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General Certificate of Education (A-level)
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Mathematics MFP1

(Specification 6360)

Further Pure 1

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	$y_{n+1} \approx y_n + h\,\mathrm{f}(x_n)$			OE
	$h\,y'(1) = 0.1 \times y'(1)$ (=0.05)	M1		Attempt to find $h\,y'(1)$. PI by eg 3.05 for $y(1.1)$
	$y(1.1) \approx 3 + 0.05 = 3.05$	A1		
	$y(1.2) \approx y(1.1) + 0.1 \times y'(1.1) = 3.05 + 0.1 \times y'(1.1)$ $\approx 3.05 + 0.1 \times \frac{1.1}{1+1.1^3}$ $\left(= 3.05 + 0.1 \times \frac{1100}{2331}\right)$	m1		Attempt to find $y(1+0.1)+0.1\times y'(1+0.1)$ must see evidence of calculation if correct ft [0.047..+c's $y(1.1)$] value not obtained
	$\approx 3.05 + 0.047(19.....)$	A1F		OE; ft on [0.047..+c's $y(1.1)$] value; PI
	≈ 3.0972 (to 4 d.p.)	A1	5	Must be 4 dp.
	Total		**5**	
2(a)	$(w=)\ \frac{-6 \pm \sqrt{36-4(34)}}{2}$ $\left\{= \frac{-6 \pm \sqrt{-100}}{2}\right\}$	M1		Correct substitution into quadratic formula OE
	$= \frac{-6 \pm 10\mathrm{i}}{2}$	B1		$\sqrt{-100} = 10\mathrm{i}$ or $\sqrt{-100}/2 = 5\mathrm{i}$
	$= -3 \pm 5\mathrm{i}$	A1	3	$-3 \pm 5\mathrm{i}$ ($p = -3$, $q = \pm 5$) NMS mark as 3/3 or 0/3
(b)(i)	$z = \mathrm{i}(1+\mathrm{i})(2+\mathrm{i}) = \mathrm{i}(2+3\mathrm{i}+\mathrm{i}^2) = 2\mathrm{i} + 3\mathrm{i}^2 + \mathrm{i}^3$	M1		Attempt to expand all brackets.
	$= 2\mathrm{i} + 3(-1) + \mathrm{i}(-1)$	B1		$\mathrm{i}^2 = -1$ used at least once
	$= -3 + \mathrm{i}$	A1	3	$-3 + \mathrm{i}$ ($a = -3$, $b = 1$)
(ii)	$z^* = -3 - \mathrm{i}$	B1F		OE Ft c's $a - b\mathrm{i}$
	$-3 + \mathrm{i} + m(-3 - \mathrm{i}) = n\mathrm{i}$ $\Rightarrow -3 - 3m = 0;\ 1 - m = n$	M1		Equating **both** real parts and the imag. parts, PI by next line
	$\Rightarrow m = -1,\ n = 2$	A1	3	Both correct
	Total		**9**	

Q	Solution	Marks	Total	Comments
3(a)	$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$	B1		OE (PI) Stated or used. A correct angle in 1st or 2nd quadrant for $\sin^{-1}(\sqrt{3}/2)$.
	$\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$	B1F		OE (PI) Stated or used. A correct ft angle in remaining quadrant for $\sin^{-1}(\sqrt{3}/2)$. B0F if angle used is in an incorrect quadrant
	$2x+\frac{\pi}{4}=2n\pi+\frac{\pi}{3}$; $2x+\frac{\pi}{4}=2n\pi+\frac{2\pi}{3}$	M1		OE Either. Ft on c's $\sin^{-1}(\sqrt{3}/2)$.
	$x=\frac{1}{2}\left(2n\pi+\frac{\pi}{3}-\frac{\pi}{4}\right)$; $x=\frac{1}{2}\left(2n\pi+\frac{2\pi}{3}-\frac{\pi}{4}\right)$	m1		Either. Correct rearrangement of $2x+\frac{\pi}{4}=2n\pi+\alpha$ to $x=\ldots$, where α is c's $\sin^{-1}(\sqrt{3}/2)$.
	GS: $x=n\pi+\frac{\pi}{24}$; $x=n\pi+\frac{5\pi}{24}$	A2,1,0	6	Both in ACF, but must now be exact and in terms of π for A2. A1 if decimal approx used.
(b)	$n=5$ (gives greatest soln $<6\pi$) $=5\pi+\frac{5\pi}{24}$	M1		Applying a correct value for n which gives greatest soln.$<6\pi$ for c's GS dep on GS, using above method, having two expressions of the form $n\pi+\lambda$, for different λ and m1 scored in (a).
	$=\frac{125\pi}{24}$	A1	2	Dep on correct full GS.
	Total		**8**	
4	$\int\frac{1}{x\sqrt{x}}\,dx=\int x^{-\frac{3}{2}}\,(dx)$	M1		$\int x^{-\frac{3}{2}}$ PI
	$=-2x^{-\frac{1}{2}}\ (+c)$	A1		ACF, can be unsimplified. Condone absence of $+c$
	$-2x^{-\frac{1}{2}}\to 0$ as $x\to\infty$	E1		OE Ft on kx^{-n}, $n>0$
	$\int_{25}^{\infty}\frac{1}{x\sqrt{x}}\,dx=\frac{2}{5}$	A1	4	
	Total		**4**	

Q	Solution	Marks	Total	Comments
5(a)	$\alpha+\beta=-2$	B1		
	$\alpha\beta=-5$	B1	2	
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-2)^2-2(-5)$	M1		OE Using correct identity for $\alpha^2+\beta^2$ with ft or correct substitution
	$=14$	A1	2	CSO A0 if $\alpha+\beta$ has wrong sign
(c)	$\alpha^3\beta+\alpha\beta^3=\alpha\beta(\alpha^2+\beta^2)$	M1		PI Seen at least once in part (c). OE eg $\alpha^3\beta+\alpha\beta^3=\alpha\beta[(\alpha+\beta)^2-2\alpha\beta]$
	S(um)$=\alpha^3\beta+\alpha\beta^3+2=(-5)(14)+2=-68$	A1F		Correct or ft c's $\alpha\beta$ × c's [answer (b)] +2
	P(roduct) $=(\alpha\beta)^4+\alpha^3\beta+\alpha\beta^3+1$ $=(-5)^4+(-5)(14)+1=556$	A1F		Correct or ft [c's $\alpha\beta$]^4 + c's $\alpha\beta$ × c's [answer (b)] +1
	x^2-Sx+P (=0)	M1		Using correct general form of LHS of eqn **with** ft substitution of c's S and P values.
	Eqn.: $x^2+68x+556=0$	A1	5	CSO ACF
	Total		**9**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathbf{X}^2 = \begin{bmatrix} 7 & 2 \\ 3 & 6 \end{bmatrix}$ **;** $(m =)\ 7$	B1	1	$(m =)7$ or 7 as top left element of $\mathbf{X}^2$
(ii)	$\mathbf{X}^3 = \begin{bmatrix} 13 & 14 \\ 21 & 6 \end{bmatrix}$ **;**	M1		At least 2 elements correct
	$7\mathbf{X} = \begin{bmatrix} 7 & 14 \\ 21 & 0 \end{bmatrix}$	B1		PI
	$\mathbf{X}^3 - 7\mathbf{X} = \begin{bmatrix} 13-7 & 14-14 \\ 21-21 & 6-0 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$	A1F		Ft on c's m value
	$= 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 6\,\mathbf{I}$	A1	4	CSO Accept either form but at least one must be shown explicitly
(b)(i)	Reflection in the x-axis	B1	1	OE
(ii)	$\mathbf{B} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$	M1		Either OE. For M mark, accept dec. equiv. (at least 3sf) for $\frac{1}{\sqrt{2}}$
	$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$	A1	2	NMS SC1 for $k = \frac{1}{\sqrt{2}}$ or better.
(iii)	$\mathbf{AB}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = k\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$	M1		Attempt to find $\mathbf{AB}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$
	$= k\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \end{bmatrix}$ $\left\{\text{or } k\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right\}$	A1		Either $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$
	$= k\begin{bmatrix} -3 \\ -1 \end{bmatrix}$	m1		Completing the matrix mult. to reach a 2×1 matrix
	(Image of P is the point) $\left(-\frac{3}{\sqrt{2}}\,,\ -\frac{1}{\sqrt{2}}\right)$	A1	4	**CSO** **SC** Wrong order, works with $\mathbf{BA}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$, mark out of a max of M1A0 m1A0
	Total		**12**	

Q	Solution	Marks	Total	Comments
7(a)	$y = ax^n \Rightarrow \log_{10} y = \log_{10} ax^n$ $\log_{10} y = \log_{10} a + \log_{10} x^n$	M1		Take logs and apply one log law in soln. correctly PI.
	$\log_{10} y = \log_{10} a + n\log_{10} x$	m1		Apply a further log law correctly.
	$Y = \log_{10} a + nX$ (which is a linear relationship between Y and X.)	A1	3	Correct eqn. with base 10 (or lg or later evidence of use of base 10 if log without base here)
(b)	n = gradient of line	M1		Stated or used. Accept $n = \pm\frac{2}{3}$ OE as evidence
	$n = -\frac{2}{3}$	A1		$n = -\frac{2}{3}$ (OE 3sf)
	$\log_{10} a = 4$	M1		Equating c's constant term [must involve a log] in c's **(a)** eqn. to the Y-intercept value PI by correct value of a
	$a = 10^4$ (= 10 000)	A1	4	
	Total		**7**	

Q	Solution	Marks	Total	Comments
8(a)	$\sum_{r=1}^{n} 2r(2r^2-3r-1) = \sum_{r=1}^{n} 4r^3 - \sum_{r=1}^{n} 6r^2 - \sum_{r=1}^{n} 2r$ $= 4\sum_{r=1}^{n} r^3 - 6\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r$	M1		Splitting up the sum into three separate sums. PI by m1 line below.
	$= 4\times\frac{1}{4}n^2(n+1)^2 - 6\times\frac{1}{6}n(n+1)(2n+1) - 2\times\frac{1}{2}n(n+1)$	m1		Substitution of the three summations from FB into $a\sum_{r=1}^{n} r^3 + b\sum_{r=1}^{n} r^2 + c\sum_{r=1}^{n} r$
	$= n^2(n+1)^2 - n(n+1)(2n+1) - n(n+1)$	A1		PI by later expressions
	$= n(n+1)[n(n+1)-(2n+1)-1]$	m1		Taking out factor $n(n+1)$ from correct expressions
	$= n(n+1)[n^2-n-2]$	A1		
	$= n(n+1)(n+1)(n-2)$ $(= n(n-2)(n+1)^2$ $(p=-2, q=1))$	A1	6	
(b)	$\sum_{r=11}^{20} 2r(2r^2-3r-1)$ $= \sum_{r=1}^{20} 2r(2r^2-3r-1) - \sum_{r=1}^{10} 2r(2r^2-3r-1)$	M1		$\sum_{r=1}^{20}\ldots - \sum_{r=1}^{10}\ldots$ PI by next line (ft c's p&q)
	$= 20(20+p)(20+q)^2 - 10(10+p)(10+q)^2$			
	$= 20\times18\times21^2 - 10\times8\times11^2 = 158760 - 9680 = 149080$	A1	2	NMS 0/2 A0 if not showing use of fully factorised form.
	Total		**8**	

Q	Solution	Marks	Total	Comments
9(a)	$y = 0,\ \frac{(x-4)^2}{4} = 1;\quad (x-4)^2 = 4$	M1		OE Sub y=0 in eqn of ellipse and either eliminate fraction **or** take sq root, condoning missing ±, ie $\frac{(x-4)}{2} = (\pm)1$
	$\Rightarrow x = 2, 6\ \ (x_A = 2,\ x_B = 6)$	A1	2	Both 2 and 6 NMS Mark as B2 or B0
(b)(i)	$\frac{(x-4)^2}{4} + (mx)^2 = 1 \quad \Rightarrow$	M1		Substitute $y=mx$ to eliminate y
	$(x-4)^2 + 4(mx)^2 = 4 \Rightarrow x^2 - 8x + 16 + 4(mx)^2 = 4$	A1		Eliminate fractions correctly and expand $(x-4)^2$ correctly
	$\Rightarrow x^2 - 8x + 16 + 4m^2x^2 - 4 = 0$ $\Rightarrow (1+4m^2)x^2 - 8x + 12 = 0$	A1	3	CSO AG
(ii)	Discriminant $b^2 - 4ac\ \ \{(-8)^2 - 4(1+4m^2)(12)\}$	M1		b^2-4ac in terms of m condone one sign or copying error OE
	For tangency, $(-8)^2 - 4(1+4m^2)(12) = 0$	A1		A correct equation with m^2 being the only unknown at any stage.
	$192m^2 - 16\ \ (=0)$	A1		OE eg $12m^2-1(=0)$ OE PI by a correct value for m condoning wrong sign
	(m>0 so) $m = \frac{1}{\sqrt{12}}$	A1	4	ACF of an **exact** value for m eg $\frac{1}{2\sqrt{3}}$, $\frac{\sqrt{3}}{6}$. Dep on prev 3 mrks
(iii)	$(1 + 4\times\left\{\frac{1}{\sqrt{12}}\right\}^2)x^2 - 8x + 12\ \ (=0)$	M1		Subst value for m in LHS of eqn (b)(i); ft on c's value of m.
	$\frac{4}{3}x^2 - 8x + 12 = 0;\qquad 4x^2 - 24x + 36 = 0$ $x^2 - 6x + 9 = 0$ $x = \frac{-(-8) \pm \sqrt{0}}{\frac{8}{3}};\qquad (x-3)^2\ \ (=0)$	m1		Valid method to solve a **correct** quadratic <u>equation</u>; as far as either correct subst into quadratic formula with $b^2 - 4ac$ evaluated to 0 or correct factorisation or correct value of x after $\frac{4}{3}x^2 - 8x + 12 = 0$ or better seen.; OE, correct use of $-b/2a$
	$x = 3$	A1		Must see earlier justification
	Coordinates of P are $\left(3,\ \frac{3}{\sqrt{12}}\right)$	A1	4	Correct coordinates with the y-coord in any correct exact form eg $\frac{\sqrt{3}}{2}$. NMS SC 1 for $\left(3,\ \frac{3}{\sqrt{12}}\right)$
	Total		**13**	
	TOTAL		**75**	