Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics MS2B

(Specification 6360)

Statistics 2B

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1(a)(i)	$X \sim \text{Po}(13)$	B1	1	Both Poisson and $\lambda = 13$
(ii)	$\text{P}(X = 20) = \text{P}(X \le 20) - \text{P}(X \le 19)$ $= 0.975(0) - 0.957(3)$ [allow $0.975 - 0.957$]	M1		Must use $\lambda = 13$ otherwise M0A0
	$= 0.0177$ (3sf)	A1	2	AWFW 0.0176 to 0.018 **or** $\text{P}(X = 20) = \dfrac{e^{-13} \times 13^{20}}{20!}$ M1 $= 0.0177$ A1
(iii)	$\text{P}(6 \le X \le 18) = \text{P}(X \le 18) - \text{P}(X \le 5)$ $= 0.930(2)$	M1		
	$-(0.0107$ **or** $0.0259)$	M1		
	$= 0.920$ (3sf)	A1	3	AWFW 0.919 to 0.92
(b)	Cars not random Cars not independent Mean and Variance of cars different / not equal	B1		Allow (number of) cars not random / not independent B1 for any one of these 3 statements Must indicate a reference to ***cars***
	Mean / Average / λ / 2.6 greater / less / smaller / different / variable / not constant / too small / too large Any contextual reason that suggests a change in traffic flow, eg due to: rush hour / congestion / traffic jams / accidents / work traffic / school traffic / peak time	B1	2	Correct comment about value of $\lambda \ne 2.6$ Any combination (one from each group): eg mean greater *due to* rush hour, **or** λ smaller *due to* congestion, **or** 2.6 too small *due to* school traffic
(c)	$Y \sim \text{Bin}(20, 0.2)$ $\text{P}(Y \ge 5) = 1 - \text{P}(Y \le 4)$ $= 1 - 0.6296$	M1		**or:** $1 - \begin{pmatrix} 0.01153 + 0.05765 + 0.13691 \\ +0.20536 + 0.21820 \end{pmatrix}$ $1 - 0.6296$ (Allow $1 - 0.8042$ seen for M1)
	$= 0.37(0)$ (3sf)	A1	2	AWFW 0.37 to 0.3704
(d)	X and Y independent	B1		Any statement which indicates two / both events are independent
	$p = 0.0177 \times 0.3704$	M1		[their (c)] $\times$[their (a)(ii)]
	$= 0.00656$ (3sf)	A1	3	AWFW 0.0065 and 0.0067
			13	

MS2B (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	Area / $\mathrm{F}(x) = 10u \times 0.01\pi$ (OE)	B1		Shown by any correct method
	$= 1 \Rightarrow u = \dfrac{10}{\pi}$	Bdep1	2	**Alternatives:** $f = \dfrac{1}{10u}$ B1
	or $u = \dfrac{10}{\pi} \Rightarrow \mathrm{F}(x) = 1$	(Bdep1)		Show $u = \dfrac{10}{\pi}$ **or** show $\dfrac{1}{10u} = 0.01\pi$ Bdep1
(ii)	$\mathrm{E}(X) = \dfrac{1}{2}(11u + u) = 6u = 6 \times \dfrac{10}{\pi} = \dfrac{60}{\pi}$	B1	1	Must be in terms of π (eg $60\pi^{-1}$)
	$\mathrm{Var}(X) = \dfrac{1}{12}(b-a)^2$ $\mathrm{Var}(X) = \dfrac{1}{12}(11u-u)^2$ $= \dfrac{1}{12} \times 100 \times \dfrac{100}{\pi^2} = \dfrac{100^2}{12\pi^2}$	B1	1	**Alternatives:** $\dfrac{10000}{12\pi^2} = \dfrac{5000}{6\pi^2} = \dfrac{2500}{3\pi^2} = \left(\dfrac{50}{\pi\sqrt{3}}\right)^2 = \dfrac{\text{(AWRT 833)}}{\pi^2}$ Must be in terms of π
(iii)	$C = \pi\left(X + \dfrac{10}{\pi}\right)$			
	$\mathrm{E}(C) = \left.\begin{array}{l}\pi \times [\text{their } \mathrm{E}(X)] + 10 \\ \pi \times \dfrac{60}{\pi} + 10\end{array}\right\}$	M1		Their **numerical** value of $\mathrm{E}(X)$ used correctly Must have a multiplier of π or 2π
	$= 70$	A1		CAO
	$\mathrm{Var}(C) = \pi^2 \times \dfrac{100^2}{12\pi^2} = \dfrac{100^2}{12}$	M1		$\pi^2 \times [\text{their } \mathrm{Var}(X) > 0]$ Must have a multiplier of π^2 or $4\pi^2$
	$= 833\frac{1}{3}$ ($833.\dot{3}$)	A1	4	**Alternatives:** $\dfrac{10000}{12} = \dfrac{5000}{6} = \dfrac{2500}{3}$ Must be exact: 833.3 gets A0
(b)	$n = 100$ and $\bar{a} = 40.5$			
	95% CI for $\mu = \left.\begin{array}{l}40.5 \pm z \times \dfrac{\sqrt{25}}{\sqrt{100}} \\ 40.5 \pm 1.0\end{array}\right\}$	B1 M1		For $z = 1.96$ $z = 1.96$ or 1.64 to 1.65 only
	$= (39.5, 41.5)$	A1	3	AWRT
	Total		**11**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3(a)	H_0: no association (between type of school and performance of 16 year olds in their GCSEs)	B1	1	H_0: type of school and performance of 16 year olds in their GCSEs independent
(b)	$\frac{(O-E)^2}{E}$ 0.195819311 0.482160711 0.003569447 1.080536181 0.062507172 1.269422099 0.785491128 0.183802623 0.541856652 0.044011976 3.274102564 4.096492891	M1		Attempt at $\frac{(O-E)^2}{E}$ (≥ 4 correct values seen to 2dp)
	$X^2 = \sum \frac{(O-E)^2}{E}$	m1		Attempt to add ≥ 8 terms
	$= 12.01977275$ $= 12.0$ (1dp)			Allow $11.9 \leq X^2 \leq 12.1 \Rightarrow$ M1m1
		A1	3	CAO
(c)	$\nu = 6 \Rightarrow \chi^2_{1\%} = 16.8(12)$	B1,B1		$\nu = 6$ can be implied by $\chi^2_{1\%} = 16.8(12)$
	No (significant evidence to suggest an) **association between** (type of) **school and** (GCSE) **performance** (of 16 year olds)	Adep1	3	Insufficient/no evidence to support Emily's belief. School and performance are independent. Correct conclusion **in context** Dep on B1M1m1B1B1 given in (a), (b), (c) **and** $11.9 \leq X^2 \leq 12.1$
(d)	More than expected gained at least / more than 5 GCSEs Fewer than expected gained at least / more than 1 GCSE but less than 5 GCSEs Fewer than expected gained no GCSEs			Since conclusion of ***no association*** between school and GCSE performance, it may be misleading to look at individual differences in any great detail
		B1	1	Any one of these 4 comments seen
(e)	$\chi^2_{10\%} = 10.6(45)$	B1		Correct value of χ^2 only
	Reject H_0 at 10% level of significance. (Evidence to suggest) an **association between** (type of) **school and** (GCSE) **performance**	Bdep1	2	Evidence to support Emily's (Joanne's) belief. (Type of) school + (GCSE) performance dependent. Dep on B1M1m1 **and** $11.9 \leq X^2 \leq 12.1$ **and** B1 in (e)
	Total		**10**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathrm{E}(X) = \sum xp$ $= \frac{3}{40} + \left(2\times\frac{6}{40}\right) + \left(3\times\frac{9}{40}\right) + \left(4\times\frac{12}{40}\right) + \left(5\times\frac{5}{20}\right) = 3.5$	B2,1	2	
(b)(i)	$\mathrm{E}\left(\frac{1}{X}\right) = \sum \frac{1}{x}\times p$			
	$= \left(1\times\frac{3}{40}\right) + \left(\frac{1}{2}\times\frac{6}{40}\right) + \left(\frac{1}{3}\times\frac{9}{40}\right) + \left(\frac{1}{4}\times\frac{12}{40}\right) + \left(\frac{1}{5}\times\frac{5}{20}\right)$	M1		At least 4 of these terms added (accept decimal equivalents)
	$= \frac{7}{20}$	A1	2	AG (allow 0.35 seen)
(ii)	$\mathrm{E}\left(\frac{1}{X^2}\right) = \sum \frac{1}{x^2}\times p$			
	$= \left(1\times\frac{3}{40}\right) + \left(\frac{1}{4}\times\frac{6}{40}\right) + \left(\frac{1}{9}\times\frac{9}{40}\right) + \left(\frac{1}{16}\times\frac{12}{40}\right) + \left(\frac{1}{25}\times\frac{5}{20}\right)$	M1		At least 4 of these terms added (accept decimal equivalents) (can be implied by $\frac{133}{800}$ seen with no other working shown)
	$= \frac{133}{800}$ (0.16625)	A1		
	$\mathrm{Var}\left(\frac{1}{X}\right) = \frac{133}{800} - \frac{49}{400}$	m1		$\left[\text{their } \mathrm{E}\left(\frac{1}{X^2}\right)\right] - \left(\frac{7}{20}\right)^2$
	$= \frac{7}{160}$	Adep1	4	AG (allow 0.04375 seen)
(c)(i)	$\begin{array}{\|c\|c\|c\|c\|c\|c\|} \hline x & 1 & 2 & 3 & 4 & 5 \\ \hline y & 40 & 20 & 13\frac{1}{3} & 10 & 8 \\ \hline p & \frac{3}{40} & \frac{6}{40} & \frac{9}{40} & \frac{12}{40} & \frac{10}{40} \\ \hline \end{array}$			
	Identifying $X = (2), 3, 4, 5$ **or** $Y = (20), 13\frac{1}{3}, 10, 8$	M1		**Alternative:** $Y < 20 \Rightarrow \frac{40}{X} < 20 \Rightarrow 40 < 20X \Rightarrow X > 2$ M1 (allow $<$ or $\le$ **and** $>$ or $\ge$ in above)
	$\mathrm{P}(X > 2) = \frac{9}{40} + \frac{12}{40} + \frac{5}{20}$ $= \mathrm{P}(Y < 20)$	A1		$\mathrm{P}(Y < 20) = \mathrm{P}(X > 2)$ $= 1 - \left(\frac{3}{40} + \frac{6}{40}\right)$ A1
	$= \frac{31}{40}$ (0.775)	A1	3	$= \frac{31}{40}$ (0.775) A1
(ii)	$\frac{9}{40}$ seen irrespective of labelling	B1		As numerator **or** final answer (0.225)
	$\mathrm{P}(X < 4 \mid Y < 20) = \frac{\frac{9}{40}}{\frac{31}{40}} = \frac{0.225}{0.775}$	M1		$= \frac{\frac{9}{40}}{(\text{their (c)(i)})}$ (or correct use of table)
	$= \frac{9}{31}$ (0.290)	A1	3	AWFW 0.29 to 0.2904
	Total		**14**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)	$Y \sim \mathrm{N}\left(\mu_y, 640^2\right)$ $n = 25$ and $\bar{y} = 19700$ H_0: $\mu_y = 20000$ H_1: $\mu_y \neq 20000$ (both)	B1		**Alternative:** $\mathrm{P}\left(\bar{Y} < 19700\right) = \mathrm{P}\left(Z < -2.34375\right)$ = 1 – 0.99036 = 0.00964 ≥ 0.005 Accept H_0
	$\bar{Y} \sim \mathrm{N}\left(20000, \dfrac{640^2}{25}\right)$ $z = \dfrac{19700 - 20000}{640/\sqrt{25}}$	M1		$(-2.35$ to $-2.34)$
	$= -2.34375$	A1		$(\pm 2.57$ to $\pm 2.58)$
	$z_{\text{crit}} = \pm 2.5758$	B1		Use of t $\Rightarrow$ max B1M1A1
	Accept H_0	Adep1		dep on B1M1B1
	Insufficient / **no evidence** (to suggest) that the **mean** (lifetime) has **changed** (from 20000 hours) **Mean** (lifetime) has **not** changed at 1% level (of significance)	Edep1	6	dep on Adep1 If incorrect hypotheses then B0 $\Rightarrow$ max M1A1B1 ie final Adep1Edep1 not available
(b)(i)	$\mu < 10000$	B1	1	
(ii)	$n = 16$ and $s = 500$; $t_{\text{crit}} = 1.753$	B1		For t_{crit} (ignore signs)
	$\text{sd}(\bar{X}) = \dfrac{500}{\sqrt{16}}$ (125)	B1		Ignore notation
	Critical value is one of: $10000 \pm 1.753 \times \dfrac{500}{\sqrt{16}}$ (considered)	M1		M0 if only considered upper value No ft on incorrect t value
	Choose 9780 (3sf)	A1		AWFW 9780 to 9781 (ignore inequality)
	($\Rightarrow$ critical region: $\bar{x} < 9780$)			If z used then max B0B1M0A0A0
	$\therefore$ Range of values for $\bar{x}$ which leads Christine **not** to reject H_0: $\mu = 10000$ is: $\bar{x} > 9780$	A1	5	Allow $\bar{x} \geq 9780$ to 9781
(iii)	No error	B1	1	Ignore any subsequent statements
	Total		**13**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\mathrm{F}(x)=\int \frac{3}{8}\left(x^2+1\right)\,\mathrm{d}x$	M1		Ignore limits
	$=\frac{3}{8}\left[\frac{x^3}{3}+x\right]$ or $=\frac{1}{8}x^3+\frac{3}{8}x$	A1		Either
	$=\frac{1}{8}x\left(x^2+3\right)$	A1	3	(including use of correct limits 0 and x **or** $+c$ used and evaluated) (AG)
(b)	$\mathrm{F}(m)=\frac{1}{2}$	B1		
	$\mathrm{F}(1)=\frac{1}{8}\times1\times4=\frac{1}{2}$	B1	2	AG
(c)	Upper quartile lies in range $1<x<2$ such that $\mathrm{F}(q)=\frac{3}{4}$			$\frac{1}{2}+\int_1^q \frac{1}{4}(5-2x)\,\mathrm{d}x=\frac{3}{4}$
	$\int_1^q \frac{1}{4}(5-2x)\,\mathrm{d}x=\frac{1}{4}$ $\left[5x-x^2\right]_1^q=1$ $5q-q^2-4=1$	M1		**Alternative:** $\int_q^2 \frac{1}{4}(5-2x)\,\mathrm{d}x=\frac{1}{4}$ $\left[5x-x^2\right]_q^2=1$ $(10-4)-\left(5q-q^2\right)=1$ $6-5q+q^2=1$
	$q^2-5q+5=0$	A1		$q^2-5q+5=0$
	$q=\frac{5\pm\sqrt{25-20}}{2}$ or $\frac{1}{2}\left(5\pm\sqrt{5}\right)$	M1		Correct use of formula (OE) to give the two surd answers to given quadratic equation
	but $1<q<2$ [or $(q<2)$]	m1		
	$\therefore\ q=\frac{1}{2}\left(5-\sqrt{5}\right)$	A1	5	Must qualify with a numerical comparison, not just quote the given answer; dep on M1; AG
(d)	$\mathrm{P}(X>1.5)=\frac{1}{2}\left[\frac{1}{2}+\frac{1}{4}\right]\times\frac{1}{2}$	M1		$\mathrm{P}(X<1.5)=0.5+\frac{1}{2}\left[\frac{3}{4}+\frac{1}{2}\right]\times\frac{1}{2}$ (M1)
	$=\frac{3}{16}$ (0.1875)	A1		$=\frac{1}{2}+\frac{1}{2}\times\frac{5}{4}\times\frac{1}{2}$ $=\frac{1}{2}+\frac{5}{16}=\frac{13}{16}$ (A1)
	$\mathrm{P}(X>q)=\frac{1}{4}$ (0.25)	B1		$\mathrm{P}(X<q)=\frac{3}{4}$ (0.75) (B1)
	$\mathrm{P}(q<X<1.5)=\frac{1}{4}-\frac{3}{16}$			$\mathrm{P}(q<X<1.5)=\frac{13}{16}-\frac{3}{4}=\frac{1}{16}$ (A1) (0.0625)
	$=\frac{1}{16}$ (0.0625)	A1	4	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(d) cont	**OR** $\int_{1\frac{1}{2}}^{2} \frac{1}{4}(5-2x)\,\mathrm{d}x = \frac{3}{16}$ etc (M1A1) **NB** statement $\mathrm{F}(1.5)-\frac{3}{4}=\frac{1}{16}$ (OE) scores 4 marks **Alternative:** $\int_{q}^{1.5} \frac{1}{4}(5-2x)\,\mathrm{d}x = \left[-\frac{1}{16}(5-2x)^2\right]_{\frac{5-\sqrt{5}}{2}}^{1.5}$ (M1) $= -\frac{1}{16}(4) - \left[-\frac{1}{16}\left(\sqrt{5}\right)^2\right]$ (sub) (A1) $= -\frac{4}{16}+\frac{5}{16}$ (A1) $= \frac{1}{16}$ (A1)			**OR** $\int_{q}^{1.5} \frac{1}{4}(5-2x)\,\mathrm{d}x = \frac{1}{4}\left[5x-x^2\right]_q^{1.5}$ (M1) (correct integration and limits) Allow use of q = 1.38 to q = 1.382 in limits for M1 Whatever follows **must** be **exact** $= \frac{1}{4}\left[(7.5-2.25)-\left(5q-q^2\right)\right]$ (A1) for use of $5q-q^2=5$ **or** showing $5q-q^2=5$ by substituting $q=\frac{1}{2}\left(5-\sqrt{5}\right)$ (A1) $= \frac{1}{4}[5.25-5] = \frac{1}{16}$ (A1) **Alternative using F(x):** for $1 \le x \le 2$ $\mathrm{F}(x) = \frac{1}{2} + \int_1^x \frac{1}{4}(5-2x)\,\mathrm{d}x$ $= \frac{1}{2}+\frac{1}{4}\left[5x-x^2\right]_1^x$ $= \frac{1}{2}+\frac{1}{4}\left[\left(5x-x^2\right)-(5-1)\right]$ $= \frac{1}{4}\left(2+5x-x^2-4\right)$ $= \frac{1}{4}\left(5x-x^2-2\right)$ (seen or used) (M1) $\mathrm{F}(1.5) = \frac{1}{4}(7.5-2.25-2) = \frac{3.25}{4}$ $= 0.8125 = \frac{13}{16}$ (A1) $\mathrm{F}(q) = \frac{1}{16}\left(50-10\sqrt{5}-\left(25-10\sqrt{5}+5\right)-8\right)$ $= \frac{12}{16}$ OE (B1) $\mathrm{P}(q < X < 1.5) = \frac{13}{16}-\frac{12}{16}=\frac{1}{16}$ (A1)
	Total		**14**	
	TOTAL		**75**	