 3$. *(3 marks)*

(b) Find the speed of P when $t = 3$. *(2 marks)*

(c) The speed of P at A is $37.5\,\mathrm{m\,s^{-1}}$. Find the time taken for the particle to reach A. *(4 marks)*

4 Two small smooth spheres, A and B, of equal radii have masses 0.3 kg and 0.2 kg respectively. They are moving on a smooth horizontal surface directly towards each other with speeds $3\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ respectively when they collide. The coefficient of restitution between A and B is 0.8.

(a) Find the speeds of A and B immediately after the collision. *(6 marks)*

(b) Subsequently, B collides with a fixed smooth vertical wall which is at right angles to the path of the sphere. The coefficient of restitution between B and the wall is 0.7.

Show that B will collide again with A. *(3 marks)*

5 A ball is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of elevation α above the horizontal so as to hit a point P on a wall. The ball travels in a vertical plane through the point of projection. During the motion, the horizontal and upward vertical displacements of the ball from the point of projection are x metres and y metres respectively.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$$ *(6 marks)*

(b) The ball is projected from a point 1 metre vertically below and R metres horizontally from the point P.

(i) By taking $g = 10\,\mathrm{m\,s^{-2}}$, show that R satisfies the equation

$$5R^2\tan^2\alpha - u^2R\tan\alpha + 5R^2 + u^2 = 0$$ *(2 marks)*

(ii) Hence, given that u and R are constants, show that, for $\tan\alpha$ to have real values, R must satisfy the inequality

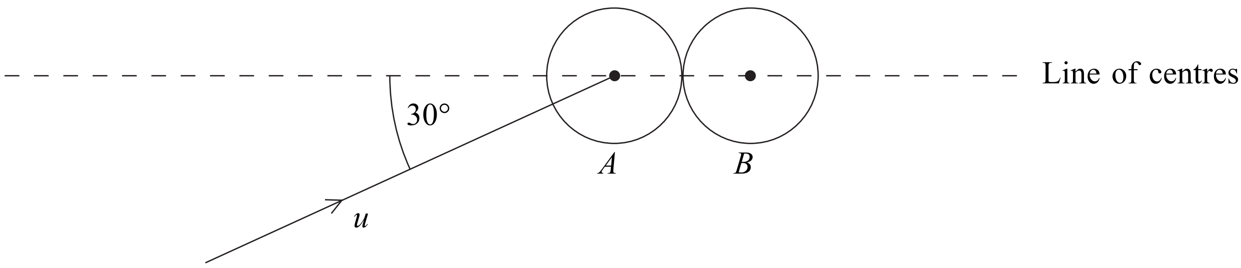
$$R^2 \leqslant \frac{u^2(u^2 - 20)}{100}$$ *(2 marks)*

(iii) Given that $R = 5$, determine the minimum possible speed of projection. *(3 marks)*

Turn over ▶

6 A smooth spherical ball, A, is moving with speed u in a straight line on a smooth horizontal table when it hits an identical ball, B, which is at rest on the table.

Just before the collision, the direction of motion of A makes an angle of 30° with the line of the centres of the two balls, as shown in the diagram.



The coefficient of restitution between A and B is e.

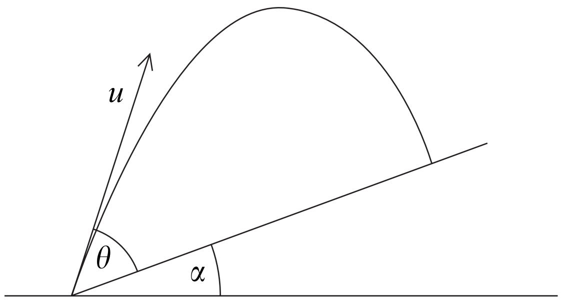
(a) Given that $\cos 30° = \dfrac{\sqrt{3}}{2}$, show that the speed of B immediately after the collision is

$$\frac{\sqrt{3}}{4}u(1+e)$$ *(5 marks)*

(b) Find, in terms of u and e, the components of the velocity of A, parallel and perpendicular to the line of centres, immediately after the collision. *(3 marks)*

(c) Given that $e = \frac{2}{3}$, find the angle that the velocity of A makes with the line of centres immediately after the collision. Give your answer to the nearest degree. *(3 marks)*

7 A particle is projected from a point on a plane which is inclined at an angle α to the horizontal. The particle is projected up the plane with velocity u at an angle θ above the plane. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Using the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, show that the range up the plane is

$$\frac{2u^2 \sin\theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$$ *(8 marks)*

(b) Hence, using the identity $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$, show that, as θ varies, the range up the plane is a maximum when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$. *(3 marks)*

(c) Given that the particle strikes the plane at right angles, show that

$$2 \tan\theta = \cot\alpha$$ *(4 marks)*

END OF QUESTIONS

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