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Mathematics MFP2

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Further Pure 2

Report on the Examination

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General

The paper seemed to provide a challenge for the very able students whilst at the same time allowing weaker students to demonstrate their basic understanding of particular topics such as hyperbolic functions, properties of roots of equations and complex numbers. The first few questions were fairly straightforward and allowed most students the opportunity to gain confidence. Proof by induction remains a weakness and whenever questions required an explanation the reasoning was often muddled. The presentation of a significant number of students was quite poor and it was difficult to read their solutions. Some students made unnecessary errors because they could not read their own writing.

Question 1

(a) Almost everyone managed to use the correct exponential forms for $\cosh x$ and $\sinh x$ and simplified their expression to the form on the right hand side of the identity. However, in order to score full marks, it was necessary to show the expression on the left hand side of the identity at least on the first line of working. The best students wrote an appropriate conclusion such as "therefore $12\cosh x - 4\sinh x = 4\mathrm{e}^{x} + 8\mathrm{e}^{-x}$ ".

(b) Most students formed a quadratic equation in e^{x} which they solved correctly. The solution $\mathrm{e}^{x} = \frac{1}{4}$ caused problems to some who were unable to write x in the form $k \ln 2$.

Question 2

(a) Very few students scored full marks for the verification. It was not sufficient to merely write $|4-4\mathrm{i}| = 4\sqrt{2}$; some indication as to how the modulus was being found needed to be shown.

Similarly " $\arg(-2+2\mathrm{i}) = -\frac{\pi}{4} = \frac{3\pi}{4}$ ", which was presented as a solution by many, was not deemed worthy of a mark for verification. A sketch with a calculation often convinced examiners that the argument had the correct value.

(b) It was pleasing to see how well students understood these basic loci, and, although some could have taken greater care drawing a circle, a freehand sketch was acceptable. The main error was in drawing the half line where quite a few did not realise that it started at $0 - \mathrm{i}$ on the Argand diagram.

(c) Those who realised that the half line passed through the centre of the circle had little trouble in finding the complex number represented by Q. Another successful approach involved forming solving a quadratic equation based on the Cartesian equations of the circle and line. A few did the hard part of the question but wrote their final answer as coordinates and so only scored the method mark.

Question 3

(a) Most students found the correct value of A, but others, usually those who omitted brackets, thought that A was equal to 1. A few students assumed that the numerator was A and simply cross multiplied to find the value of A, but this approach did not score full marks.

(b) The method of differences was generally understood. The printed answer no doubt helped many to amend their solution when they realised that they needed to divide their sum by 5 in order to obtain the given answer. Some did this legitimately and scored full marks; others used dubious reasoning and lost at least the final mark.

(c) The basic idea of a limit, as used in MFP1, was required here in order to find the value of the sum as n tended to infinity. A common wrong answer was $\frac{1}{3}$ and was often as a result of the earlier working where the factor of $\frac{1}{5}$ had been omitted. Other common wrong answers were 1, 0 and infinity.

Question 4

(a)(i) This question was answered much better than in recent years, with very few making sign errors in either the sum or the product of the roots.

(a)(ii) Almost everyone realised how to factorise the expression and to substitute the numerical values from the previous parts.

(b)(i) In contrast, very few explanations were considered good enough to earn the mark in this part. A proof by contradiction was the best approach, reasoning that if the roots α, β and γ were all real then $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ could not be negative and hence if $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = -4$ then α, β and γ could not possibly all be real.

(b)(ii) Some careless arithmetic was seen in this part and others were unable to form a correct identity in which to substitute values. There were two values for k, but for some reason some students thought that k had to be positive.

Question 5

(a) Most students used $x = \tanh y$ and the given exponential form to obtain an expression in terms of x for e^{2y} and hence the printed result. Quite a few students thought that $\tanh^{-1} y = \dfrac{\sinh^{-1} y}{\cosh^{-1} y}$ and made no progress. Those students who started with $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ were also rarely successful.

(b) The majority of students used the chain rule to differentiate the result from part **(a)**. Those with greater insight used the law of logarithms to obtain the difference of two logarithms and the new form was then much easier to differentiate. Poor algebraic manipulation meant that some were forced to fudge their working to obtain the printed answer.

(c) This part of the question was the best source of marks for most students, though some were unable to integrate $\dfrac{x}{1 - x^2}$ correctly. Those whose calculus was sound had no trouble expressing the final answer in the given form.

Question 6

(a) Most students were able to establish the given answer for the arc length, although some neglected to insert the limits and others omitted the $\mathrm{d}t$ on the integral and so lost the final mark. Others were unable to find the correct value of A when trying to simplify an expression inside the square root sign.

(b) Students resorted to a variety of substitutions in order to find the value of the integral. Quite a few used hyperbolic functions but were then unable to integrate their resulting expression of the form $k \cosh^2\theta \sinh\theta$; others used a substitution such as $v^2 = t^2 + 16$ quite effectively. However, many students did not change their limits when making a substitution, writing expressions such as $\int_0^3 \frac{3}{2} u^{\frac{1}{2}} \, \mathrm{d}u$ and this was penalised.

It had been expected that most students would have been able to write down the answer to the integral as $\left(t^2 + 16\right)^{\frac{3}{2}}$ before evaluating the limits. It was then necessary to show some working such as $s = 125 - 64 = 61$, since the numerical value of the answer was given in the question.

Question 7

(a)(i) The algebra was handled quite well here, but having obtained a simplified expression such as $9(k^2+k+1)$, it was necessary to make a statement that the expression was divisible by 9.

(a)(ii) Very few students scored full marks for the proof by induction. Most students were confused as to what they were trying to prove. A large number of students' attempts contained expressions such as "M9" without defining their notation. Some even wrote things like "M9+M9=M18". Examiners were often not convinced that students were actually trying to show that p(k+1) was divisible by 9, particularly when they omitted key phrases such as "assume that p(k) is a multiple of 9". Even the mark for the case when n=1 was rarely earned since students omitted to write a conclusion such as "therefore p(1) is a multiple of 9" after showing that p(1)=9.
Many students seemed only familiar with proof by induction involving formulae for sums of series as they wrote things such as "adding the (k+1)th term" or "LHS= ..." and "RHS= …" or "formula is true" with no reference to divisibility at all.

(b) It was necessary for students to show that $\mathrm{p}(n)=(n-1)^3+n^3+(n+1)^3$ could be simplified to $3n^3+6n$ in order to convince examiners that they were not working back from the printed answer. Clearly the simplification of p(n) may have appeared in an earlier part of the question, in which case credit was given for simply writing $\mathrm{p}(n)=3n^3+6n$ here.
Most students who found this expression for p(n) legitimately were able to deduce that since p(n) was a multiple of 9 then the printed expression was a multiple of 3.

Question 8

(a) Many students seemed unable to find the correct argument of the complex number. Students would be well advised to make a small sketch of the Argand diagram in order to determine the correct quadrant for the angle θ .

(b)(i) An incorrect value of θ in part **(a)** prevented a large number of students from obtaining the correct roots of the cubic equation. Most students had the correct modulus and realised the need to divide an argument by 3, but did not always apply de Moivre's Theorem correctly.

(b)(ii) It was possible for students to score full marks here if they had the correct modulus of the 3 roots, provided they recognised that the points P, Q and R were the vertices of an equilateral triangle. Poor trigonometrical skills prevented most students from obtaining the correct value for the area of the triangle.

(c) It was necessary to state explicitly that the sum of the roots of the cubic equation was zero. By considering the sum of the roots and then taking real parts of both sides of the equation the printed result followed. Better students stated explicitly that $\cos\left(-\frac{4\pi}{9}\right)=\cos\left(\frac{4\pi}{9}\right)$ at some stage in their proof.

Mark Ranges and Award of Grades
Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website: http://www.aqa.org.uk/over/stat.html

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Convert raw marks into Uniform Mark Scale (UMS) marks by using the link below.

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