



General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics

MPC1

Unit Pure Core 1

Monday 11 January 2010 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $p(x)$ is given by $p(x) = x^3 - 13x - 12$.

- (a) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. (2 marks)
- (b) Express $p(x)$ as the product of three linear factors. (3 marks)

2 The triangle ABC has vertices $A(1, 3)$, $B(3, 7)$ and $C(-1, 9)$.

- (a) (i) Find the gradient of AB . (2 marks)
- (ii) Hence show that angle ABC is a right angle. (2 marks)
- (b) (i) Find the coordinates of M , the mid-point of AC . (2 marks)
- (ii) Show that the lengths of AB and BC are equal. (3 marks)
- (iii) Hence find an equation of the line of symmetry of the triangle ABC . (3 marks)

3 The depth of water, y metres, in a tank after time t hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t, \quad 0 \leq t \leq 4$$

(a) Find:

(i) $\frac{dy}{dt}$; (3 marks)

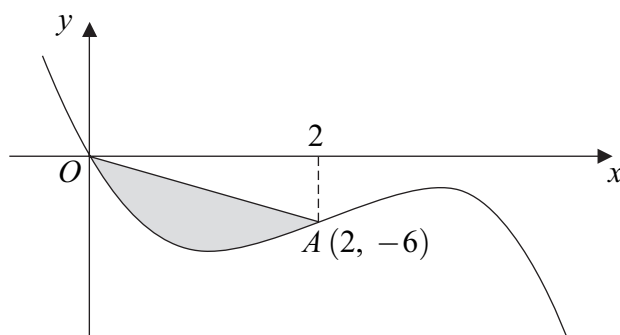
(ii) $\frac{d^2y}{dt^2}$. (2 marks)

(b) Verify that y has a stationary value when $t = 2$ and determine whether it is a maximum value or a minimum value. (4 marks)

(c) (i) Find the rate of change of the depth of water, in metres per hour, when $t = 1$. (2 marks)

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when $t = 1$. (1 mark)

- 4 (a) Show that $\frac{\sqrt{50} + \sqrt{18}}{\sqrt{8}}$ is an integer and find its value. (3 marks)
- (b) Express $\frac{2\sqrt{7} - 1}{2\sqrt{7} + 5}$ in the form $m + n\sqrt{7}$, where m and n are integers. (4 marks)
- 5 (a) Express $(x - 5)(x - 3) + 2$ in the form $(x - p)^2 + q$, where p and q are integers. (3 marks)
- (b) (i) Sketch the graph of $y = (x - 5)(x - 3) + 2$, stating the coordinates of the minimum point and the point where the graph crosses the y -axis. (3 marks)
- (ii) Write down an equation of the tangent to the graph of $y = (x - 5)(x - 3) + 2$ at its vertex. (2 marks)
- (c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = (x - 5)(x - 3) + 2$. (3 marks)
- 6 The curve with equation $y = 12x^2 - 19x - 2x^3$ is sketched below.



The curve crosses the x -axis at the origin O , and the point $A(2, -6)$ lies on the curve.

- (a) (i) Find the gradient of the curve with equation $y = 12x^2 - 19x - 2x^3$ at the point A . (4 marks)
- (ii) Hence find the equation of the normal to the curve at the point A , giving your answer in the form $x + py + q = 0$, where p and q are integers. (3 marks)
- (b) (i) Find the value of $\int_0^2 (12x^2 - 19x - 2x^3) dx$. (5 marks)
- (ii) Hence determine the area of the shaded region bounded by the curve and the line OA . (3 marks)

Turn over for the next question

Turn over ►

7 A circle with centre C has equation $x^2 + y^2 - 4x + 12y + 15 = 0$.

(a) Find:

(i) the coordinates of C ; *(2 marks)*

(ii) the radius of the circle. *(2 marks)*

(b) Explain why the circle lies entirely below the x -axis. *(2 marks)*

(c) The point P with coordinates $(5, k)$ lies outside the circle.

(i) Show that $PC^2 = k^2 + 12k + 45$. *(2 marks)*

(ii) Hence show that $k^2 + 12k + 20 > 0$. *(1 mark)*

(iii) Find the possible values of k . *(4 marks)*

END OF QUESTIONS