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General Certificate of Education (A-level)
January 2011

Mathematics MD02

(Specification 6360)

Decision 2

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

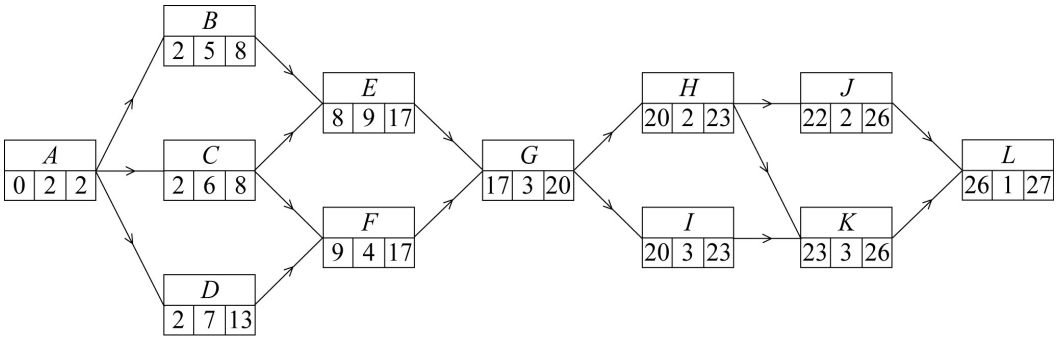
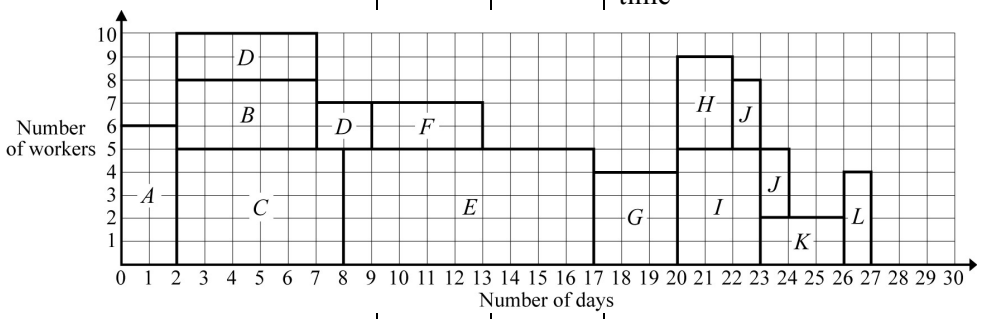
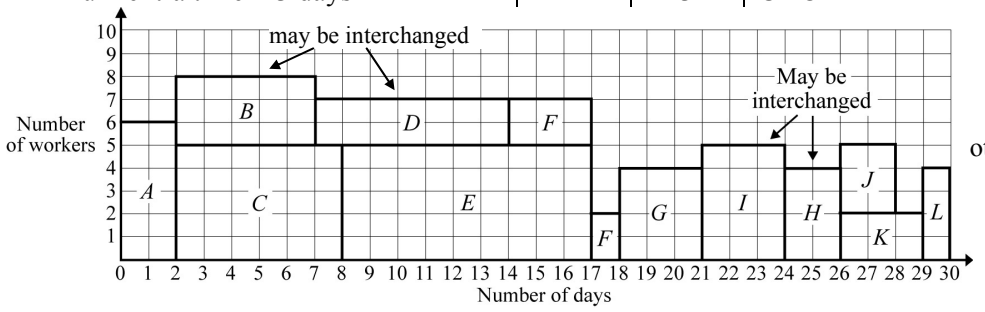
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

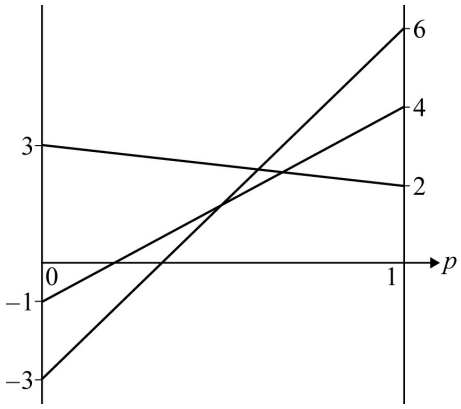
MD02

Q	Solution	Marks	Total	Comments
1				
(a)	Forward pass	M1		up to one slip ft
	Correct	A1		
	Backward pass	M1		up to one slip ft
	Correct	A1	4	
(b)(i)	Critical path *A C E G I K L*	B1		
(ii)	Float for *D* = 13 – 2 – 7	M1		'their 13' – 'their 2' – 7
	= 4 days	A1	3	
(c)	*A C E G I K L* correct durations and heights	M1 A1		one slip in duration or height correct
	D and *B* and *F* correct (no "holes")	B1		
	H and *J* correct (no "holes")	B1	4	withhold final mark earned if not clear which activities are taking place at any time
				
(d)	Correctly dealing with *D*, *B* and *F*	B1		ft 1 slip
	Correctly dealing with *H* and *J*	B1		ft 1 slip
	Minimum extra time = 3 days	B1	3	CAO
	 other possibilities			
	Total		**14**	

MD02 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	4 8 12 2 6 0 5 12 4 8 11 10 8 3 8 2 9 3 5 1 *n* *n* *n* *n* *n*	B1	1	
(ii)	No of rows = no of columns Hungarian algorithm minimises $20 - x$ gives measure of criteria not met which needs minimising	E1 E1 E1	3	square matrix by adding extra row (total score) points lost (in each entry)
(b)(i)	~~2~~ 6 10 ~~0~~ ~~4~~ ~~0~~ 5 12 ~~4~~ ~~8~~ ~~8~~ 7 5 ~~0~~ ~~5~~ ~~1~~ 8 2 ~~4~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~	M1 A1√	 2	reducing rows column reduction leaves matrix unchanged (*p* = 4, *q* = 5) (ft one slip)
(ii)	Zeros covered with 4 lines shown	B1		row 5 and columns 1, 4 and 5
	~~2~~ 4 8 ~~0~~ 4 ~~0~~ 3 10 ~~4~~ 8 ~~8~~ 5 3 ~~0~~ 5 ~~1~~ ~~6~~ ~~0~~ ~~4~~ ~~0~~ ~~2~~ ~~0~~ ~~0~~ 2 2	M1 A1		subtract 2 from all uncovered and add 2 to double covered (condone one slip) (follow through their *p* and *q*)
	2 1 5 (0) 1 (0) 0 7 4 5 8 2 (0) 0 2 4 6 0 7 (0) 5 (0) 0 5 2	M1 A1		augment (at least) one more time (condone one slip) may put line through second row and not first column M1 0 2 6 0 2 / 0 3 10 6 8 / 6 3 1 0 3 / 1 6 0 6 0 / 2 0 0 4 2 → 0 1 5 0 1 / 0 2 9 6 7 / 6 2 0 0 2 / 2 6 0 7 0 / 3 0 0 5 2 A1 any correct final matrix requiring zeros to be covered by 5 lines
	1D, 2A, 3C, 4E is matching	B1	6	(field B unused)
(iii)	(18 + 20 + 12 + 19 =) 69	B1	1	
	Total		**13**	

MD02 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	Row minima 2, –3, x	B1	1	} Check for answers written on table
(ii)	Column maxima 3, 6, 4	B1		
	Max (row min) = 2 Min (col max) = 3 Or $2 \neq 3$	M1		Condone Best (worst) =2 etc Worst (best) =3
	Since $2 \neq 3 \rightarrow$ no stable solution	A1cso	3	Both lines and statement must score previous B1, B1
(b)	$x < 2, x + 3 < 6, 3 < 4$ $\rightarrow$ R_1 dominates R_3 } Either of these	B1	1	hence Rhona should not play R_3
(c)(i)	Let Rhona play R_1 with prob p and R_2 with prob $1 - p$			
	When C plays C_1: exp value $= 2p + 3(1-p)$			$= 3 - p$
	C_2: $6p - 3(1-p)$			$= -3 + 9p$
	C_3: $4p - (1-p) = -1 + 5p$	M1		any two correct unsimplified
		A1		all correct unsimplified
		M1		drawing two of their expected values for $0 \leq p \leq 1$ both vertical axes using same scale condone use of horizontal lines in paper
		A1		all three correct lines must see numbers on at least one vertical axis
	$3 - p = -1 + 5p$	M1		choosing highest point of region
	$\rightarrow p = \frac{2}{3}$	A1		
	$\rightarrow$ Rhona plays R_1 $\frac{2}{3}$ of time and R_2 $\frac{1}{3}$ of time	E1√	7	ft their p
(ii)	Value of game $= 3 - \frac{2}{3} = \frac{7}{3}$	B1	1	or $-1 + \frac{10}{3} = \frac{7}{3}$
	Total		**13**	

MD02 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{4}{-1}=-4;\ \frac{10}{2}=5;\ \frac{21}{4}=5\frac{1}{4}$ 5 is smallest **positive** ratio	E1		Must see 5 and $5\frac{1}{4}$ plus correct statement
	Pivot = 2	B1	2	
(ii)	$1\ \ 0\ \ -\frac{1}{2}\ \ 5\ \ 0\ \ \frac{3}{2}\ \ 0\ \ 15$	M1		row operations (even with wrong pivot)
	$0\ \ 0\ \ \frac{3}{2}\ \ 3\ \ 1\ \ \frac{1}{2}\ \ 0\ \ 9$	A1		1st, 2nd or last row correct
	$0\ \ 1\ \ \frac{1}{2}\ \ 2\ \ 0\ \ \frac{1}{2}\ \ 0\ \ 5$	A1		another of these correct
	$0\ \ 0\ \ 0\ \ -5\ \ 0\ \ -2\ \ 1\ \ 1$	A1		all correct (condone multiples of rows)
	Negative value in top row (→ optimum not reached)	E1	5	must have negative value in their top row
(b)(i)	New pivot is 'their $\frac{3}{2}$' in y-column PI	M1		or multiple of this
	$1\ \ 0\ \ 0\ \ 6\ \ \frac{1}{3}\ \ \frac{5}{3}\ \ 0\ \ 18$	A1		1st, 3rd or 4th row correct
	$0\ \ 0\ \ 1\ \ 2\ \ \frac{2}{3}\ \ \frac{1}{3}\ \ 0\ \ 6$	A1		another of these rows correct
	$0\ \ 1\ \ 0\ \ 1\ \ -\frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 2$			
	$0\ \ 0\ \ 0\ \ -5\ \ 0\ \ -2\ \ 1\ \ 1$	A1	4	all correct (condone multiples of rows)
(ii)	Optimum value of P reached	E1		must have no negative values in top row
	$P = 18$	B1√		ft their tableau
	$x=2, y=6, z=0$	B1√		$s=0, t=0, u=1$ (no more than 2 slips in final tableau for ft)
	$4x+2y+3z\le 21$ still has slack	B1	4	Tableau must indicate u is only slack variable
	Total		**15**	

MD02 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Completing stage 2 values (condone correct unsimplified) (all 7 values)	B1		
	At least 6 values calculated at stage 3 (M0 for 10 or more values)	M1		
	Using only their minimum F or G value from stage 2	m1		
	All 9 stage 3 values correct	A1		
	Using minima (at least 3) from A, B, C, D stage 3 in stage 4	M1		
	All correct in stage 4	A1	6	

Stage	State	From	Value	
1	I	T	-7	
	J	T	-6	
	K	T	-5	
2	E	I	$-7 - 4 = -11$	←
	F	I	$-7 - 3 = -10$	←
		J	$-6 - 2 = -8$	
	G	I	$-7 + 4 = -3$	
		J	$-6 + 7 = 1$	
		K	$-5 - 1 = -6$	←
	H	K	$-5 + 4 = -1$	←
3	A	E	$-11 + 5 = -6$	
		G	$-6 - 2 = -8$	←
	B	E	$-11 - 2 = -13$	
		F	$-10 - 4 = -14$	←
	C	F	$-10 + 6 = -4$	
		G	$-6 - 3 = -9$	←
		H	$-1 - 5 = -6$	
	D	G	$-6 - 5 = -11$	←
		H	$-1 - 3 = -4$	
4	S	A	$-8 + 23 = 15$	
		B	$-14 + 28 = 14$	←
		C	$-9 + 25 = 16$	
		D	$-11 + 25 = 14$	←

(b)	Minimum cost of ticket (£)14	B1√		ft their lowest stage 4 value
	Path $S\ B\ F\ I\ T$	B1		one correct path
	$S\ D\ G\ K\ T$	B1	3	2nd correct path and no others
	Total		**9**	

MD02 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$SP \geq 12$ $SQ \geq 10$ $SR \geq 17$ $YT \geq 18$ $ZT \geq 17$	B1 B1	 2	S in correct place, (arrows) and capacities T in correct place, (arrows) and capacities
(b)	$S\ P\ U\ Y\ T$ 10 $S\ R\ V\ W\ Z\ T$ 8	B1 B1	 2	
(c)(i)	Initial flow forward and backward PU 2 and 10 ; UY 0 and 10 RV 0 and 8 ; VW 1 and 8 ; WZ 2 and 8	 B1 B1	 2	withhold one B1 if paths to S and T not updated
(ii)	Two correct routes and flows on Figure 6 Correct additional flows Max flow = 33 Adjustment of at least 4 edges corresponding to flows (forward and backward) Correct final flows forward and backward (must score A1 for table)	M1 A1 M1 A1cso	 4	$S\ P\ U\ Y\ T$ 10 $S\ R\ Y\ W\ Z\ T$ 8 $S\ P\ U\ X\ Y\ T$ 2 $S\ Q\ V\ U\ X\ Y\ T$ 6 $S\ R\ W\ X\ Z\ T$ 5 $S\ R\ W\ Z\ T$ 2 (other possibilities) edges UY, UX, WX and WZ will be saturated $XY + XZ = 13$ in back flow
(d)	Cut with value 33 is through UY, UX, WX and WZ	B1	1	
	Total		**11**	
	TOTAL		**75**	