



**General Certificate of Education (A-level)**  
**June 2012**

**Mathematics**

**MM03**

**(Specification 6360)**

**Mechanics 3**

***Mark Scheme***

---

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: [aqa.org.uk](http://aqa.org.uk)

Copyright © 2012 AQA and its licensors. All rights reserved.

**Copyright**

AQA retains the copyright on all its publications. However, registered schools/colleges for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

## Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

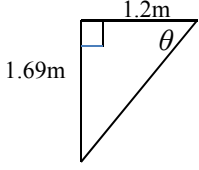
## MM03

Q	Solution	Marks	Total	Comments
1(a)	$I = \int_0^{0.5} 4 \times 10^4 t^2 (1 - 2t) dt$ $= 4 \times 10^4 \left[ \frac{1}{3} t^3 - \frac{1}{2} t^4 \right]_0^{0.5}$ $= 417 \text{ (or } \frac{1250}{3}) \text{ Ns}$	M1 A1 A1F A1F	4	Attempt to integrate Use of correct limits, PI Correct integration Accept 416.6 or 416.7
(b)	$416.6 = 60v + 60 \times 5$ $v = 1.94$	M1A1F A1F	3	A1F correct sign AWRT 1.94, accept 1.95 ISW
<b>Total</b>			<b>7</b>	
2	<p>Dimension of <math>g</math> is <math>LT^{-2}</math>            Dimension of <math>s</math> is L            Dimension of <math>h</math> is L            Dimension of <math>m_1</math> and <math>m_2</math> is M</p> <p>Dimension of <math>\frac{g}{s} [s(m_1 + m_2) + \frac{hm_1^2}{m_1 + m_2}]</math> is</p> $\frac{LT^{-2}}{L} [LM + \frac{LM^2}{M}] \cong MLT^{-2} + MLT^{-2}$ $\cong MLT^{-2}$ <p>which is a force</p>	<p>{ B1</p> <p>M1</p> <p>A1</p> <p>B1</p>	4	B1 for dimensions of the five quantities  Correct substitution of dimensions
<b>Total</b>			<b>4</b>	

## MM03

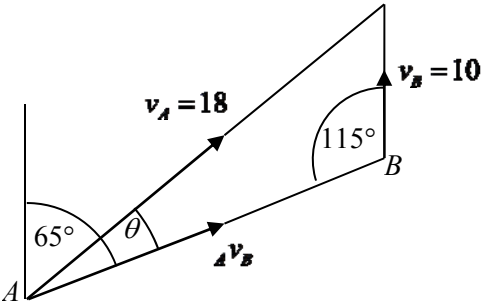
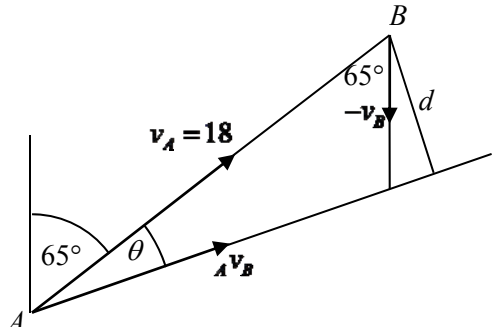
Q	Solution	Marks	Total	Comments
3(a)	$x = ut \cos \alpha$	M1		
	$t = \frac{x}{u \cos \alpha}$	A1		
	$y = -\frac{1}{2}gt^2 + ut \sin \alpha$	M1		Must have correct signs
	$y = -\frac{1}{2}g\left(\frac{x}{u \cos \alpha}\right)^2 + u\left(\frac{x}{u \cos \alpha}\right)\sin \alpha$	M1		
	$y = -\frac{gx^2}{2u^2 \cos^2 \alpha} + \frac{x \sin \alpha}{\cos \alpha}$			
	$y = -\frac{gx^2}{2u^2}(1 + \tan^2 \alpha) + x \tan \alpha$	A1		
	$k = -\frac{10(2k)^2}{2u^2}(1 + \tan^2 \alpha) + 2k \tan \alpha$	M1		
	$u^2 = -20k(1 + \tan^2 \alpha) + 2u^2 \tan \alpha$			
	$20k \tan^2 \alpha - 2u^2 \tan \alpha + u^2 + 20k = 0$	A1	7	AG
	(b)	Pass through P $\Rightarrow$ Discriminant $\geq 0$		
	$(-2u^2)^2 - 4(20k)(u^2 + 20k) \geq 0$	M1A1		OE must be seen
	$4u^4 - 80ku^2 - 1600k^2 \geq 0$			
	$u^4 - 20ku^2 - 400k^2 \geq 0$	A1	3	AG
	<b>Total</b>		<b>10</b>	

## MM03

Q	Solution	Marks	Total	Comments
4(a)	 <p> <math>\theta = \tan^{-1} \frac{1.69}{1.2} = 54.623^\circ</math>  <math>u \cos 60^\circ = v \cos 54.623^\circ</math>  <math>eu \sin 60^\circ = v \sin 54.623^\circ</math>  <math display="block">e = \frac{v \sin 54.623^\circ}{\frac{v \cos 54.623^\circ}{\cos 60^\circ} \times \sin 60^\circ}</math> <math>e = 0.813 \text{ or } 0.812</math> </p>	B1 M1 M1 m1 A1	5	AWRT $55^\circ$ $v = 0.864u$ OE, dependent on both M1s ISW
(b)	$I = 0.15u \sin 60^\circ + 0.15v \sin 54.623^\circ$ $= 0.15u \sin 60^\circ + 0.15 \times \frac{u \cos 60^\circ}{\cos 54.623^\circ} \times \sin 54.623^\circ$ $= 0.236u$	M1A1 m1 A1	4	Single angle values needed for A1 AG (condone 0.2355 or negative result)
(c)	<p>Attempt at considering motion parallel or perpendicular to AC</p> $t = \frac{1.2}{u \cos 60^\circ}$ $t = \frac{12}{5u} \text{ or } \frac{2.4}{u}$ <p>Alternative :</p> $CP = \frac{1.2}{\cos 54.623^\circ} \quad (= 2.072703844 \text{ m})$ $t = \frac{\frac{1.2}{\cos 54.623^\circ}}{\frac{u \cos 60^\circ}{\cos 54.623^\circ}}$ $= \frac{12}{5u} \text{ or } \frac{2.4}{u}$	M1 M1 A1 (M1) (M1) (A1)	3	OE, No ISW     (OE), No ISW
(d)	<p>Velocity (momentum) parallel to the cushion is unchanged, or, Restitution only affects motion perpendicular to the cushion</p>	E1	1	Accept 'horizontal component of velocity is unchanged'
<b>Total</b>			<b>13</b>	

Q	Solution	Marks	Total	Comments
5(a)	$0 = 15t \sin 30 - \frac{1}{2}g \cos 25t^2$	M1A1	4	Accept wrong angle(s) for M1 but <b>not</b> sin and cos in wrong places
	$t = \frac{15 \sin 30}{\frac{1}{2}g \cos 25}$	M1		
	$t = 1.69 \text{ sec.}$	A1F		AWRT 1.69
(b)	$\perp$ to plane $\dot{y} = 15 \sin 30 - g \cos 25 \times \frac{15 \sin 30}{\frac{1}{2}g \cos 25}$	M1	8	Or $-7.51$ , ft from their answer in (a)
	$\dot{y} = -7.5 \text{ ms}^{-1}$	A1F		
	$\parallel$ to plane $\dot{x} = 15 \cos 30 - g \sin 25 \times \frac{15 \sin 30}{\frac{1}{2}g \cos 25}$	M1		
	$\dot{x} = 5.995766 \text{ or } 6.00 \text{ ms}^{-1}$	A1F		
	Restitution: Rebound $\dot{y} = \frac{2}{3} \times 7.5 = 5 \text{ ms}^{-1}$	M1		
	$\dot{x}$ unchanged	B1		
	Speed of rebound $= \sqrt{5.995766^2 + 5^2}$ $= 7.81 \text{ ms}^{-1}$	m1 A1F		
	<b>Total</b>		<b>12</b>	

## MM03

Q	Solution	Marks	Total	Comments
6(a)	 $\frac{\sin \theta}{10} = \frac{\sin 115^\circ}{18}$ $\theta = 30.2^\circ$ <p>Bearing = <math>035^\circ</math></p>	B1  M1 A1 A1	4	For any appropriate diagram PI by correct method   Accept $034.8^\circ$
(b)(i)	 $A v_B^2 = 18^2 + 10^2 - 2(18)(10) \cos 65^\circ$ $A v_B = 16.4881 \quad \text{ms}^{-1}$ $\frac{\sin 65^\circ}{16.4881} = \frac{\sin \theta}{10}$ $\theta = 33.3446^\circ$ $d = 12 \times \sin 33.3446^\circ$ $d = 6.60 \quad \text{km}$	B1  M1 A1 M1 A1F m1 A1F	7	For any appropriate diagram PI by correct method   OE  OE Dependent on the previous two M1s (AWRT 6.6 km)
(ii)	$t = \frac{12 \times \cos 33.3446^\circ}{16.4881} = 0.607987 \quad \text{hours}$ <p>( = 36.5 min)</p>	M1 A1F A1F	3	Or 0.608 hours LHS values Correct time
<b>Total</b>			<b>14</b>	



## MM03

**Q6 (b)(i) Alternative:**

$$r_A = [(18\cos 25)\mathbf{i} + (18\sin 25)\mathbf{j}]t$$

$$r_B = [(12\cos 25)\mathbf{i} + (12\sin 25)\mathbf{j}] + 10\mathbf{j}t$$

$${}_A r_B = (-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}$$

$$|{}_A r_B|^2 = (-12\cos 25 + 18t\cos 25)^2 + (-12\sin 25 + 18t\sin 25 - 10t)^2$$

$$\frac{d|{}_A r_B|^2}{dt} = (36\cos 25)(-12\cos 25 + 18t\cos 25) + (36\sin 25 - 20)(-12\sin 25 + 18t\sin 25 - 10t) = 0$$

$$t = 0.608$$

$$d = 6.60 \text{ km} \quad \text{or} \quad 6.6 \text{ km}$$

The corresponding marks awarded for finding the closest approach time:

$$\frac{d|{}_A r_B|^2}{dt} = (36\cos 25)(-12\cos 25 + 18t\cos 25) + (36\sin 25 - 20)(-12\sin 25 + 18t\sin 25 - 10t) = 0$$

$$t = 0.608 \quad (\text{or better})$$

M1 for both

m1 A1

A1

m1

A1

A1

M1 A1

A1

**(b)(i) Alternative (Not in the specification):**

$${}_A r_B = (-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}$$

M1 A1

$$[(-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}] \cdot [(18\sin 65)\mathbf{i} + (18\cos 65 - 10)\mathbf{j}] = 0$$

$$(-12\cos 25 + 18t\cos 25)(18\sin 65) + (-12\sin 25 + 18t\sin 25 - 10t)(18\cos 65 - 10) = 0$$

$$271.85t = 165.27$$

$$t = 0.608 \quad (\text{or better})$$

$$d = 6.60 \text{ km} \quad \text{or} \quad 6.6 \text{ km}$$

The corresponding marks awarded for finding the closest approach time:

$$(-12\cos 25 + 18t\cos 25)(18\sin 65) + (-12\sin 25 + 18t\sin 25 - 10t)(18\cos 65 - 10) = 0$$

$$271.85t = 165.27$$

$$t = 0.608 \quad (\text{or better})$$

**(b)(ii) FT from their answers in part (b)(i)**

m1

A1

m1

A1

A1

M1

A1

A1

## MM03

Q	Solution	Marks	Total	Comments
7(a)	$2m(3i + j) + m(2i - 5j) = 2mv_A + m(2i + j)$ $8i - 3j = 2v_A + (2i + j)$ $v_A = 3i - 2j$	M1A1  A1	3	
(b)	$I = m(2i + j) - m(2i - 5j)$ $I = 6mj$	M1A1 A1	3	AG
(c)	$I = 6mj \Rightarrow$ Line of centres along $j$ Restitution along $j$ : $1 + 2 = e(5 + 1)$ $e = 0.5$ Accept energy methods	B1 M1A1 A1	4	PI
(d)	${}_A v_B = i - 3j$ ${}_A r_B = -0.1j + (i - 3j)t$ $1.1^2 = t^2 + (-0.1 - 3t)^2$ $10t^2 + 0.6t - 1.2 = 0$ $t = \frac{-0.6 \pm \sqrt{0.6^2 - 4(10)(-1.2)}}{2(10)} \quad (= 0.31770677)$ $t = 0.318$ or $0.317$ sec.	M1A1 M1  m1 A1	5	OE  Dependent on both M1s
	<b>Total</b>		<b>15</b>	
	<b>TOTAL</b>		<b>75</b>	