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| For Examiner's Use | |
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| Examiner's Initials | |
| Question | Mark |
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General Certificate of Education
Advanced Subsidiary Examination
January 2012

# Mathematics MFP1

## Unit Further Pure 1

**Tuesday 17 January 2012 9.00 am to 10.30 am**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

**1** The quadratic equation

$$2x^2 + 7x + 8 = 0$$

has roots $\alpha$ and $\beta$.

**(a)** Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

**(b)** Show that $\alpha^2 + \beta^2 = \frac{17}{4}$. *(2 marks)*

**(c)** Find a quadratic equation, with integer coefficients, which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$$ *(5 marks)*

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**2** Show that only one of the following improper integrals has a finite value, and find that value:

**(a)** $\int_{8}^{\infty} x^{-\frac{2}{3}}\,\mathrm{d}x$;

**(b)** $\int_{8}^{\infty} x^{-\frac{4}{3}}\,\mathrm{d}x$. *(5 marks)*

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**3 (a)** Solve the following equations, giving each root in the form $a + b\mathrm{i}$:

**(i)** $x^2 + 9 = 0$; *(1 mark)*

**(ii)** $(x + 2)^2 + 9 = 0$. *(1 mark)*

**(b) (i)** Expand $(1 + x)^3$. *(1 mark)*

**(ii)** Express $(1 + 2\mathrm{i})^3$ in the form $a + b\mathrm{i}$. *(3 marks)*

**(iii)** Given that $z = 1 + 2\mathrm{i}$, find the value of

$$z^* - z^3$$ *(2 marks)*

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**4 (a)** Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r^2(4r-3) = kn(n+1)(2n^2-1)$$

where $k$ is a constant. *(5 marks)*

**(b)** Hence evaluate

$$\sum_{r=20}^{40} r^2(4r-3)$$ *(2 marks)*

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**5** The diagram below (not to scale) shows a part of a curve $y = \mathrm{f}(x)$ which passes through the points $A(2, -10)$ and $B(5, 22)$.

**(a) (i)** On the diagram, draw a line which illustrates the method of linear interpolation for solving the equation $\mathrm{f}(x) = 0$. The point of intersection of this line with the $x$-axis should be labelled $P$. *(1 mark)*

**(ii)** Calculate the $x$-coordinate of $P$. Give your answer to one decimal place. *(3 marks)*

**(b) (i)** On the same diagram, draw a line which illustrates the Newton–Raphson method for solving the equation $\mathrm{f}(x) = 0$, with initial value $x_1 = 2$. The point of intersection of this line with the $x$-axis should be labelled $Q$. *(1 mark)*

**(ii)** Given that the gradient of the curve at $A$ is $8$, calculate the $x$-coordinate of $Q$. Give your answer as an exact decimal. *(2 marks)*

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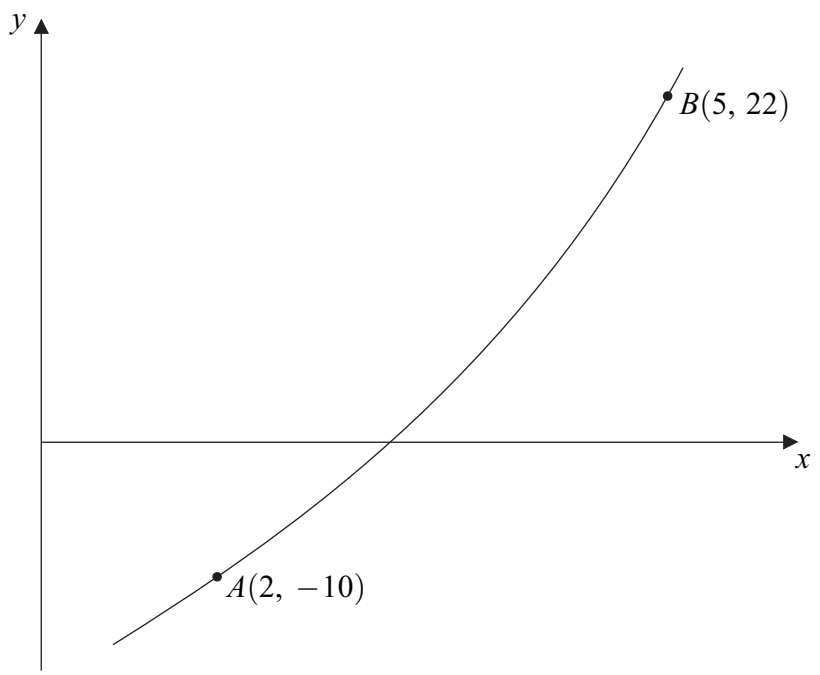


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**6** Find the general solution of each of the following equations:

**(a)** $\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$; *(4 marks)*

**(b)** $\tan^2\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{3}$. *(3 marks)*

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**7** A hyperbola $H$ has equation

$$\frac{x^2}{9} - y^2 = 1$$

**(a)** Find the equations of the asymptotes of $H$. *(1 mark)*

**(b)** The asymptotes of $H$ are shown in the diagram opposite. On the same diagram, sketch the hyperbola $H$. Indicate on your sketch the coordinates of the points of intersection of $H$ with the coordinate axes. *(3 marks)*

**(c)** The hyperbola $H$ is now translated by the vector $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$.

**(i)** Write down the equation of the translated curve. *(2 marks)*

**(ii)** Calculate the coordinates of the two points of intersection of the translated curve with the line $y = x$. *(4 marks)*

**(d)** From your answers to part **(c)(ii)**, **deduce** the coordinates of the points of intersection of the original hyperbola $H$ with the line $y = x - 3$. *(2 marks)*

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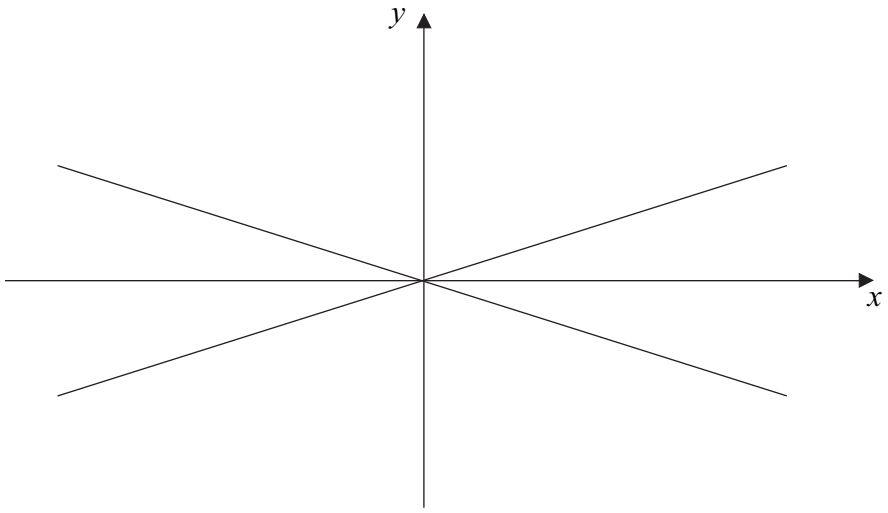
**(b)**



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**8** The diagram below shows a rectangle $R_1$ which has vertices (0, 0), (3, 0), (3, 2) and (0, 2).

**(a)** On the diagram, draw:

**(i)** the image $R_2$ of $R_1$ under a rotation through 90° clockwise about the origin; *(1 mark)*

**(ii)** the image $R_3$ of $R_2$ under the transformation which has matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$ *(3 marks)*

**(b)** Find the matrix of:

**(i)** the rotation which maps $R_1$ onto $R_2$; *(1 mark)*

**(ii)** the combined transformation which maps $R_1$ onto $R_3$. *(3 marks)*

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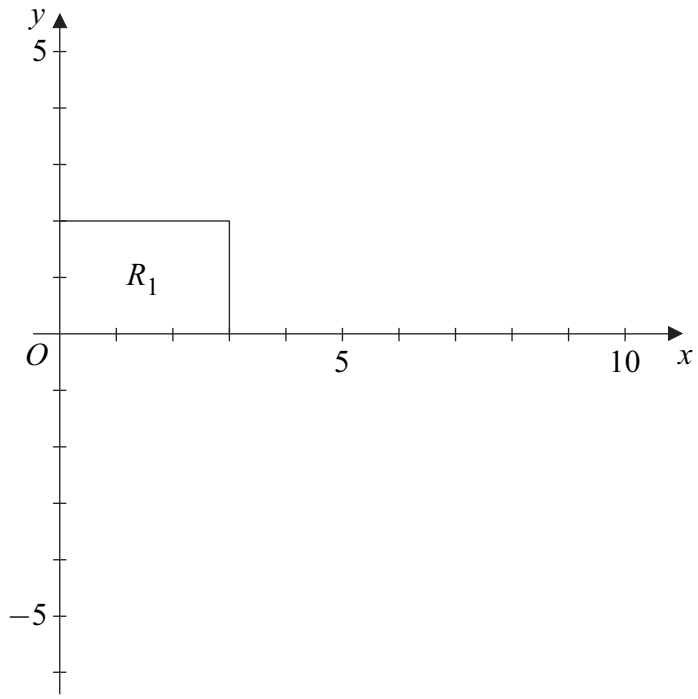
**(a)**

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**9** A curve has equation

$$y = \frac{x}{x-1}$$

**(a)** Find the equations of the asymptotes of this curve. *(2 marks)*

**(b)** Given that the line $y = -4x + c$ intersects the curve, show that the $x$-coordinates of the points of intersection must satisfy the equation

$$4x^2 - (c+3)x + c = 0$$ *(3 marks)*

**(c)** It is given that the line $y = -4x + c$ is a tangent to the curve.

**(i)** Find the two possible values of $c$.

(No credit will be given for methods involving differentiation.) *(3 marks)*

**(ii)** For each of the two values found in part **(c)(i)**, find the coordinates of the point where the line touches the curve. *(4 marks)*

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**END OF QUESTIONS**

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