



**General Certificate of Education**

**Mathematics 6360**

**MD02      Decision 2**

**Mark Scheme**

*2009 examination - June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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**Key to mark scheme and abbreviations used in marking**

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

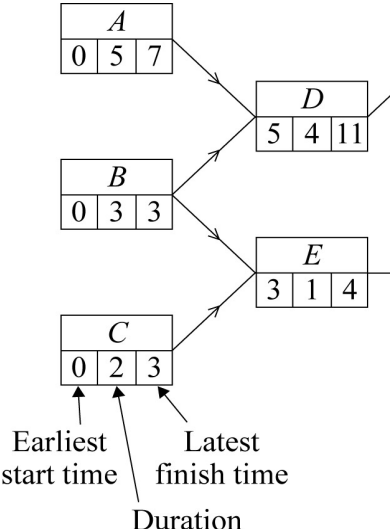
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

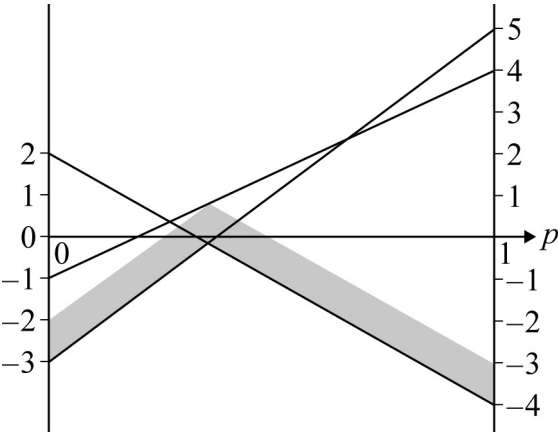
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MD02**

Q	Solution	Marks	Total	Comments
1				
(a)	Network attempted (3 more activities) Up to 2 slips (boxes or connections) Correct network	M1 A1 A1	3	SCA Condone missing arrows if sequence is clear
(b)(i)	Forward pass Correct	M1 A1	2	up to 1 slip ft
(ii)	Backward pass Correct	M1 A1	2	up to 1 slip ft
(c)	Minimum completion time 22 days  Critical path <i>B E G H I K</i>	B1  B1	2	Must be stated – not simply in <i>K</i> box and no others
(d)(i)	New start time for <i>H</i> is 15 days New start time for <i>I</i> is 16 days	M1 A1	2	For <i>H</i> , their ( <i>F</i> earliest time 9) + (2 + 4) both correct
(ii)	Minimum delay is 2 days	B1	1	Condone new completion time 24 days
	<b>Total</b>		<b>12</b>	

**MD02 (cont)**

Q	Solution	Marks	Total	Comments
2(a)	(For each outcome) Rowena's gain + Colin's gain = 0	E1	1	One player's loss is other's gain
(b)	(Column maxima 2, 5, 4) ⇒ min(col max)=2 (OE but strict) ⇒ Colin's play-safe strategy is $C_1$	E1 B1	2	Withhold E mark if any value incorrect; accept column minimax = 2
(c)	$R_3$ is dominated by $R_1$	E1	1	$-5 < -4$ ; $4 < 5$ and $3 < 4$ E0 if $R_2$ mentioned as well
(d)	Let Rowena play $R_1$ with prob $p$ and $R_2$ with prob $1 - p$ Expected gain when Colin plays $C_1 : -4p + 2(1 - p) = 2 - 6p$ $C_2 : 5p - 3(1 - p) = -3 + 8p$ $C_3 : 4p - (1 - p) = -1 + 5p$  Plot expected gains against $p$ for $0 \leq p \leq 1$	M1 A1  M1		attempt at least 2 with one correct all 3 correct unsimplified  All 3 drawn ft their exp gains
		A1		correct
	⇒ $2 - 6p = -3 + 8p$	M1		Using "correct" equation Choosing highest point of region
	⇒ $p = \frac{5}{14}$	A1		
	Therefore Rowena plays $R_1$ with prob $\frac{5}{14}$ and $R_2$ with prob $\frac{9}{14}$	E1✓	7	ft their $p$
	<b>Total</b>		<b>11</b>	

## MD02 (cont)

Q	Solution	Marks	Total	Comments
3(a)	Hungarian algorithm minimises.	E1	2	Or changes maximising to minimising problem
	$17-x$ gives measure of criteria not met (which need minimising in order to maximise scores)	E1		Explanation of what each new entry or $17-x$ represents (as something which can be minimised)
(b)	$\begin{array}{ccccc} 4 & 4 & 8 & 7 & 4 \\ 4 & 3 & 5 & 0 & 2 \\ 1 & 7 & 9 & 3 & 3 \\ 6 & 3 & 5 & 1 & 7 \\ 5 & 3 & 3 & 4 & 2 \end{array}$	B1	3	array with $17-x$ values
	$\begin{array}{ccccc} 0 & 0 & 4 & 3 & 0 & 0 & 0 & 3 & 3 & 0 \\ 4 & 3 & 5 & 0 & 2 & 4 & 3 & 4 & 0 & 2 \\ 0 & 6 & 8 & 2 & 2 & \rightarrow & 0 & 6 & 7 & 2 & 2 \\ 5 & 2 & 4 & 0 & 6 & 5 & 2 & 3 & 0 & 6 \\ 3 & 1 & 1 & 2 & 0 & 3 & 1 & 0 & 2 & 0 \end{array}$	M1		reduce rows first – condone one slip
		A1		then columns; AG
(c)	Top and bottom rows and 1 <sup>st</sup> & 4 <sup>th</sup> columns covered	B1	3	Zeros covered with 2 horizontal and 2 vertical lines
	$\begin{array}{ccccc} 2 & 0 & 3 & 5 & 0 \\ 4 & 1 & 2 & 0 & 0 \\ 0 & 4 & 5 & 2 & 0 \\ 5 & 0 & 1 & 0 & 4 \\ 5 & 1 & 0 & 4 & 0 \end{array}$	M1		augment by subtracting 2 from each uncovered and adding 2 to each double covered – condone one slip (may earn if 4 different lines are drawn)
(d)	T1, R2, V3, U4, S5	M1	4	3 items correctly matched
	T1, U2, V3, S4, R5	A1		First matching correct
		M1		3 items correct in second matching
		A1		Second matching correct and no other matches attempted
(e)	Maximum total score = 74	B1	1	
<b>Total</b>			<b>13</b>	

## MD02 (cont)

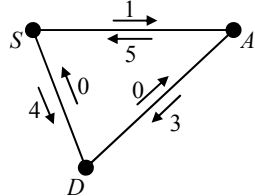
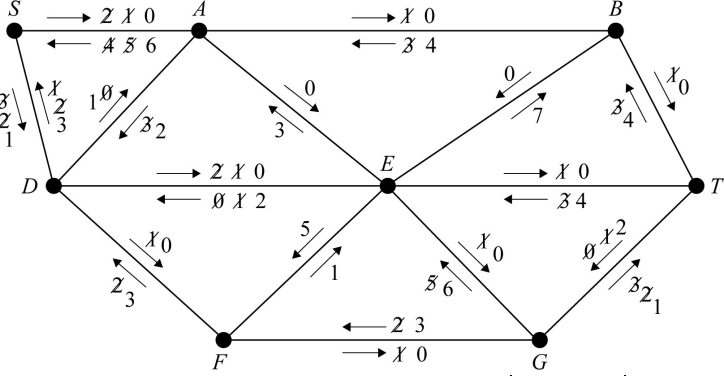
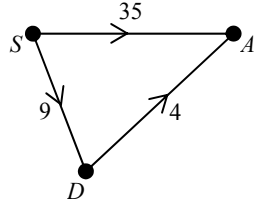
Q	Solution	Marks	Total	Comments																																
4(a)	$x+2y+3z \leq 7$ $2x+y+4z \leq 10$	B1	1	Exactly this																																
(b)(i)	Pivot is 2 in $x$ -column	B1		Must be ringed or clearly indicated or stated – <b>not</b> simply implied																																
	<table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="padding-right: 10px;"><math>P</math></td> <td style="padding-right: 10px;"><math>x</math></td> <td style="padding-right: 10px;"><math>y</math></td> <td style="padding-right: 10px;"><math>z</math></td> <td style="padding-right: 10px;"><math>s</math></td> <td style="padding-right: 10px;"><math>t</math></td> <td style="padding-right: 10px;"><math>value</math></td> <td></td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> <td><math>8-k</math></td> <td>0</td> <td>2</td> <td>20</td> <td>A1</td> </tr> <tr> <td>0</td> <td>0</td> <td><math>1\frac{1}{2}</math></td> <td>1</td> <td>1</td> <td><math>-\frac{1}{2}</math></td> <td>2</td> <td>A1</td> </tr> <tr> <td>0</td> <td>1</td> <td><math>\frac{1}{2}</math></td> <td>2</td> <td>0</td> <td><math>\frac{1}{2}</math></td> <td>5</td> <td></td> </tr> </table>	$P$	$x$	$y$	$z$	$s$	$t$	$value$		1	0	1	$8-k$	0	2	20	A1	0	0	$1\frac{1}{2}$	1	1	$-\frac{1}{2}$	2	A1	0	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	5		M1		row operations (even with incorrect pivot) condone one slip
$P$	$x$	$y$	$z$	$s$	$t$	$value$																														
1	0	1	$8-k$	0	2	20	A1																													
0	0	$1\frac{1}{2}$	1	1	$-\frac{1}{2}$	2	A1																													
0	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	5																														
		A1		Top or 2 <sup>nd</sup> row correct using correct pivot																																
		A1	4	All correct (condone multiples of rows)																																
(ii)	$8-k < 0$	M1		Their $f(k) < 0$																																
	$\Rightarrow k > 8$	A1	2	SC B1 for $k \geq 9$																																
(c)(i)	New pivot from $z$ -column in second row	B1 $\checkmark$		Stated or possibly implied from tableau																																
	<table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="padding-right: 10px;"><math>P</math></td> <td style="padding-right: 10px;"><math>x</math></td> <td style="padding-right: 10px;"><math>y</math></td> <td style="padding-right: 10px;"><math>z</math></td> <td style="padding-right: 10px;"><math>s</math></td> <td style="padding-right: 10px;"><math>t</math></td> <td style="padding-right: 10px;"><math>value</math></td> <td></td> </tr> <tr> <td>1</td> <td>0</td> <td>4</td> <td>0</td> <td>2</td> <td>1</td> <td>24</td> <td>M1</td> </tr> <tr> <td>0</td> <td>0</td> <td><math>1\frac{1}{2}</math></td> <td>1</td> <td>1</td> <td><math>-\frac{1}{2}</math></td> <td>2</td> <td>A1</td> </tr> <tr> <td>0</td> <td>1</td> <td><math>-2\frac{1}{2}</math></td> <td>0</td> <td>-2</td> <td><math>1\frac{1}{2}</math></td> <td>1</td> <td>A1</td> </tr> </table>	$P$	$x$	$y$	$z$	$s$	$t$	$value$		1	0	4	0	2	1	24	M1	0	0	$1\frac{1}{2}$	1	1	$-\frac{1}{2}$	2	A1	0	1	$-2\frac{1}{2}$	0	-2	$1\frac{1}{2}$	1	A1	M1		row operations using “their” <b>correct pivot</b> condone 1 slip
$P$	$x$	$y$	$z$	$s$	$t$	$value$																														
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0	1	$-2\frac{1}{2}$	0	-2	$1\frac{1}{2}$	1	A1																													
		A1		one row (other than pivotal row) correct																																
		A1	4	all correct (condone multiples of rows)																																
(ii)	$P = 24$	B1 $\checkmark$		Provided no negatives in top row																																
	Optimum now reached	E1		Or $P_{\max} = \dots$																																
	$x = 1, y = 0, z = 2$	B1 $\checkmark$		Only ft if no more than 2 slips in final tableau																																
			3																																	
	<b>Total</b>		<b>14</b>																																	

## MD02 (cont)

Q	Solution	Marks	Total	Comments																																																																																																																								
5(a)	Completing stage 2 values (condone unsimplified)	B1	7	<table border="1"> <thead> <tr> <th>Stage</th> <th>State</th> <th>From</th> <th>Value</th> <th></th> </tr> </thead> <tbody> <tr> <td rowspan="2">1</td> <td><i>K</i></td> <td><i>T</i></td> <td>7</td> <td></td> </tr> <tr> <td><i>L</i></td> <td><i>T</i></td> <td>8</td> <td></td> </tr> <tr> <td rowspan="2">2</td> <td><i>H</i></td> <td><i>K</i></td> <td><math>-2 + 7 = 5</math></td> <td></td> </tr> <tr> <td><i>I</i></td> <td><i>K</i></td> <td><math>4 + 7 = 11</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>L</i></td> <td><math>-1 + 8 = 7</math></td> <td></td> </tr> <tr> <td rowspan="2">3</td> <td><i>J</i></td> <td><i>L</i></td> <td><math>5 + 8 = 13</math></td> <td></td> </tr> <tr> <td><i>D</i></td> <td><i>H</i></td> <td><math>4 + 5 = 9</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>I</i></td> <td><math>2 + 11 = 13</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>E</i></td> <td><math>7 + 5 = 12</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>I</i></td> <td><math>-9 + 11 = 2</math></td> <td></td> </tr> <tr> <td></td> <td></td> <td><i>F</i></td> <td><math>-4 + 11 = 7</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>J</i></td> <td><math>9 + 13 = 22</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>G</i></td> <td><math>-7 + 11 = 4</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>J</i></td> <td><math>-8 + 13 = 5</math></td> <td>*</td> </tr> <tr> <td rowspan="2">4</td> <td><i>A</i></td> <td><i>D</i></td> <td><math>-2 + 13 = 11</math></td> <td>*</td> </tr> <tr> <td></td> <td><i>E</i></td> <td><math>5 + 12 = 17</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>F</i></td> <td><math>-8 + 22 = 14</math></td> <td></td> </tr> <tr> <td></td> <td></td> <td><i>B</i></td> <td><math>-1 + 12 = 11</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>F</i></td> <td><math>-7 + 22 = 15</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>G</i></td> <td><math>-3 + 5 = 2</math></td> <td></td> </tr> <tr> <td rowspan="2">5</td> <td><i>C</i></td> <td><i>G</i></td> <td><math>5 + 5 = 10</math></td> <td></td> </tr> <tr> <td><i>S</i></td> <td><i>A</i></td> <td><math>1 + 17 = 18</math></td> <td>*</td> </tr> <tr> <td></td> <td></td> <td><i>B</i></td> <td><math>2 + 15 = 17</math></td> <td></td> </tr> <tr> <td></td> <td></td> <td><i>C</i></td> <td><math>6 + 10 = 16</math></td> <td></td> </tr> </tbody> </table>	Stage	State	From	Value		1	<i>K</i>	<i>T</i>	7		<i>L</i>	<i>T</i>	8		2	<i>H</i>	<i>K</i>	$-2 + 7 = 5$		<i>I</i>	<i>K</i>	$4 + 7 = 11$	*			<i>L</i>	$-1 + 8 = 7$		3	<i>J</i>	<i>L</i>	$5 + 8 = 13$		<i>D</i>	<i>H</i>	$4 + 5 = 9$	*			<i>I</i>	$2 + 11 = 13$	*			<i>E</i>	$7 + 5 = 12$	*			<i>I</i>	$-9 + 11 = 2$				<i>F</i>	$-4 + 11 = 7$	*			<i>J</i>	$9 + 13 = 22$	*			<i>G</i>	$-7 + 11 = 4$	*			<i>J</i>	$-8 + 13 = 5$	*	4	<i>A</i>	<i>D</i>	$-2 + 13 = 11$	*		<i>E</i>	$5 + 12 = 17$	*			<i>F</i>	$-8 + 22 = 14$				<i>B</i>	$-1 + 12 = 11$	*			<i>F</i>	$-7 + 22 = 15$	*			<i>G</i>	$-3 + 5 = 2$		5	<i>C</i>	<i>G</i>	$5 + 5 = 10$		<i>S</i>	<i>A</i>	$1 + 17 = 18$	*			<i>B</i>	$2 + 15 = 17$				<i>C</i>	$6 + 10 = 16$	
	Stage	State			From	Value																																																																																																																						
1	<i>K</i>	<i>T</i>	7																																																																																																																									
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(b)	Maximum profit £18m Sequence of actions <i>SAEHKT</i>	B1 B1	2	condone 18																																																																																																																								
<b>Total</b>			<b>9</b>																																																																																																																									



MD02 (cont)

Q	Solution	Marks	Total	Comments										
6(a)	Value of cut = $30 - 10 + 12 + 20 = 52$	M1 A1	2	Full marks for correct answers without working										
(b)	$AE = 9;$ $EF = 5;$ $FG = 4$	B1 B1 B1	3											
(c)(i)	Attempt at forward and backward flows $SA$ 2 & 4; $AB$ 1 & 3; $BT$ 1 & 3 $SD$ 3 & 1; $DA$ 0 & 3; $AE$ 0 & 3 $BE$ 0 & 7; $DE$ 2 & 0; $ET$ 1 & 3 $FD$ 2 & 1; $EF$ 5 & 1; $EG$ 1 & 5 $FG$ 1 & 2; $GT$ 3 & 0	M1 A1 A1	3	At least 5 pairs correct 10 pairs correct all correct										
(ii)	First flow augmenting path and correct flow on table Table correct Adjusting flows – forward and back Correct	M1 A1 M1 A1	4	May end up with 										
				<table border="1"> <thead> <tr> <th>Path</th> <th>Extra flow</th> </tr> </thead> <tbody> <tr> <td>SABT</td> <td>1</td> </tr> <tr> <td>SADET</td> <td>1</td> </tr> <tr> <td>SDFGT</td> <td>1</td> </tr> <tr> <td>SDEGT</td> <td>1</td> </tr> </tbody> </table> <p style="text-align: right;">Or SDET</p>	Path	Extra flow	SABT	1	SADET	1	SDFGT	1	SDEGT	1
Path	Extra flow													
SABT	1													
SADET	1													
SDFGT	1													
SDEGT	1													
(d)	Max flow of 44 shown on figure 5	M1 A1	2	up to 2 slips all correct May have 										
(e)	Cut through their saturated arcs Cut passes through $AB, AE, DE$ and $DF$	M1 A1	2	Or $BT, ET, EG,$ and $FG$										
<b>Total</b>			<b>16</b>											
<b>TOTAL</b>			<b>75</b>											