

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



General Certificate of Education
Advanced Level Examination
January 2012

Mathematics

MFP2

Unit Further Pure 2

Friday 20 January 2012 1.30 pm to 3.00 pm

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J A N 1 2 M F P 2 0 1

Answer **all** questions in the spaces provided.

1 (a) Show, by means of a sketch, that the curves with equations

$$y = \sinh x$$

and

$$y = \operatorname{sech} x$$

have exactly one point of intersection.

(4 marks)

(b) Find the x -coordinate of this point of intersection, giving your answer in the form $a \ln b$.

(4 marks)

QUESTION
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*QUESTION
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- 2 (a)** Draw on an Argand diagram the locus L of points satisfying the equation $\arg z = \frac{\pi}{6}$.
(1 mark)
- (b) (i)** A circle C , of radius 6, has its centre lying on L and touches the line $\operatorname{Re}(z) = 0$.
Draw C on your Argand diagram from part **(a)**. (2 marks)
- (ii)** Find the equation of C , giving your answer in the form $|z - z_0| = k$. (3 marks)
- (iii)** The complex number z_1 lies on C and is such that $\arg z_1$ has its least possible value.
Find $\arg z_1$, giving your answer in the form $p\pi$, where $-1 < p \leq 1$. (2 marks)

QUESTION
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3 A curve has cartesian equation

$$y = \frac{1}{2} \ln(\tanh x)$$

(a) Show that

$$\frac{dy}{dx} = \frac{1}{\sinh 2x} \quad (4 \text{ marks})$$

(b) The points A and B on the curve have x -coordinates $\ln 2$ and $\ln 4$ respectively. Find the arc length AB , giving your answer in the form $p \ln q$, where p and q are rational numbers. (8 marks)

QUESTION
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4 The sequence u_1, u_2, u_3, \dots is defined by

$$u_1 = \frac{3}{4} \quad u_{n+1} = \frac{3}{4 - u_n}$$

Prove by induction that, for all $n \geq 1$,

$$u_n = \frac{3^{n+1} - 3}{3^{n+1} - 1} \quad (6 \text{ marks})$$

QUESTION
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5

Find the smallest positive integer values of p and q for which

$$\frac{\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right)^p}{\left(\cos \frac{\pi}{12} - i \sin \frac{\pi}{12}\right)^q} = i \quad (7 \text{ marks})$$

QUESTION
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6 (a) Express $7 + 4x - 2x^2$ in the form $a - b(x - c)^2$, where a , b and c are integers. (2 marks)

(b) By means of a suitable substitution, or otherwise, find the exact value of

$$\int_1^{\frac{5}{2}} \frac{dx}{\sqrt{7 + 4x - 2x^2}} \quad (6 \text{ marks})$$

QUESTION
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7 The numbers α , β and γ satisfy the equations

$$\alpha^2 + \beta^2 + \gamma^2 = -10 - 12i$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 5 + 6i$$

(a) Show that $\alpha + \beta + \gamma = 0$. (2 marks)

(b) The numbers α , β and γ are also the roots of the equation

$$z^3 + pz^2 + qz + r = 0$$

Write down the value of p and the value of q . (2 marks)

(c) It is also given that $\alpha = 3i$.

(i) Find the value of r . (3 marks)

(ii) Show that β and γ are the roots of the equation

$$z^2 + 3iz - 4 + 6i = 0$$
 (2 marks)

(iii) Given that β is real, find the values of β and γ . (3 marks)

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8 (a) Write down the five roots of the equation $z^5 = 1$, giving your answers in the form $e^{i\theta}$, where $-\pi < \theta \leq \pi$. (1 mark)

(b) Hence find the four linear factors of

$$z^4 + z^3 + z^2 + z + 1 \quad (3 \text{ marks})$$

(c) Deduce that

$$z^2 + z + 1 + z^{-1} + z^{-2} = \left(z - 2 \cos \frac{2\pi}{5} + z^{-1} \right) \left(z - 2 \cos \frac{4\pi}{5} + z^{-1} \right) \quad (4 \text{ marks})$$

(d) Use the substitution $z + z^{-1} = w$ to show that $\cos \frac{2\pi}{5} = \frac{\sqrt{5} - 1}{4}$. (6 marks)

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QUESTION
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QUESTION
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END OF QUESTIONS



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ANSWER IN THE SPACES PROVIDED**

