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General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$\mathrm{f}\left(\frac{1}{3}\right)=3\times\frac{1}{27}+8\times\frac{1}{9}-3\times\frac{1}{3}-5$	M1		Use $\frac{1}{3}$ in evaluating $\mathrm{f}(x)$
	$=-5$	A1	2	No ISW Evidence of Remainder Theorem
(b)	$\begin{array}{r} x^2+3x \\ 3x-1\overline{)3x^3+8x^2-3x-5} \\ 3x^3\ -x^2 \qquad\qquad \\ 9x^2-3x \qquad \\ 9x^2-3x \qquad \end{array}$	M1		Division with x^2 and an x term seen; x^2+px
	$a=1 \quad b=3$ or $x^2+3x+\frac{c}{3x-1}$	A1		Explicit or in expression
	$c=-5$	B1		Condone $+\frac{-5}{3x-1}$
	Alternative			
	$\frac{(3x-1)(x^2+px)}{3x-1}-\frac{5}{3x-1}$	(M1)		Split fraction and attempt factors
	$x^2+3x \qquad -\frac{5}{3x-1}$	(A1) (B1)		$a=1 \quad b=3$ $c=-5$
	Alternative			
	$\mathrm{f}(x)=3ax^3+(3b-a)x^2-bx+c$	(M1)		Multiply by $(3x-1)$ and attempt to collect terms
	$a=1 \quad b=3$ $c=-5$	(A1) (B1)		
	Alternative			
	$\mathrm{f}(x)=(ax^2+bx)(3x-1)+c$	(M1)		Multiply by $(3x-1)$ and attempt to find a, b, c: substitute 3 values of x and form 3 simultaneous equations, and attempt to solve; or substitute 3 values of x into given equation
	$x=0\Rightarrow c=-5$ $x=1\Rightarrow 2a+2b+c=3$ $x=2\Rightarrow 20a+10b+c=45$	(B1)		
	$a=1 \quad b=3$	(A1)	3	
	Total		**5**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{1}{t^2}$ $\frac{\mathrm{d}y}{\mathrm{d}t}=1-\frac{1}{2t^2}$	B1B1		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-\frac{1}{2t^2}}{-\frac{1}{t^2}}$ $\left(=\frac{2t^2-1}{-2}\right)$	M1		Their $\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$; condone 1 slip
		A1		CSO; ISW
	Alternative			
	$y=\frac{1}{x}+\frac{x}{2}$	(B1)		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x^2}+\frac{1}{2}$	(B1)		
	Substitute $x=\frac{1}{t}$	(M1)		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-t^2+\frac{1}{2}$	(A1)	4	CSO
(b)	$t=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$	M1		Substitute $t=1$ in $\frac{\mathrm{f}(t)}{\mathrm{g}(t)}\not\equiv k$
	$m_T=-\frac{1}{2}\Rightarrow m_n=2$	B1F		F on $m_T\neq 0$; if in $t\rightarrow$numerical later
	$(x,y)=\left(1,\frac{3}{2}\right)$	B1		PI $\frac{3}{2}=m(\times 1)+c$
	$\left(y-\frac{3}{2}\right)=2(x-1)$ **or** $y=2x+c,\ c=-\frac{1}{2}$	A1	4	ISW, CSO (a) and (b) all correct
(c)	$y=\frac{1}{\frac{1}{t}}+\frac{1}{2}\times\frac{1}{t}$	M1		Attempt to use $t=\frac{1}{x}$ to eliminate t t, or equivalent
	$=\frac{1}{x}+\frac{x}{2}$	A1		
	$2xy=2+x^2\Rightarrow x^2-2xy+2=0$	A1		Correct algebra to AG with $k=2$ allow $k=2$ stated $k=2$, no working or from $\left(1,\frac{3}{2}\right)$: 0/3
	Alternative **or**			
	$\left(\frac{1}{t}\right)^2-2\left(\frac{1}{t}\right)\left(t+\frac{1}{2t}\right)$ | $xy=\frac{1}{t}\left(t+\frac{1}{2t}\right)$	(M1)		Substitute and multiply out
	$=-2$ | $=1+\frac{x^2}{2}$	(A1)		Eliminate t
	$\Rightarrow x^2-2xy+2=0$	(A1)	3	Conclusion, $k=2$
			11	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$(1-x)^{-1}=1+(-1)(-x)+\frac{1}{2}(-1.-2)(-x)^2$	M1		$1\pm x+kx^2$
	$=1+x+x^2$	A1	2	Fully simplified
(b)(i)	$3x-1=A(2-3x)+B(1-x)$	M1		
	$x=1 \qquad x=\frac{2}{3}$	m1		Use 2 values of x or equate coefficients and solve $-3A-B=3 \quad 2A+B=-1$ condone coefficient errors
	$A=-2 \quad B=3$	A1	3	Both values NMS 3/3 if both correct, 1/3 if one correct
(ii)	$\left(\frac{3x-1}{(1-x)(2-3x)}=\frac{-2}{1-x}+\frac{3}{2-3x}\right)$			
	$\frac{-2}{1-x}=-2-2x-2x^2$	B1F		F on $(1-x)^{-1}$ and A
	$\frac{1}{2-3x}=\frac{1}{2}\left(1-\frac{3}{2}x\right)^{-1}$	B1		
	$=(p)\left(1+kx+(kx)^2\right)$	M1		p, k = candidate's $\frac{1}{2},\frac{3}{2}$, $k\neq\pm1$
	$=(p)\left(1+\frac{3}{2}x+\frac{9}{4}x^2\right)$	A1		Use (a) or start binomial again; condone missing brackets, and one sign error
	$\frac{3x-1}{(1-x)(2-3x)}=-2(1-x)^{-1}+3(2-3x)^{-1}$	M1		Valid combination of both expansions
	$=-\frac{1}{2}+\frac{1}{4}x+\frac{11}{8}x^2$	A1		CSO
	Alternative			
	$(2-3x)^{-1}=\frac{1}{2}\left(1-\frac{3}{2}x\right)^{-1}$	(B1)		k = candidate's $\frac{3}{2}$ $\quad k\neq\pm1$
	$(1-kx)^{-1}=1+kx+(kx)^2$	(M1)		Use (a) or start binomial again; condone missing brackets and one error
	$=1+\frac{3}{2}x+\frac{9}{4}x^2$	(A1)		
	$\frac{3x-1}{(1-x)(2-3x)}=(3x-1)(1-x)^{-1}(2-3x)^{-1}$	(M1)		$(3x-1)\times$ both expansions
	$\frac{3x-1}{(1-x)(2-3x)}=-\frac{1}{2}+\frac{1}{4}x+\frac{11}{8}x^2$	(m1) (A1)	6	Multiply out; collect terms to form $a+bx+cx^2$ CSO
	Alternative for $(2-3x)^{-1}$			Using $(a+bx)^n$
	$2^{-1}+(-1)(2)^{-2}(-3x)+\frac{(-1)(-2)(2)^{-3}(-3x)^2}{2}$	(M1)		Condone missing brackets, and 1 error
	$=\frac{1}{2}+\frac{3}{4}x+\frac{9}{8}x^2$	(A1) (A1)		First two terms x^2 term

MPC4 (cont)

Q	Solution	Marks	Total	Comments
(c)	$-2 < 3x < 2$	M1		PI, or any equivalent form
	$\Rightarrow -\frac{2}{3} < x < \frac{2}{3}$	A1	2	Condone ≤; accept $\pi \geq \frac{2}{3}$ or $x \geq -\frac{2}{3}$ CSO; allow $\lvert \pm x \rvert \leq \frac{2}{3}$, or $x < \frac{2}{3}$ and $x > -\frac{2}{3}$
	Total		**13**	
4(a)(i)	$A = 12499$	B1	1	Stated in (i) or (ii)
(ii)	$k^{36} = \frac{7000}{\text{their } A}$	M1		$p = \frac{7000}{12499} = 0.560044803$
	$k = \sqrt[36]{0.56(00448...)} = 0.9840251(26)$ or $\left(0.56(00448...)\right)^{\frac{1}{36}}$ or $k = \sqrt[36]{\frac{7000}{12499}}$ $k = 0.984025$	A1	2	Correct expression for k or 7th dp seen. $k = 10^{\frac{1}{36}\log p}$ or $k = 10^{-0.00699...}$ $k = \mathrm{e}^{\frac{1}{36}\ln p}$ or $k = \mathrm{e}^{-0.016103...}$ AG
(b)	$k^{t} = \frac{5000}{\text{their } A}$	M1		$\frac{5000}{12499} = 0.400032...$; condone 4999
	$t\log(k) = \log\left(\frac{5000}{A}\right) \quad (t = 56.89)$	m1		Correct use of logs
	$n = 57$	A1		n integer; n = 57 CAO
	Alternative ; trial and improvement on $5000 = 12499 \times 0.984025^{t}$			
	2 values of $t \geq 40$	(M1)		
	1 value of t $50 < t < 60$	(m1)		
	$n = 57$	(A1)	3	
	Special case, answer only $n = 57$ 3/3 $n = 56$ 0/3 $n = 56.9$ 2/3			
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5	$8x+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=3y+3x\dfrac{\mathrm{d}y}{\mathrm{d}x}$			
	$8x$ and $4\to0$	B1		
	$2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$	B1		
	$3y+3x\ \dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1		Two terms with one $\dfrac{\mathrm{d}y}{\mathrm{d}x}$
	at $(1,3)$ (gradient) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{3}$	A1	5	CSO
	Total		**5**	
6(a)(i)	$\cos 2x=2\cos^2 x-1$	B1		Seen in question, in consistent variable
	$3\left(2\cos^2 x-1\right)+7\cos x+5$	M1		Substitute candidate's $\cos 2x$ in terms of $\cos x$
	$6\cos^2 x+7\cos x+2(=0)$	A1	3	
(ii)	$(2\cos x+1)(3\cos x+2)$	M1		Attempt factors; formula ('a' and 'c' correct; allow one slip)
	$\cos x=-\dfrac{1}{2}\qquad \cos x=-\dfrac{2}{3}$	A1	2	Accept $-0.5,-0.67$ $x=\cos^{-1}\left(-\dfrac{1}{2}\right)$; $\cos^{-1}\left(-\dfrac{2}{3}\right)$
(b)(i)	$R=\sqrt{58}$	B1		Accept 7.6 or better
	$\alpha=\sin^{-1}\left(\dfrac{3}{\text{their } R}\right)$	M1		OE $\alpha=\sin^{-1}\left(\dfrac{3}{7}\right)$
	$=23.2^{\circ}$	A1	3	AWRT 23.2° (23.1985…)
(ii)	$\alpha+\theta=\sin^{-1}\left(\dfrac{4}{\text{their } R}\right)$	M1		Candidate's R, α
	$\theta=8.5^{\circ}$	A1F		F on α, AWRT, condone 8.6
	$\theta=125.1^{\circ}$	A1	3	Two solutions only, but ignore out of range
(c)(i)	$h^2=1+\left(2\sqrt{2}\right)^2$	M1		Pythagoras with h or sec x
	$h=3\Rightarrow\cos\beta=\dfrac{1}{3}$	A1	2	AG
(ii)	$\sin 2\beta=2\sin\beta\cos\beta$	M1		
	$\sin 2\beta=\dfrac{4}{9}\sqrt{2}$	A1	2	CSO; accept $p=\dfrac{4}{9}$ (not 0.444…)
	Total		**15**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\left(AB^2=\right)(4-3)^2+(0--2)^2+(1-5)^2$	M1		Condone one sign error in one bracket
	$AB=\sqrt{21}$	A1	2	Accept 4.58 or better
(b)	$4=6+2\lambda \quad\Rightarrow\quad \lambda=-1$	M1		$\lambda=-1$
	$0=-1+(-1)\times(-1)$ $1=5+(-1)\times 4$	A1	2	$\lambda=-1$ confirmed in other two equations Accept for M1A1 $\begin{bmatrix}6\\-1\\5\end{bmatrix}-\begin{bmatrix}2\\-1\\4\end{bmatrix}=\begin{bmatrix}4\\0\\1\end{bmatrix}$ M1 condone 1 slip
	Special case $\begin{bmatrix}6\\-1\\5\end{bmatrix}+\lambda\begin{bmatrix}2\\-1\\4\end{bmatrix}=\begin{bmatrix}4\\0\\1\end{bmatrix}, \quad \lambda\begin{bmatrix}2\\-1\\4\end{bmatrix}=\begin{bmatrix}-2\\1\\-4\end{bmatrix}$ $\lambda=-1$	(B2)		
(c)	$\begin{bmatrix}3\\-2\\5\end{bmatrix}+\mu\begin{bmatrix}-1\\3\\8\end{bmatrix}=\begin{bmatrix}6\\-1\\5\end{bmatrix}+\lambda\begin{bmatrix}2\\-1\\4\end{bmatrix}$	M1		Equate vector equations PI by two equations in λ or μ
	$\left.\begin{aligned}3-\mu&=6+2\lambda\\-2+3\mu&=-1-\lambda\end{aligned}\right\\}$ eliminate λ *or* μ	m1		Form (any) two simultaneous equations and solve for λ or μ
	$\lambda=-2$ or $\mu=1$	A1		
	C has coordinates $(2, 1, -3)$	A1		CAO condone $\begin{bmatrix}2\\1\\-3\end{bmatrix}$
	$BC^2=(2-4)^2+(0-1)^2+(1--3)^2$ $BC=\sqrt{21}$	M1		Use C to find BC or AC or to find two angles
	$AB=BC \quad (=\sqrt{21})$	A1	6	$AB = BC$ or $\angle A=\angle C\ (=20.2°)$ stated
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\int x \, \mathrm{d}x = \int 150 \cos 2t \, \mathrm{d}t$	B1		Correct separation; condone missing ∫ signs; must see d*x*, d*t*
	$\frac{1}{2}x^2 = 75 \sin 2t \quad (+C)$	B1B1		Correct integrals Accept $\frac{1}{2} \times 150$
	$\left(20, \frac{\pi}{4}\right) \quad \frac{1}{2} \times 20^2 = 75 \sin\left(2 \times \frac{\pi}{4}\right) + C$	M1		C present. Use $\left(20, \frac{\pi}{4}\right)$ to find C
	$C = 125$	A1F		F on $x^2 = k \sin 2t$
	$x^2 = 150 \sin 2t + 250$	A1	6	Correct integrals and evaluation of C
(b)(i)	$t = 13 \quad x^2 = 150 \sin 26 + 250 \; (= 364.38)$	M1		Evaluate $x^2 = \mathrm{f}\,(13)$; $x^2 = k \sin 2t + c$ with numerical k and t
	$x = 19.1 \; (\mathrm{cm})$	A1	2	AWRT
(ii)	$\left.\begin{array}{l} x = 11 \quad \sin 2t = -\frac{129}{150} \quad (= -0.86) \\ \text{or} \quad 2t = -1.035.., 4.176... \end{array}\right\}$	M1		
	$t = 2.1 (\text{seconds})$	A1	2	AWRT
	Total		**10**	
	TOTAL		**75**	