



**General Certificate of Education
June 2010**

Mathematics

MFP4

Further Pure 4

Mark Scheme

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
✓ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1(a)	$\begin{vmatrix} 3 & 4 & -1 \\ -1 & 2 & 2 \\ 1 & 4 & 1 \end{vmatrix} = 6 + 8 + 4 + 2 - 24 + 4$ <p>or $3(2 - 8) - 4(-1 - 2) - 1(-4 - 2)$ etc or $3(2 - 8) + 1(4 + 4) + 1(8 + 2)$ etc Correctly shown = 0</p> <p>Or $3\mathbf{p} + 4\mathbf{q} = 5\mathbf{r}$</p>	M1 A1 (M1) (A1)	2	Good attempt at det M0 for $ \dots = 0$ and no working
(b)	<p>For attempt at 2 of $(\pm)\overline{PQ}$, \overline{PR}, \overline{QR}</p> <p>Area $\Delta PQR = \frac{1}{2} \overline{QP} \times \overline{QR}$ e.g.</p> $= \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 2 & -3 \\ 2 & 0 & -2 \end{vmatrix} = \frac{1}{2} \pm(4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}) $ $= \frac{1}{2}\sqrt{4^2 + 2^2 + 4^2}$ $= 3$	M1 M1 M1 A1	4	Formula used with attempt at a vector product of any 2 of the above (ignore missing $\frac{1}{2}$ for now) Method for finding magnitude of their relevant vector CSO
Total			6	
2(a)	$\mathbf{AB} = \begin{bmatrix} 2x+1 & 2x-1 \\ 8 & 4 \end{bmatrix}$	M1 A1	2	Good attempt (at least one entry in R_1 ✓) All four correct
(b)	$\mathbf{B}^T \mathbf{A}^T = (\mathbf{AB})^T \quad \text{Or} \quad \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ x & 3 \end{bmatrix}$ $= \begin{bmatrix} 2x+1 & 8 \\ 2x-1 & 4 \end{bmatrix}$ <p>$2x+1 = 4 - 4x$ Or $2x-1 = 8x-4$ $x = \frac{1}{2}$</p> <p>Checking/noting $x = \frac{1}{2}$ in other eqn.</p>	M1 A1 ✓ M1 A1 B1	5	Ft their (a) Or CAO Ft previous answers CAO <i>Visibly</i>
Total			7	
3(a)	<p>Clearly identifying $\mathbf{n} = \begin{bmatrix} 9 \\ -8 \\ 72 \end{bmatrix}$</p> $d = \begin{bmatrix} 9 \\ -8 \\ 72 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 10 \\ 1 \end{bmatrix} = 10$	B1 M1 A1	3	
(b)	<p>Use of $\frac{\text{Sc.prod. of normals}}{\text{prod. of their moduli}}$</p> <p>$N^r = 73$ $D^r = 73\sqrt{3}$ or $\sqrt{15987}$ $\cos\theta = \frac{1}{\sqrt{3}}$</p>	M1 B1 ✓ B1 ✓ A1	4	Must be $(9\mathbf{i} - 8\mathbf{j} + 72\mathbf{k})$, $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ or their \mathbf{n} from (a) Ft their \mathbf{n} from (a) only CAO Allow unsimplified exact forms
Total			7	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$(\mathbf{v} =) \pm (\mathbf{a} - \mathbf{b}) = \pm \begin{bmatrix} 1 \\ -5 \\ 4 \end{bmatrix}$ $\mathbf{u} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} \text{ or } \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$	M1 A1 B1	3	M1 A0 if $\pm \overline{AB}$ found but not stated/shown this is \mathbf{v}
(b)	$\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -3 \\ 2-t & t & 5 \end{vmatrix} = \begin{bmatrix} 3t+5 \\ 3t-16 \\ 3t-2 \end{bmatrix}$	M1 A3,2,1	4	
(c) (i)	$\mathbf{a} \bullet \mathbf{b} \times \mathbf{c} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} 3t+5 \\ 3t-16 \\ 3t-2 \end{bmatrix} = 77$	M1		Or starting again: $\begin{vmatrix} 3 & -4 & 1 \\ 2 & 1 & -3 \\ 2-t & t & 5 \end{vmatrix}$
(ii)	<p>C never lies in plane of O, A, B (or is a fixed distance from it) or Vol. //ppd. $OABC$ always = 77 or Vol. tetrahdrn. $OABC$ always = $\frac{77}{6}$ or O is never in plane of A, B, C or $\overline{OA}, \overline{OB}, \overline{OC}$ never co-planar</p>	A1 B1	2 1	CAO Any suitable geometrical comment
Total			10	Vectors \checkmark ; points \times
5	$\Delta = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$ $= \begin{vmatrix} x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \\ yz & z(x-y) & y(x-z) \end{vmatrix}$ $= (y-x)(z-x) \begin{vmatrix} x & 1 & 1 \\ x^2 & y+x & z+x \\ yz & -z & -y \end{vmatrix}$ $= (y-x)(z-x) \begin{vmatrix} x & 1 & 0 \\ x^2 & y+x & z-y \\ yz & -z & z-y \end{vmatrix}$ $= (y-x)(z-x)(z-y) \begin{vmatrix} x & 1 & 0 \\ x^2 & y+x & 1 \\ yz & -z & 1 \end{vmatrix}$ $= (x-y)(y-z)(z-x)(xy + yz + zx)$	M1 M1 A1 A1 M1 A1 M1 A1	8	By $C_2' = C_2 - C_1$ (eg) $C_3' = C_3 - C_1$ (eg) First two factors extracted (what's left has to be correct also) By $C_3' = C_3 - C_2$ (e.g.) 3rd factor extracted Further R/C ops or expansion of remaining det (almost a dM1) CAO up to equivalents due to re-positioning of the signs
Total			8	Alternatives using <i>Cyclic Symmetry</i> and the <i>Factor Theorem</i> are fine

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\bullet = \sqrt{6^2 + 2^2 + 9^2}$ attempted <i>and</i> $\pm \left(\frac{6}{\bullet}, \frac{2}{\bullet}, \frac{-9}{\bullet} \right)$ $\bullet = 11$ and all correct	M1 A1	2	$\pm (0.545, 0.182, -0.818)$ ok
(ii)	Either $\begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix} = -3 \begin{bmatrix} 6 \\ 2 \\ -9 \end{bmatrix}$ Explaining that d.v. of L is in dirn. of Π 's nml. $\Rightarrow L \perp^r \Pi$ Or $\begin{bmatrix} 6 \\ 2 \\ -9 \end{bmatrix} \bullet \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} = 0$ and $\begin{bmatrix} 6 \\ 2 \\ -9 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix} = 0$ Explaining that d.v. of L is \perp^r to 2 (non-//) vectors in $\Pi \Rightarrow L \perp^r \Pi$	M1 A1 B1 (M1) (A1)	3	Correct vector product only here Not just stating
(b)	E.g. $6 \times \textcircled{1} - \textcircled{2}: 46 = 34p + 27q$ $2 \times \textcircled{1} - \textcircled{3}: -57 = 21p + 6q$ $\textcircled{2} - 3 \times \textcircled{3}: -217 = 29p - 9q$ $2 \times \textcircled{4} + 9 \times \textcircled{5}: 605 = -121p$ $p = -5, q = 8, r = -1$	M1 A1 A1 M1 A1	5	Eliminating r from any pair of eqns. Any 2 correct eqns (1 mark each) Solving a 2×2 system (any means) in order to get values for p, q, r All 3 ✓ CAO
(c)	$7 + 6t = -2 + 5\lambda + \mu$ (i) $8 + 2t = 0 + 3\lambda + 6\mu$ $50 - 9t = -25 + 4\lambda + 2\mu$ $9 = -6t + 5\lambda + \mu$ $\rightarrow 8 = -2t + 3\lambda + 6\mu$ $75 = 9t + 4\lambda + 2\mu$ i.e. the above system with $p = -t, q = \lambda$ and $r = \mu$	M1 A1	2	Including re-arrangement attempt
	(ii) Subst ^e . $t = 5$ into L 's eqn. Or $\lambda = 8$ and $\mu = -1$ into Π 's eqn. $P = (37, 18, 5)$	M1 A1	2	CAO
	Total		14	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	Evals $\lambda = 27$ 1 Evecs $(\alpha) \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $(\beta) \begin{bmatrix} -1 \\ 3 \end{bmatrix}$	B1 B1 B1 B1	4	Both Correctly matched up with evals (look out for λ_1, v_1 notations) Ft $4y = x$ if evecs mis-matched
(ii)	$y = -3x$ from $\lambda = 1$	B1 ✓ B1	2	Must say why they have chosen this one
(b)	$U^{-1} = \frac{1}{13} \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}$ $M = U D U^{-1}$ $= \frac{1}{13} \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 27 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}$ $= \frac{1}{13} \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 81 & 27 \\ -1 & 4 \end{bmatrix}$ or $\frac{1}{13} \begin{bmatrix} 108 & -1 \\ 27 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}$ $= \begin{bmatrix} 25 & 8 \\ 6 & 3 \end{bmatrix}$	B1 B1 M1 A1 A1	5	Det; mtx Including attempt to multiply (at least U D ...) Ft incorrect/missing U^{-1} for one product; ignore missing $\frac{1}{13}$ until the end CAO
(c)	$M^n = U D^n U^{-1}$ $D^n = \begin{bmatrix} 27^n & 0 \\ 0 & 1 \end{bmatrix}$ $M^n(1,1) = \frac{1}{13}(12 \times 27^n + 1)$ So $4 \times 3 \times 3^{3n} + 1 = 4 \times 3^{3n+1} + 1$ div. by 13 Since M has all integer elements, each element of Mⁿ is an integer also	M1 B1 A1 E1 E1	5	Including attempt to multiply Legitimately so from their working, from fact that the element is an integer Explaining <i>why</i> it must be an integer
	Total		16	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
8	$\det \mathbf{W} = 12.36 + 9.16 = 576 = k^2$ $\Rightarrow k = 24$ $\frac{1}{24} \mathbf{W} = \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}$ $\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x + \frac{2}{3}y \\ \frac{3}{2}y - \frac{3}{8}x \end{bmatrix}$ <p>Equating this to $\begin{bmatrix} x \\ y \end{bmatrix} =$</p> $y = \frac{3}{4}x$ <p>ALT. 1</p> $\frac{1}{24} \mathbf{W} = \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}$ $\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} x \\ mx \end{bmatrix} = \begin{bmatrix} (\frac{1}{2} + \frac{2}{3}m)x \\ (\frac{3}{2}m - \frac{3}{8})x \end{bmatrix}$ <p>Setting $y' = mx'$ and solving for m</p> $y = \frac{3}{4}x$ <p>ALT. 2</p> $\lambda^2 - 2\lambda + 1 = 0$ $\Rightarrow \lambda = 1 \text{ (twice)}$ $\lambda = 1 \Rightarrow \begin{aligned} -\frac{1}{2}x + \frac{2}{3}y &= 0 \\ -\frac{3}{8}x + \frac{1}{2}y &= 0 \end{aligned}$ $y = \frac{3}{4}x$	<p>M1 A1</p> <p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(B1)</p> <p>(M1) (A1)</p> <p>(M1) (A1)</p> <p>(M1) (A1) (A1)</p>	<p>2</p> <p>5</p>	<p>Attempt at det. = k^2</p> <p>CAO</p> <p>Get $(4m - 3)^2 = 0$ CAO</p> <p>This may simply be stated or assumed</p>
Total			7	
TOTAL			75	