

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



General Certificate of Education  
Advanced Level Examination  
January 2013

# Mathematics

# MFP2

## Unit Further Pure 2

Wednesday 23 January 2013 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Time allowed

- 1 hour 30 minutes

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J A N 1 3 M F P 2 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

**1 (a)** Show that

$$12 \cosh x - 4 \sinh x = 4e^x + 8e^{-x} \quad (2 \text{ marks})$$

**(b)** Solve the equation

$$12 \cosh x - 4 \sinh x = 33$$

giving your answers in the form  $k \ln 2$ . (5 marks)

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QUESTION  
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2 Two loci,  $L_1$  and  $L_2$ , in an Argand diagram are given by

$$L_1 : |z + 6 - 5i| = 4\sqrt{2}$$

$$L_2 : \arg(z + i) = \frac{3\pi}{4}$$

The point  $P$  represents the complex number  $-2 + i$ .

- (a) Verify that the point  $P$  is a point of intersection of  $L_1$  and  $L_2$ . (2 marks)
- (b) Sketch  $L_1$  and  $L_2$  on one Argand diagram. (6 marks)
- (c) The point  $Q$  is also a point of intersection of  $L_1$  and  $L_2$ . Find the complex number that is represented by  $Q$ . (2 marks)

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**3 (a)** Show that  $\frac{1}{5r-2} - \frac{1}{5r+3} = \frac{A}{(5r-2)(5r+3)}$ , stating the value of the constant  $A$ .  
(2 marks)

**(b)** Hence use the method of differences to show that

$$\sum_{r=1}^n \frac{1}{(5r-2)(5r+3)} = \frac{n}{3(5n+3)} \quad (4 \text{ marks})$$

**(c)** Find the value of

$$\sum_{r=1}^{\infty} \frac{1}{(5r-2)(5r+3)} \quad (1 \text{ mark})$$

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**4** The roots of the equation

$$z^3 - 5z^2 + kz - 4 = 0$$

are  $\alpha$ ,  $\beta$  and  $\gamma$ .

**(a) (i)** Write down the value of  $\alpha + \beta + \gamma$  and the value of  $\alpha\beta\gamma$ . *(2 marks)*

**(ii)** Hence find the value of  $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ . *(2 marks)*

**(b)** The value of  $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$  is  $-4$ .

**(i)** Explain why  $\alpha$ ,  $\beta$  and  $\gamma$  cannot all be real. *(1 mark)*

**(ii)** By considering  $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$ , find the possible values of  $k$ . *(4 marks)*

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5 (a) Using the definition  $\tanh y = \frac{e^y - e^{-y}}{e^y + e^{-y}}$ , show that, for  $|x| < 1$ ,

$$\tanh^{-1} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right) \quad (3 \text{ marks})$$

(b) Hence, or otherwise, show that  $\frac{d}{dx} (\tanh^{-1} x) = \frac{1}{1-x^2}$ . (3 marks)

(c) Use integration by parts to show that

$$\int_0^{\frac{1}{2}} 4 \tanh^{-1} x \, dx = \ln \left( \frac{3^m}{2^n} \right)$$

where  $m$  and  $n$  are positive integers. (5 marks)

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**6** A curve is defined parametrically by

$$x = t^3 + 5, \quad y = 6t^2 - 1$$

The arc length between the points where  $t = 0$  and  $t = 3$  on the curve is  $s$ .

- (a) Show that  $s = \int_0^3 3t\sqrt{t^2 + A} dt$ , stating the value of the constant  $A$ . (4 marks)
- (b) Hence show that  $s = 61$ . (4 marks)

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8 (a) Express  $-4 + 4\sqrt{3}i$  in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta \leq \pi$ . (3 marks)

(b) (i) Solve the equation  $z^3 = -4 + 4\sqrt{3}i$ , giving your answers in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta \leq \pi$ . (4 marks)

(ii) The roots of the equation  $z^3 = -4 + 4\sqrt{3}i$  are represented by the points  $P$ ,  $Q$  and  $R$  on an Argand diagram.

Find the area of the triangle  $PQR$ , giving your answer in the form  $k\sqrt{3}$ , where  $k$  is an integer. (3 marks)

(c) By considering the roots of the equation  $z^3 = -4 + 4\sqrt{3}i$ , show that

$$\cos \frac{2\pi}{9} + \cos \frac{4\pi}{9} + \cos \frac{8\pi}{9} = 0 \quad (4 \text{ marks})$$

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**END OF QUESTIONS**

