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Mathematics MFP3

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Further Pure 3

Report on the Examination

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General

Presentation of work was generally very good and most candidates completed their solution to a question at the first attempt. However, not all candidates followed all the instructions printed on the front cover of the question-answer booklet, in particular:

- Use black ink or black ball-point pen.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.

Teachers may also wish to emphasise the following points to their students in preparation for future examinations in this module:

- Candidates should continue to be encouraged to draw relevant sketches to help them plan solutions to polar coordinates questions which do not have full sketches provided.

- The general solution of the differential equation $a\frac{\mathrm{d}^2u}{\mathrm{d}x^2}+b\frac{\mathrm{d}u}{\mathrm{d}x}+cu=\mathrm{f}(x)$ will be in the form $u = \mathrm{g}(x)$; it will not involve the variable y directly.

- A general solution to a first order differential equation should have one arbitrary constant and the general solution to a second order differential equation should have two arbitrary constants.

Question 1

Numerical solutions of first order differential equations continue to be a good source of marks for all candidates. Once again, this topic was the best answered on the paper. The most common error involved an incorrect evaluation of $\mathrm{f}[1.1, y(1.1)]$. Almost all candidates gave their final answer to the required degree of accuracy.

Question 2

This question, which tested limits, proved to be the least well answered question on the paper. Usually candidates have been guided into the method on previous papers and, without the guidance of earlier parts to the question, the majority of candidates just presented solutions based on replacing x by 0 and claiming that $0/0$ was some definite value. The most successful candidates started by finding the series expansion of $\sqrt{4+x}$ and then divided both the numerator and denominator by x before taking the limit. A small minority of candidates started by multiplying the numerator and denominator by $\sqrt{4+x}+2$ and usually went on to show that the value of the limit was $1/4$. Answers with no method seen were awarded no credit.

Question 3

The majority of candidates scored full marks for solving this second order differential equation, subject to the given boundary conditions, correctly. Other than arithmetic and algebraic slips, the main source of misunderstanding was to give the complementary function as $A\mathrm{e}^{-x}(\cos 3x + \sin 3x)$, which lead to a general solution of a second order differential equation with only one arbitrary constant.

Question 4

Most candidates were able to show that they knew how to find and use an integrating factor to solve a first order differential equation. The method of integration by parts was generally both spotted and applied correctly, although some failed to include the constant of integration in their general solution, thus ending with a general solution of a first order differential equation which contained no arbitrary constant.

In part (b) the examiners expected candidates to pay particular attention to the limit of $x\ln x$ (or $x^3\ln x$) as x tended to 0. Also some candidates were less than convincing in dealing with the term C/x^2 as x tended to 0. In general, part (b) was not answered as well as part (a).

Question 5

Explanations for why the integral was improper were frequently less than convincing. The examiners were ideally looking for the response 'The interval of integration is infinite. '

In part (b) a significant minority could not reach the correct integral in terms of u. Errors of the type $\frac{1}{x^2+3e^{4x}} = x^{-2} + 3e^{-4x}$, seen in a number of solutions, were disappointing.

In part (c), the improvement in answers to questions on this topic seen in recent question papers was not maintained. There was a smaller proportion of candidates starting off correctly by, for example, writing $\int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2+3e^{4x}}\,dx = \lim_{a\to\infty}\int_{\frac{1}{2}}^{a} \frac{x(1-2x)}{x^2+3e^{4x}}\,dx$.

Question 6

In part (a) candidates almost always used the chain rule correctly to find $\frac{dy}{dx}$ and frequently obtained a correct expression for $\frac{d^2y}{dx^2}$. A common error in finding the next two derivatives was to differentiate $\sec^2 2x$ incorrectly as $2\sec^2 2x\tan 2x$. This single error was not overly penalised with many such candidates losing just the one mark in part (a).

However, in part (b) it was disappointing to see so many candidates stating that the fourth derivative at $x = 0$ was -32 even though their expressions for $\frac{d^4y}{dx^4}$ clearly did not give rise to this value. Such candidates would have been better advised to look for their earlier error rather than make too much 'use' of the printed answer. Those who used the follow through value from their incorrect expression for $\frac{d^4y}{dx^4}$ generally scored 2 of the 3 marks for part (b), one more than those who stated -32 incorrectly.

In the final part of the question, in general only the more able candidates recognized and applied the 'Hence' by using '$\ln(\sec^2 2x) = -2\ln(\cos 2x)$'.

Question 7

In part (a), those candidates who started by writing $\frac{\mathrm{d}u}{\mathrm{d}x} = y + x\frac{\mathrm{d}y}{\mathrm{d}x}$ and then moved on to find $\frac{\mathrm{d}^2u}{\mathrm{d}x^2}$ generally presented a convincing solution to show that the given substitution transformed one differential equation into the other.

The above average candidates generally showed that they had a thorough understanding of the method required to find the general solution of the differential equation although a few marks were lost through careless slips, most notably, due to '$9(px + q) = 9px+q$'.

Three more serious errors/lack of understanding seen, which led to a greater loss of marks, are illustrated by (**I**) $y = \mathrm{e}^{-3x}(Ax + B)$ or (**II**) $y = \mathrm{e}^{-3u}(Au+B)$ or (**III**) $u = A\mathrm{e}^{-3x}$ as the CF for $\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 6\frac{\mathrm{d}u}{\mathrm{d}x} + 9u = 18x$.

Question 8

Most candidates had a good understanding of the method required to find the area of the region bounded by the curve. It was pleasing to see that the method for integrating $\cos^2\theta$ was well known, with many candidates scoring at least 4 of the 6 marks.

In part (b)(i) those candidates who started by finding the polar equation for the circle were generally more successful than those who started by finding the cartesian equation of the curve. Some candidates incorrectly stated the polar equation of the circle as $r = 4$, presumably just by consideration of the radius rather than any thought about the centre of the circle.

The final part of this final question proved to be very demanding for almost all candidates. Many answers displayed a lack of understanding of the term 'segment', with most solutions involving a perimeter of a region which consisted of the two radii and either an arc length (i.e. perimeter of a sector) or the length of AB (i.e. perimeter of a triangle). The few successful candidates noted that the circle passed through the pole, O, drew the circle and curve on the same diagram and found the size of angle AMB, in radians, with M being the centre of the circle. They then found the arc length using MPC2 work and added it to the length of AB found in part (b)(i).

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results statistics page of the AQA Website. UMS conversion calculator www.aqa.org.uk/umsconversion