

General Certificate of Education
January 2008
Advanced Subsidiary Examination



MATHEMATICS
Unit Decision 1

MD01

Tuesday 15 January 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 2, 4 and 5 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD01.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

- 1 Five people, A , B , C , D and E , are to be matched to five tasks, J , K , L , M and N . The table shows the tasks that each person is able to undertake.

Person	Task
A	J, N
B	J, L
C	L, N
D	M, N
E	K, M

- (a) Show this information on a bipartite graph. (2 marks)
- (b) Initially, A is matched to task N , B to task J , C to task L , and E to task M .

Complete the alternating path $D-M \dots$, from this initial matching, to demonstrate how each person can be matched to a task. (3 marks)

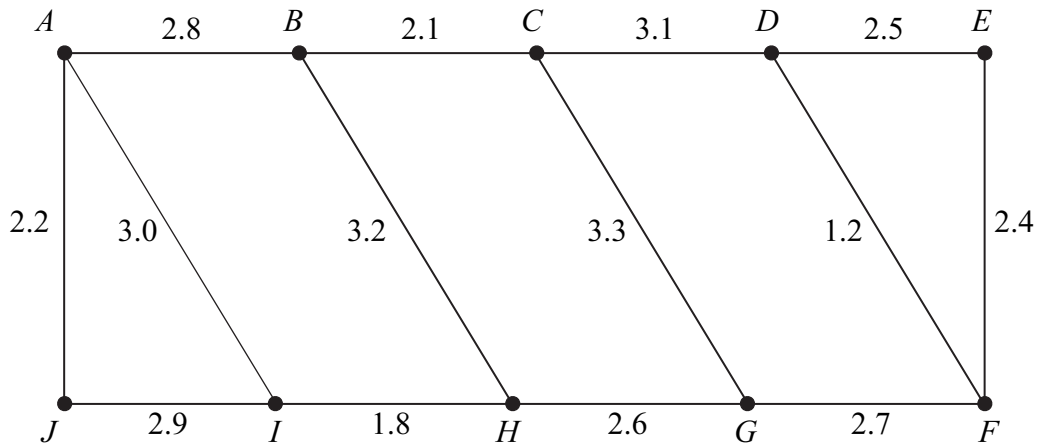
- 2 [Figure 1, printed on the insert, is provided for use in this question.]

The feasible region of a linear programming problem is represented by

$$\begin{aligned} x + y &\leq 30 \\ 2x + y &\leq 40 \\ y &\geq 5 \\ x &\geq 4 \\ y &\geq \frac{1}{2}x \end{aligned}$$

- (a) On **Figure 1**, draw a suitable diagram to represent these inequalities and indicate the feasible region. (5 marks)
- (b) Use your diagram to find the maximum value of F , on the feasible region, in the case where:
- (i) $F = 3x + y$; (2 marks)
- (ii) $F = x + 3y$. (2 marks)

- 3 The diagram shows 10 bus stops, A, B, C, \dots, J , in Geneva. The number on each edge represents the distance, in kilometres, between adjacent bus stops.



The city council is to connect these bus stops to a computer system which will display waiting times for buses at each of the 10 stops. Cabling is to be laid between some of the bus stops.

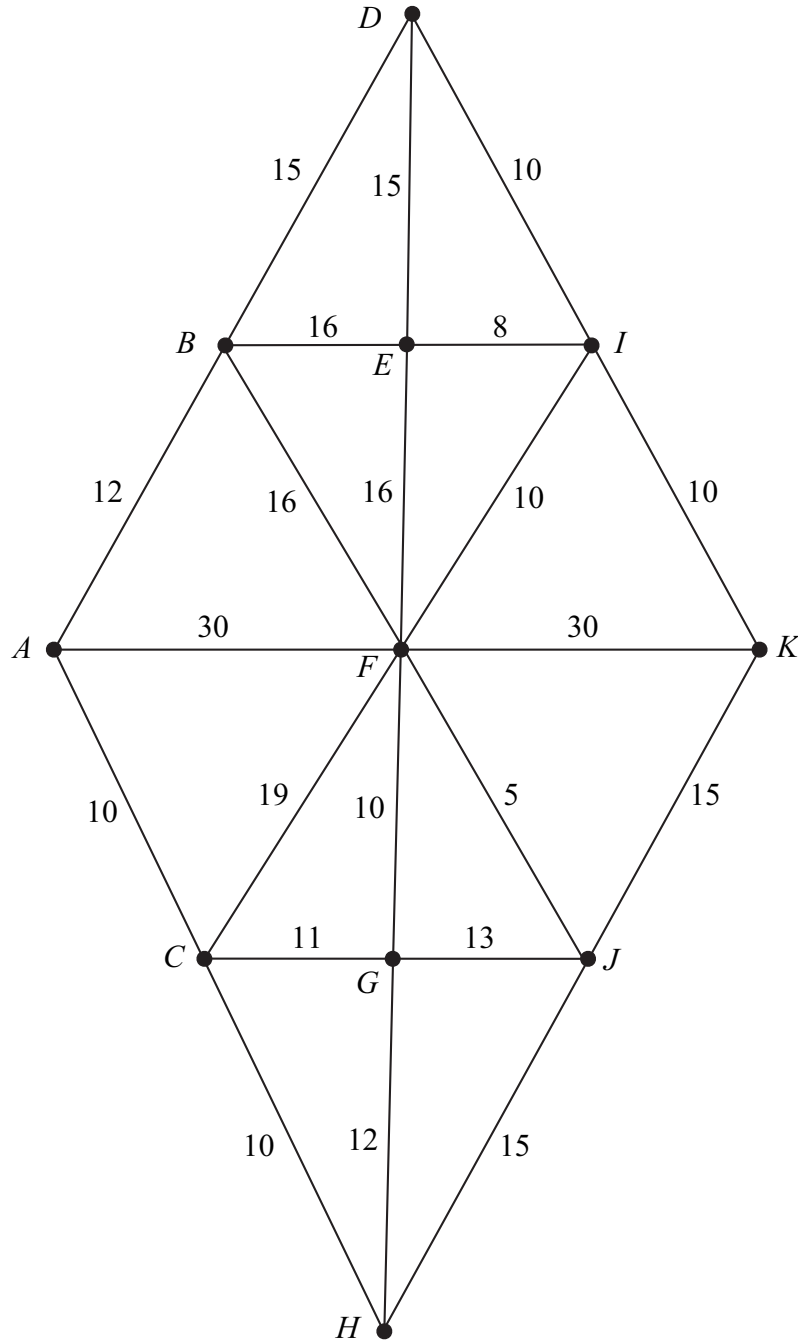
- Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the 10 bus stops. *(5 marks)*
- State the minimum length of cabling needed. *(1 mark)*
- Draw your minimum spanning tree. *(2 marks)*
- If Prim's algorithm, starting from A , had been used to find the minimum spanning tree, state which edge would have been the final edge to complete the minimum spanning tree. *(2 marks)*

Turn over for the next question

Turn over ►

4 [Figure 2, printed on the insert, is provided for use in this question.]

The network shows 11 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.



The total of all of the times is 308 minutes.

- (a) (i) Use Dijkstra's algorithm on **Figure 2** to find the minimum time to travel from A to K . (6 marks)
- (ii) State the corresponding route. (1 mark)
- (b) Find the length of an optimum Chinese postman route around the network, starting and finishing at A . (The minimum time to travel from D to H is 40 minutes.) (5 marks)

5 [Figure 3, printed on the insert, is provided for use in this question.]

(a) James is solving a travelling salesperson problem.

(i) He finds the following upper bounds: 43, 40, 43, 41, 55, 43, 43.

Write down the best upper bound.

(1 mark)

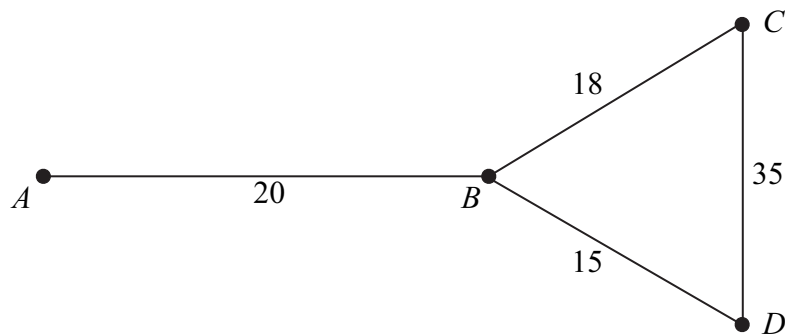
(ii) James finds the following lower bounds: 33, 40, 33, 38, 33, 38, 38.

Write down the best lower bound.

(1 mark)

(b) Karen is solving a different travelling salesperson problem and finds an upper bound of 55 and a lower bound of 45. Write down an interpretation of these results. (1 mark)

(c) The diagram below shows roads connecting 4 towns, A , B , C and D . The numbers on the edges represent the lengths of the roads, in kilometres, between adjacent towns.



Xiong lives at town A and is to visit each of the other three towns before returning to town A . She wishes to find a route that will minimise her travelling distance.

(i) Complete **Figure 3**, on the insert, to show the shortest distances, in kilometres, between **all** pairs of towns. (2 marks)

(ii) Use the nearest neighbour algorithm on **Figure 3** to find an upper bound for the minimum length of a tour of this network that starts and finishes at A . (3 marks)

(iii) Hence find the actual route that Xiong would take in order to achieve a tour of the same length as that found in part (c)(ii). (2 marks)

Turn over ►

- 6 A student is solving cubic equations that have three different positive integer solutions.

The algorithm that the student is using is as follows:

```
Line 10    Input  $A, B, C, D$ 
Line 20    Let  $K = 1$ 
Line 30    Let  $N = 0$ 
Line 40    Let  $X = K$ 
Line 50    Let  $Y = AX^3 + BX^2 + CX + D$ 
Line 60    If  $Y \neq 0$  then go to Line 100
Line 70    Print  $X$ , "is a solution"
Line 80    Let  $N = N + 1$ 
Line 90    If  $N = 3$  then go to Line 120
Line 100   Let  $K = K + 1$ 
Line 110   Go to Line 40
Line 120   End
```

- (a) Trace the algorithm in the case where the input values are:

(i) $A = 1, B = -6, C = 11$ and $D = -6$; *(4 marks)*

(ii) $A = 1, B = -10, C = 29$ and $D = -20$. *(4 marks)*

- (b) Explain where and why this algorithm will fail if $A = 0$. *(2 marks)*

7 The numbers 17, 3, 16 and 4 are to be sorted into ascending order.

The following four methods are to be compared: bubble sort, shuttle sort, Shell sort and quick sort (with the first number used as the pivot).

A student uses each of the four methods and produces the correct solutions below. Each solution shows the order of the numbers after each pass.

Solution 1 17 3 16 4
 3 17 16 4
 3 16 17 4
 3 4 16 17

Solution 2 17 3 16 4
 16 3 17 4
 3 4 16 17

Solution 3 17 3 16 4
 3 16 4 17
 3 16 4 17
 3 4 16 17

Solution 4 17 3 16 4
 3 16 4 17
 3 4 16 17
 3 4 16 17

- (a) Write down which of the four solutions is the bubble sort, the shuttle sort, the Shell sort and the quick sort. *(3 marks)*
- (b) For each of the four solutions, write down the number of comparisons and swaps (exchanges) on the first pass. *(8 marks)*

Turn over for the next question

Turn over ►

- 8 Each day, a factory makes three types of hinge: basic, standard and luxury. The hinges produced need three different components: type A , type B and type C .

Basic hinges need 2 components of type A , 3 components of type B and 1 component of type C .

Standard hinges need 4 components of type A , 2 components of type B and 3 components of type C .

Luxury hinges need 3 components of type A , 4 components of type B and 5 components of type C .

Each day, there are 360 components of type A available, 270 of type B and 450 of type C .

Each day, the factory must use at least 720 components in total.

Each day, the factory must use at least 40% of the total components as type A .

Each day, the factory makes x basic hinges, y standard hinges and z luxury hinges.

In addition to $x \geq 0$, $y \geq 0$, $z \geq 0$, find five inequalities, each involving x , y and z , which must be satisfied. Simplify each inequality where possible. *(8 marks)*

END OF QUESTIONS

Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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Insert

Insert for use in **Questions 2, 4 and 5.**

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ►

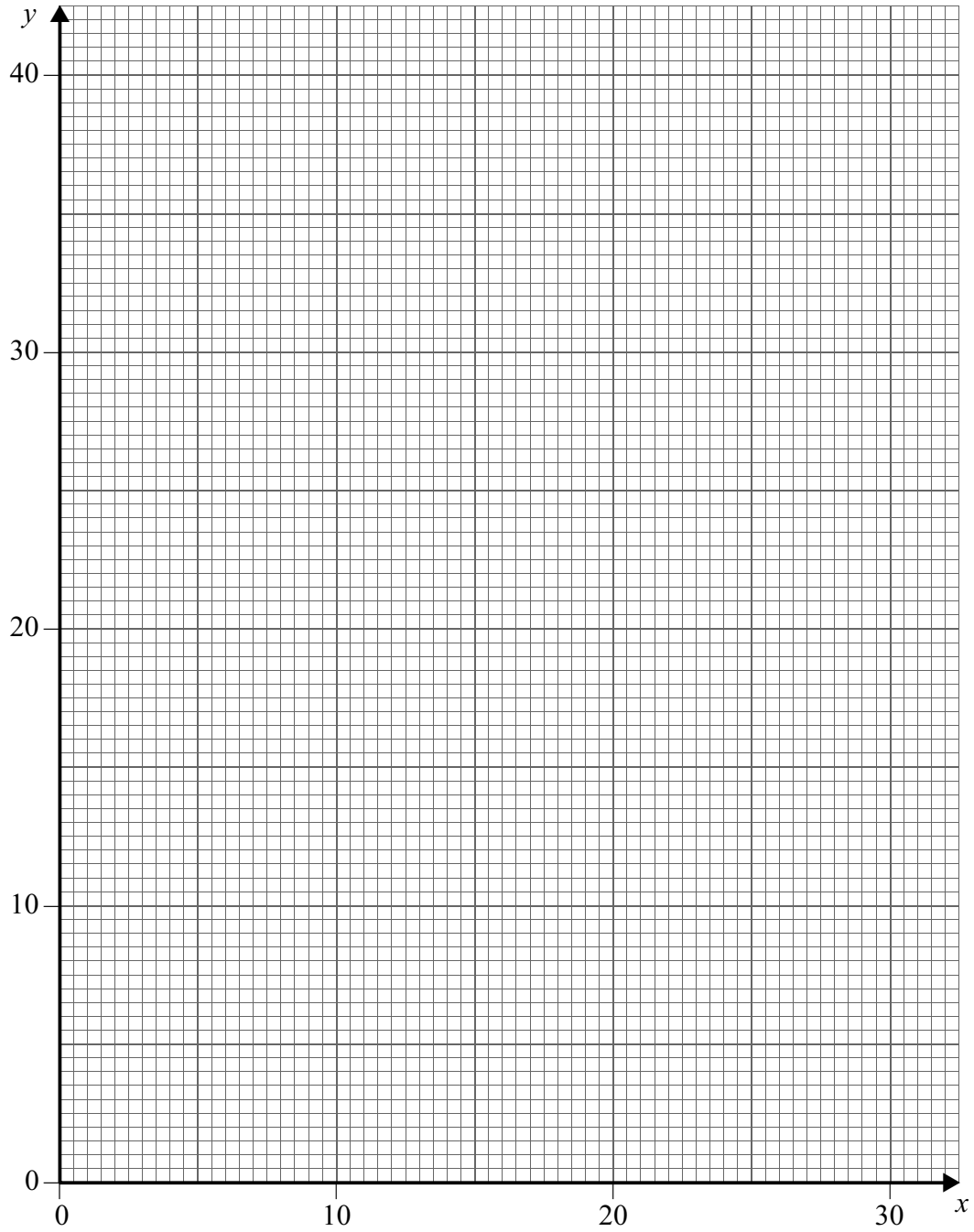
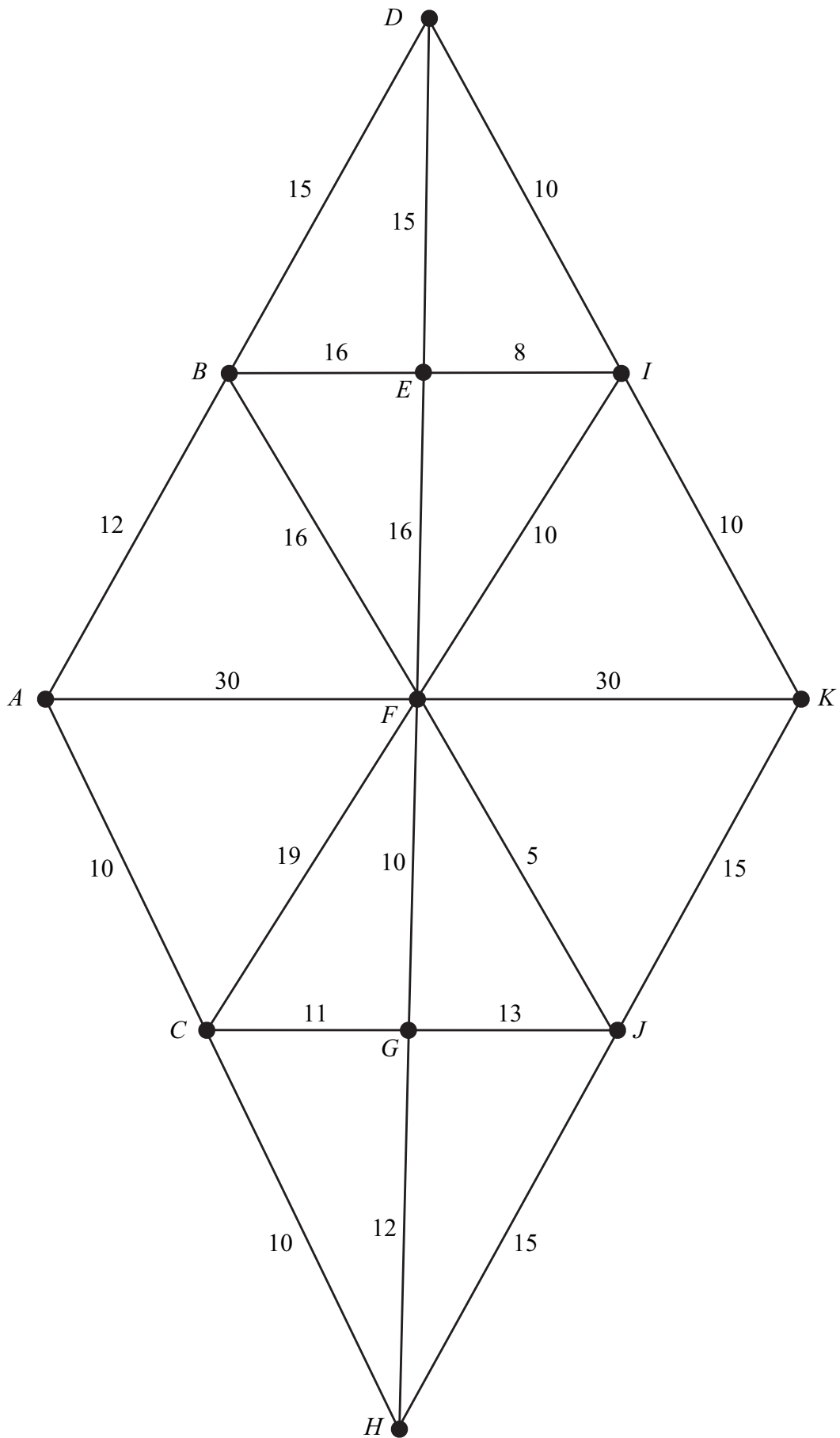
Figure 1 (for use in Question 2)

Figure 2 (for use in Question 4)



Turn over ►

Figure 3 (for use in Question 5)

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>A</i>	—		38	
<i>B</i>		—		
<i>C</i>	38		—	
<i>D</i>				—