# GCE

# MATHEMATICS

## MFP2 Further Pure 2
## Report on the Examination

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## General

The paper seemed to provide a challenge for the very able students whilst at the same time allowing weaker students to show their basic understanding of particular topics such as complex numbers, properties of roots of cubic equations and hyperbolic functions. Whenever an explanation was required, the responses were usually quite poor. Proofs very rarely had a concluding statement and the presentation of solutions from a significant number of students fell short of the standard expected for this level of examination.
A slipshod approach was seen by many who omitted $\mathrm{d}x$ etc from their integrals and who confused the limits when the method of integration by substitution was used.

## Question 1

(a) Almost everyone managed to draw a circle for the locus with the vast majority realizing that the centre was at $6i$. Some drew a circle with its centre below the real axis and a few thought that the centre lay on the real axis. It was necessary to indicate that the radius of the circle was $3$ and most did this by showing the values where the circle cut the imaginary axis. Others indicated the radius of $3$ in a variety of ways but quite a few neglected to make this clear to examiners.

(b) (i) Several students wrote $9i$ instead of $9$ as the maximum value of the modulus of $z$.
It was evident that some had no idea what the question was really asking.

(ii) Most students realised the need to draw a tangent to the circle but weaker students could not cope with the basic trigonometry required to find a relevant angle on their diagram. Nevertheless a large number had no problem in finding the correct maximum value of the argument of $z$.

## Question 2

(a)(i) The graphs of $\sinh x$ and $\cosh x$ usually had the correct basic shape; however, some drew the $\sinh x$ curve with vertical asymptotes or with a zero gradient at the origin and several students failed to indicate that the minimum point of
$y = \cosh \mathrm{x}$ was at $(0,1)$.

(ii) Very few students gave a satisfactory explanation here with many referring to the two graphs drawn in part (a)(i) having a "single point of intersection", which is not true.

It was necessary to state that the equation $\cosh x = 0$ has no solutions and that, by considering the graph of $\sinh x$, the equation $\sinh x = -k$ has exactly one solution. It was good to see students referring to $\sinh x$ as being a one-one-function, but many failed to write a suitable conclusion with regard to the given equation having a single solution.

(b) Those who differentiated the given expression for $y$ usually obtained the correct derivative. Some tried to use the double angle formula for $\cosh 2x$ but, if they did this incorrectly, then they were not eligible for full marks even if they obtained the correct expression $6\cosh x + \sinh 2x$. Some used the result from part (a)(ii) to deduce that $C$ has only one stationary point. Others reinvented the wheel by stating that $\cosh x \neq 0$ and then showed that $\sinh x = -3$, before concluding that the curve had a single stationary point. The vast majority failed to make any statement of this form and lost a mark. Those who tried to convert into exponential form usually made no progress beyond the first two marks for differentiation. Having found that $\sinh x = -3$, it was expected that students would have shown that $\cosh^2 x = 10$ by using the identity $\cosh^2 x = 1 + \sinh^2 x$ , but many were very dependent on their calculator and some did not provide enough working to convince the examiners that the *y*-coordinate was an integer. At least one student solved the problem effectively by writing the equation of the curve as $y = (\sinh x + 3)^2 - 8$.

## Question 3

Some students had been well taught in the setting out of proofs by induction; others sadly floundered as if they were trying to remember the words of a song but could only hum the tune. Several students tried **adding** the next term instead of realizing that this proof involved the $n$th term of a sequence. Quite a few weaker students, who realised what they wanted to achieve, were defeated by the double quotient and could not cope with the algebra. Many who wrote $\frac{3+1}{3-1} = \frac{4}{2} = 2$ failed to make a further statement such as "the formula is true when $n=1$" . Those students who use expressions such as "$P(k)$ is true" are well advised to define what they mean by the proposition $\mathrm{P}(k)$. Several students who set out their proof quite well spoiled their solution by writing "therefore true for all $n \geq 1$ ".

## Question 4

(a) This part of the question required stamina in expanding then simplifying the expression for $\mathrm{f}(r) - \mathrm{f}(r-1)$. Some looked in vain for a common factor such as $(2r-1)$. Those who made careful use of brackets and who set out their solution neatly were usually successful in showing that the resulting expression was equivalent to $(2r-1)^3$.

(b) The method of differences was generally used effectively. Some realised that the sum reduced to $\mathrm{f}(2n) - \mathrm{f}(n)$. Others used the method of differences on two separate series and also obtained $(2n)^2(2(2n)^2 - 1) - n^2(2n^2 - 1)$ . Quite a few students did not use brackets and failed to obtain this precise expression. This prevented some from obtaining the printed answer without fudging their work.

## Question 5

This question was answered much better than similar questions in recent years, with very few making sign errors in either the sum or the product of the roots.

(a) (i) Almost everyone wrote down the correct product of the roots as a complex number. Very few forgot the associated minus sign.

(ii) The printed answer clearly helped with the product $(-2+3\mathrm{i})(1+2\mathrm{i})$ causing few problems. No doubt many did this calculation on a calculator and so in future questions involving algebra rather than simply numbers might be set.

(iii) Some used simultaneous equations to find the real and imaginary parts of the complex number, $\alpha$ . Others wrote $\alpha = \dfrac{37-3\mathrm{i}}{8+\mathrm{i}}$ and many were able to write this in the correct form using a calculator. It had been expected that students would have shown their working when evaluating $\dfrac{37-3\mathrm{i}}{8+\mathrm{i}} \times \dfrac{8-\mathrm{i}}{8-\mathrm{i}}$. The correct answer was obtained by the majority of students.

(b) Those who had the correct value of $\alpha$ were usually successful in finding the value of $p$.
A small minority wrote $p = 3$, sometimes after obtaining $-p = 3$ .

(c) Some careless arithmetic was seen in this part but most students found the correct value of $q$. The majority found the sum of three separate products and a few realised that they could use the value of $\beta\gamma$ found earlier and evaluated an expression of the form $\beta\gamma + \alpha(\beta + \gamma)$ , which was slightly more efficient.

## Question 6

a) Almost everyone managed to use the correct exponential forms for $\cosh x$ and $\sinh x$ and simplified their expression to the form on the right hand side of the identity. However, in order to score full marks, it was necessary to show the expression on the left hand side of the identity at least on the first line of working. The best students wrote an appropriate conclusion such as " therefore $\dfrac{1}{5\cosh x - 3\sinh x} = \dfrac{\mathrm{e}^x}{4+\mathrm{e}^{2x}}$ ". Practically every student showed that $m$ = 4.

(b) Most students carried out the first stage of the substitution process, differentiating $\mathrm{e}^x$ , in order to transform the integral. The majority obtained an integral of the form $\int \dfrac{1}{4+u^2}\,\mathrm{d}u$ although quite a few omitted the d$u$ and this was penalised. Others retained the limits for $x$ on the integral with respect to $u$ and this was penalised also. The vast majority did the integration well and obtained the printed answer. Students must realise that when they are asked to obtain a printed answer then they must include vital steps such as writing the value of the integral as $\frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}\frac{1}{2}$ before writing down the printed answer.

## Question 7

(a)(i) The differentiation caused more difficulties than had been anticipated. Many students did not use the product rule on the first expression and those who did rarely had both terms correct. Even the derivative of $\sinh^{-1} 2u$ was done badly with many omitting the "2" in the numerator.

(ii) Those who did not obtain $k = 4$ were still able to score full marks here if they used their value of $k$ or even "$k$" in their anti-differentiation.

(b)(i) Students needed to show the correct value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ before substituting into the formula for the curved surface area in order to score full marks. Despite the printed answer, quite a few omitted the limits or the d$x$ or failed to show the product $2\times\frac{1}{2}$ in their working and lost an easy accuracy mark.

(ii) Once again the integration by substitution caused problems for many who omitted the factor $\frac{\pi}{4}$ or who did not have the correct limits for $u$. Because of errors in finding the value of $k$ in part (a)(i) only the best students scored full marks here. Some credit was given for using their previous results correctly even though students did not have the correct value of $k$.

## Question 8

(a)(i) It had been expected that students would have written a correct identity using de Moivre's theorem at the beginning of their solution, but only the best students wrote $(\cos\theta + \mathrm{i}\sin\theta)^4 = \cos 4\theta + \mathrm{i}\sin 4\theta$ as a starting point. The binomial expansion was usually correct, although many did not use brackets correctly to indicate powers of i. The printed answer for $\cos 4\theta$ helped many to recover from sign errors in their expansion, though at least one " i " was often left in the expression offered for $\sin 4\theta$.

(ii) Most students wrote down an expression for $\tan 4\theta$ by writing the quotient of their previous results for $\sin 4\theta$ and $\cos 4\theta$. Many students failed to justify the leap from this expression to the printed expression involving $\tan\theta$. It was necessary to indicate that the numerator and denominator had been divided throughout by $\cos^4\theta$. Nevertheless, the majority of students scored full marks for both sections of part (a) and the printed answers clearly helped.

(b) The crux of the proof was in realizing that $\tan 4\theta = 1$ when $\theta = \frac{\pi}{16}$ so as to obtain the given quartic equation in $t$. Very few were able to give a convincing reason as to why $\tan\frac{\pi}{16}$ was a root of the equation in $t$. No doubt influenced by the printed solution in the final part of the question, most gave their values of the other roots as $\tan\frac{3\pi}{16}, \tan\frac{5\pi}{16}$ and $\tan\frac{7\pi}{16}$ instead of the correct expressions involving trigonometric functions.

(c) It was necessary for students to state explicitly that $\tan\frac{9\pi}{16} = -\tan\frac{7\pi}{16}$ and that $\tan\frac{13\pi}{16} = -\tan\frac{3\pi}{16}$ in order to score full marks. It was also necessary for examiners to see the sum of the roots written as $-4$ and the sum of the products of pairs of roots equal to $-6$ whilst showing that the sum of the squares of the roots was equal to $28$. Very few students scored full marks in this part of the question which was targeted at the very best students. Either the question had proved very difficult or students had run out of time at this stage of the examination.

## Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.

## Converting Marks into UMS marks

Convert raw marks into Uniform Mark Scale (UMS) marks by using the link below.

**UMS conversion calculator** www.aqa.org.uk/umsconversion