Version 1.0

General Certificate of Education
June 2010

Mathematics MS03

Statistics 3

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

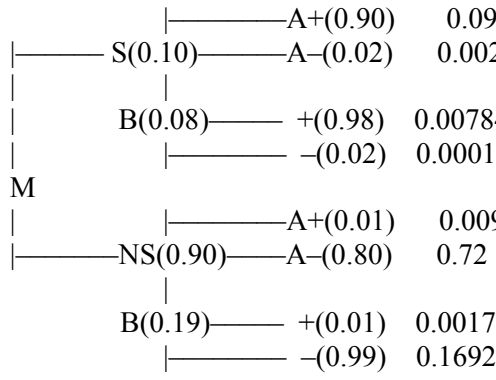
MS03

Q	Solution	Marks	Total	Comments
1	H_0: $\rho = 0$ H_1: $\rho \neq 0$	B1		Both
	SL $\alpha = 0.05$ (5%)			
	CV $r = (\pm)$ **0.404**	B1		AWRT (0.4044) H_1: $\rho > 0 \Rightarrow r = 0.3438$
	Calculated $r = 0.336 <$ Tabulated r	M1		Comparison
	No evidence, at 5% level, **of a correlation** between stem length and cup diameter of matsutake mushrooms	A1F	4	F on CV At 5% level, **accept hypothesis of no correlation**
	Total		**4**	
2(a)	99% $\Rightarrow$ z = **2.57 to 2.58**	B1		AWFW (2.5758)
	CI for $\mu_R - \mu_D$ is $(\bar{x}_R - \bar{x}_D) \pm z \times \sqrt{\frac{s_R^2}{n_R} + \frac{s_D^2}{n_D}}$	M1 A1		Form Allow $\left(\frac{ns^2}{n-1}\right)$ or $(n-1)$ Correct expression
	ie $(225-219) \pm 2.5758\sqrt{\frac{5^2}{50} + \frac{8^2}{75}}$	A1F		Or equivalent F on z only
	ie **6 ± 3 or (3, 9)** **Note:** Use of pooled $s^2 = 5961/123 = 48.46341 \Rightarrow$ $6 \pm 3.3 \Rightarrow$ max of B1 M1 A0 A1F A0 (3)	A1	5	CAO/AWRT or AWRT
(b)	**CI does not** include **0/zero**	B1F		F on (a)
	Evidence of a **difference** in mean weights	B1F dep	2	F on (a) Dependent on CI but not on 0/zero
(c)	Price, size, quality, taste, presentation, organic, marketing, stall position, etc	B1	1	Any sensible reason
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3	H_0: $\lambda_T = \lambda_S$ H_1: $\lambda_T > \lambda_S$	B1		Both
	SL $\alpha = 0.02$ (2%)			
	CV z = **2.05 to 2.06**	B1		AWFW (2.0537)
	or H_1 $\lambda_T \neq \lambda_s$ $\Rightarrow$ z = **2.32 to 2.33**	(B1)		AWFW (2.3263)
	$\bar{s} = \frac{940}{40}$ = **23.5** $\bar{t} = \frac{1560}{60}$ = **26**	B1		Both CAO; may be implied
	Pooled value, $\bar{p} = \frac{2500}{100}$ = **25**	B1		CAO
	$z = \frac{\lvert 23.5 - 26 \rvert}{\sqrt{25\left(\frac{1}{40} + \frac{1}{60}\right)}}$ **or** $z = \frac{\lvert 23.5 - 26 \rvert}{\sqrt{\left(\frac{23.5}{40} + \frac{26}{60}\right)}}$	M1		
	z = **2.44 to 2.45 or z = 2.47 to 2.48**	A1		Either AWFW (2.449 or 2.474)
	Evidence, at 2% level, **to agree** with Tina's claim	A1F	7	F on CV and z-value
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4(a)		B1 B1 B1 B1 (B2,1)	4	S and NS with Ps or %s 2 × (A+ and A–) with Ps or %s 2 × (B) with Ps or %s 2 × (B+ and B–) with Ps or %s Basic shape with labels, but without Ps or %s
	Note: The following BF and AF marks are dependent on an essentially correctly-shaped tree diagram			
(b)(i)				
(A)	P(S and –) = 0.002 + 0.00016 = **0.00216**	B1F		F on (a); otherwise CAO
(B)	P(NS and +) = 0.009 + 0.00171 = **0.01071**	B1F	2	F on (a); otherwise AWRT 0.0107
(ii)	E(N) = 10000 × [(A) + (B)]	M1		Or equivalent
	= 128.6 to 128.7 ⇒ **130**	A1F	2	CAO
(c)(i)	$P(S \mid +) = \frac{P(S \text{ and } +)}{P(+)} =$	M1		Used
	$\frac{0.09+0.00784}{0.09+0.00784+0.009+0.00171}=\frac{0.09784}{0.10855}$	A1F		F on (a) Otherwise correct
	= 0.901 to 0.902	A1		AWRT (0.90134)
(ii)	$P(NS \mid -) = \frac{P(NS \text{ and } -)}{P(-)} =$	(M1)		Used; only if not scored in (i)
	$\frac{0.72+0.16929}{0.002+0.00016+0.72+0.16929}=\frac{0.88929}{0.89145}$	A1F		F on (a) and/or denominator (c)(i) Otherwise correct
	= 0.997 to 0.998	A1	5	AWFW (0.99758)
	Special cases: Only numerators correct ⇒ (M1) B1 B1 Only denominators correct ⇒ (M1) B1 B1			
	Total		**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$E(T) = 2 \times 350 + 2 \times 210 = \mathbf{1120}$	B1		CAO
	$\text{Cov}(W, H) = \sqrt{5 \times 4} \times 0.75 =$	M1		Used; may be implied
	3.34 to 3.36	A1		AWFW (3.3541)
	$\text{Var}(T) = (2^2 \times 5) + (2^2 \times 4) + (2 \times 2 \times 2 \times 3.3541)$	M1		Used Ignore 3rd expression
	$= 20 + 16 + 26.8328 =$ **62.7 to 62.9**	A1	5	AWFW (62.8328)
(b)	$L = T_1 + T_2 + T_3 + T_4$			
	Mean of $L =$ **4480**	B1F		CAO; F on E(T)
	Variance of $L =$ **4** $\times$ Var(T)	M1		
	= **250.8 to 251.6**	A1F		(251.3312) Either AWFW; F on Var (T)
	SD of $L =$ **15.8 to 15.9**			(15.8534)
	$\text{P}(L < 4500) = \text{P}\left(Z < \frac{4500 - 4480}{\sqrt{251.3312}}\right)$	M1		Standardising 4500 using C's mean and SD
	$= \text{P}(Z < 1.25 \text{ to } 1.27)$			
	= 0.894 to 0.898	A1		AWFW (0.89645)
	Alternative Solution: Use of $\bar{T}$ rather than L			
	Mean of $\bar{T} =$ **1120**	(B1F)		CAO; F on E(T)
	Variance of $\bar{T}$ = Var(T) ÷ **4**	(M1)		
	= **15.6 to 15.8**	(A1F)		(15.7082) Either AWFW; F on Var(T)
	SD of $\bar{T} =$ **3.95 to 3.97**			(3.9634)
	$\text{P}(\bar{T} < 1125) = \text{P}\left(Z < \frac{1125 - 1120}{\sqrt{15.7082}}\right)$	(M1)		Standardising 1125 using C's mean and SD
	$= \text{P}(Z < 1.25 \text{ to } 1.27)$			
	= **0.894 to 0.898**	(A1)	5	AWFW (0.89645)
	Total		**10**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\hat{p} = \frac{28}{175} = \mathbf{0.16}$	B1		CAO; or equivalent
	95% $\Rightarrow$ z = **1.96**	B1		AWRT
	Approximate CI for p is $\hat{p} \pm z \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	M1		Used
	ie $0.16 \pm 1.96\sqrt{\frac{0.16 \times 0.84}{175}}$	A1F		Or equivalent F on $\hat{p}$ and z
	ie **0.16 ± 0.054 or (0.106, 0.214)**	A1	5	CAO/AWRT or AWRT (0.0543)
(ii)	**CI does** include **0.2** (20%)	B1F		F on (i)
	No evidence to support councils' claim	B1F dep	2	F on (i) Dependent on CI and on 0.2
(b)(i)	H_0: p = 0.40 (40%) H_1: p < 0.40	B1		Both
	Using B (50, 0.4) (40%)	M1		May be implied
	P($X \leq 16$) = **0.156**	A1		AWRT (0.1561)
	Calculated probability > 0.10 (10%)	M1		Comparison
	No evidence, at 10% level, **to support** council's claim **Special Case:** Normal approximation z = **–1.15**(47) B1 CV = **–1.28**(16) B1 Conclusion B1F Max of 4 marks	A1F	5	F on probability v 0.10 or 0.05 At 10% level, **accept** (at least) **40%** Allow B1 for hypotheses p = 0.123 to 0.125 v 0.10 B1 B1 F on z and CV
(ii)	Require P($X \leq x$) $\leq$ 0.10	M1		May be implied Ignore any reasoning if '15' stated
	$\Rightarrow$ CV = **15** (CR $\leq$ 15)	A1	2	CAO; or equivalent
(iii)	P(Type II error) = P(accept H_0 | H_0 false)	B1		Stated or used; or equivalent
	= P(X > CV or $X \geq$ CV)	M1		Attempt at a probability > or ≥ C's CV from (ii)
	= 1 – (**0.8369 or 0.7481**)	M1		Ignore '1 –'
	= **0.163**	A1	4	AWRT
	Total		**18**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7	$X \sim \text{Po}(\lambda)$			
(a)(i)	$\text{E}(X) = \sum_{x=0}^{\infty} x \times \frac{\text{e}^{-\lambda}\lambda^x}{x!} =$	M1		Used; ignore limits until A1
	$\lambda\text{e}^{-\lambda} \times \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} =$	M1		Factor of at least λ Division of $x!$ by x
	$\lambda\text{e}^{-\lambda} \times \text{e}^{\lambda} = \boldsymbol{\lambda}$	A1	3	AG; fully correct solution
(ii)	$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$ $= \text{E}[X(X-1)] + \text{E}(X) - [\text{E}(X)]^2$	M1		Used (Other derivations are possible)
	$= \lambda^2 + \lambda - \lambda^2 = \boldsymbol{\lambda}$	A1	2	CAO
(b)(i)	$\text{P}(X = m) \geq \text{P}(X = m-1)$ and $\text{P}(X = m) \geq \text{P}(X = m+1) \Rightarrow$			
	$\frac{\text{e}^{-\lambda}\lambda^m}{m!} \geq \frac{\text{e}^{-\lambda}\lambda^{m-1}}{(m-1)!}$ and $\frac{\text{e}^{-\lambda}\lambda^m}{m!} \geq \frac{\text{e}^{-\lambda}\lambda^{m+1}}{(m+1)!}$	M1 M1		Use of $\text{Po}(\lambda)$ for $x = m$ Either inequality (accept = sign)
	$\boldsymbol{m \leq \lambda}$ **and** $\boldsymbol{m \geq \lambda - 1}$	A1	3	AG; fully correct solution
(ii)	Given $\lambda = 4.9 \Rightarrow m =$ **4**	B1		CAO
	$\text{P}(X = 4) = \frac{\text{e}^{-4.9}4.9^4}{4!} =$ **0.178 to 0.179**	B1	2	AWFW (0.178867)
(c)	Given $\text{SD}(Y) = 15.5 \Rightarrow$ $\lambda = \text{Var}(\text{Y}) =$ **15.5^2 = 240.25**	B1		Either CAO
	Mode, $d =$ **240**	B1F		F on λ providing an integer value
	$\text{P}(Y_\text{P} \geq d) = \text{P}(Y_\text{N} > d - 0.5) =$	B1		Accept use of 'd'
	$\text{P}\left(Z > \frac{239.5 - 240.25}{15.5}\right) =$	M1		Standardising $(d - 0.5, d$ or $d + 0.5)$ with 15.5^2 and 15.5; do **not** accept use of 'd'
	$\text{P}(Z > -0.05) =$ **0.515 to 0.52**	A1	5	AWFW (0.5193)
	Total		**15**	
	TOTAL		**75**	