

General Certificate of Education
June 2008
Advanced Level Examination



MATHEMATICS
Unit Mechanics 3

MM03

Friday 23 May 2008 9.00 am to 10.30 am

For this paper you must have:

- a 12-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The speed, $v \text{ m s}^{-1}$, of a wave travelling along the surface of a sea is believed to depend on
- the depth of the sea, $d \text{ m}$,
 - the density of the water, $\rho \text{ kg m}^{-3}$,
 - the acceleration due to gravity, g , and
 - a dimensionless constant, k

so that

$$v = kd^\alpha \rho^\beta g^\gamma$$

where α , β and γ are constants.

By using dimensional analysis, show that $\beta = 0$ and find the values of α and γ . (6 marks)

- 2 The unit vectors \mathbf{i} and \mathbf{j} are directed due east and due north respectively.

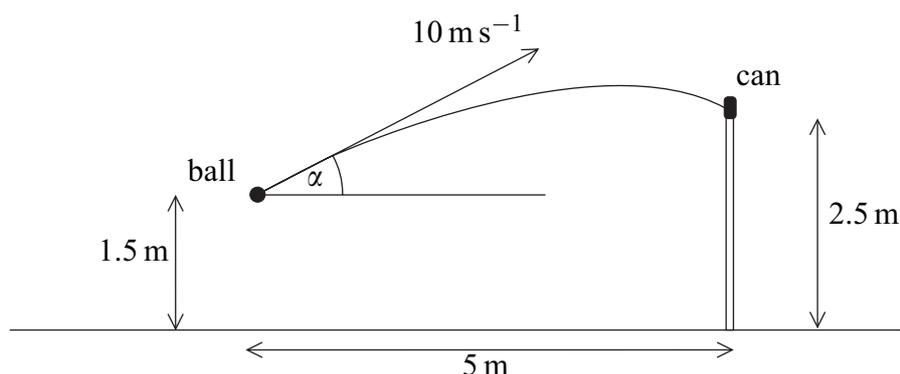
Two runners, Albina and Brian, are running on level parkland with constant velocities of $(5\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$ and $(3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$ respectively. Initially, the position vectors of Albina and Brian are $(-60\mathbf{i} + 160\mathbf{j}) \text{ m}$ and $(40\mathbf{i} - 90\mathbf{j}) \text{ m}$ respectively, relative to a fixed origin in the parkland.

- (a) Write down the velocity of Brian relative to Albina. (2 marks)
 - (b) Find the position vector of Brian relative to Albina t seconds after they leave their initial positions. (3 marks)
 - (c) Hence determine whether Albina and Brian will collide if they continue running with the same velocities. (3 marks)
- 3 A particle of mass 0.2 kg lies at rest on a smooth horizontal table. A horizontal force of magnitude F newtons acts on the particle in a constant direction for 0.1 seconds. At time t seconds,

$$F = 5 \times 10^3 t^2, \quad 0 \leq t \leq 0.1$$

Find the value of t when the speed of the particle is 2 m s^{-1} . (4 marks)

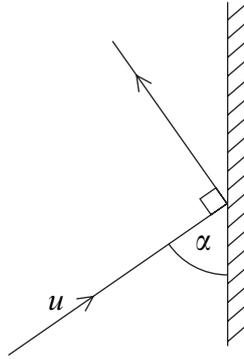
- 4 Two smooth spheres, A and B , have equal radii and masses m and $2m$ respectively. The spheres are moving on a smooth horizontal plane. The sphere A has velocity $(4\mathbf{i} + 3\mathbf{j})$ when it collides with the sphere B which has velocity $(-2\mathbf{i} + 2\mathbf{j})$. After the collision, the velocity of B is $(\mathbf{i} + \mathbf{j})$.
- (a) Find the velocity of A immediately after the collision. (3 marks)
- (b) Find the angle between the velocities of A and B immediately after the collision. (3 marks)
- (c) Find the impulse exerted by B on A . (3 marks)
- (d) State, as a vector, the direction of the line of centres of A and B when they collide. (1 mark)
- 5 A boy throws a small ball from a height of 1.5 m above horizontal ground with initial velocity 10 m s^{-1} at an angle α above the horizontal. The ball hits a small can placed on a vertical wall of height 2.5 m, which is at a horizontal distance of 5 m from the initial position of the ball, as shown in the diagram.



- (a) Show that α satisfies the equation
- $$49 \tan^2 \alpha - 200 \tan \alpha + 89 = 0 \quad (7 \text{ marks})$$
- (b) Find the **two** possible values of α , giving your answers to the nearest 0.1° . (3 marks)
- (c) (i) To knock the can off the wall, the horizontal component of the velocity of the ball must be greater than 8 m s^{-1} .
- Show that, for one of the possible values of α found in part (b), the can will be knocked off the wall, and for the other, it will **not** be knocked off the wall. (3 marks)
- (ii) Given that the can is knocked off the wall, find the direction in which the ball is moving as it hits the can. (4 marks)

- 6 A small smooth ball of mass m , moving on a smooth horizontal surface, hits a smooth vertical wall and rebounds. The coefficient of restitution between the wall and the ball is $\frac{3}{4}$.

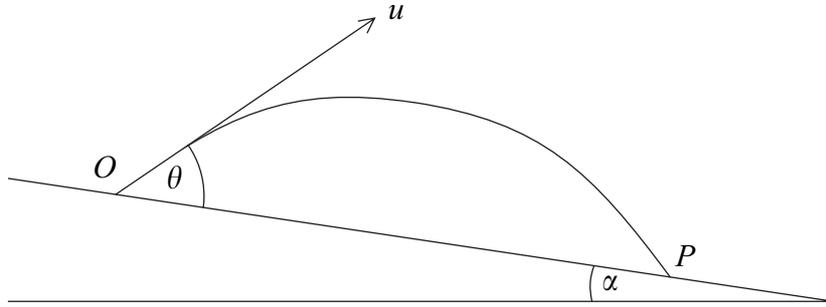
Immediately before the collision, the ball has velocity u and the angle between the ball's direction of motion and the wall is α . The ball's direction of motion immediately after the collision is at right angles to its direction of motion before the collision, as shown in the diagram.



- (a) Show that $\tan \alpha = \frac{2}{\sqrt{3}}$. (5 marks)
- (b) Find, in terms of u , the speed of the ball immediately after the collision. (2 marks)
- (c) The force exerted on the ball by the wall acts for 0.1 seconds.

Given that $m = 0.2 \text{ kg}$ and $u = 4 \text{ m s}^{-1}$, find the average force exerted by the wall on the ball. (6 marks)

- 7 A projectile is fired with speed u from a point O on a plane which is inclined at an angle α to the horizontal. The projectile is fired at an angle θ to the inclined plane and moves in a vertical plane through a line of greatest slope of the inclined plane. The projectile lands at a point P , lower down the inclined plane, as shown in the diagram.



- (a) Find, in terms of u , g , θ and α , the greatest perpendicular distance of the projectile from the plane. (4 marks)
- (b) (i) Find, in terms of u , g , θ and α , the time of flight from O to P . (2 marks)
- (ii) By using the identity $\cos A \cos B + \sin A \sin B = \cos(A - B)$, show that the distance OP is given by $\frac{2u^2 \sin \theta \cos(\theta - \alpha)}{g \cos^2 \alpha}$. (6 marks)
- (iii) Hence, by using the identity $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ or otherwise, show that, as θ varies, the maximum possible distance OP is $\frac{u^2}{g(1 - \sin \alpha)}$. (5 marks)

END OF QUESTIONS

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