GCE
MATHEMATICS
MFP3 Further Pure 3
Report on the Examination

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General

Students appeared to be well prepared for the examination and they were able to tackle all that they could do without there being any apparent evidence of shortage of time.

Teachers may wish to emphasise the following points to their students in preparation for future examinations in this unit:

- Differentiating $y\frac{\mathrm{d}y}{\mathrm{d}x}$ with respect to x gives $y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ which is NOT the same as $y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. (See Q6(b))
- When finding a particular integral the coefficients of each of the terms in the trial must be found. (See Question 7(b))
- The polar coordinates of points on a curve, whose polar equation and relevant domain are given, must be within the domain stated. (See Question 8(a))

Question 1

This question on numerical solutions of first order differential equations again proved to be a good source of marks for many students. Almost all students found the correct value for k_1 but a significant minority did not get the correct value for k_2. Such students generally either scored a total of 1 mark or 3 marks for the question. The greater award was normally given to those who showed a correct substitution into the formula for k_2, for example $0.2\times(2.2-1-0.346...)\sqrt{2.2+1+0.346...}$, before stating and using the wrong value for k_2 in the improved Euler formula. Those who just stated and used the wrong value of k_2 did not score these extra two marks. Almost all students gave their final answer to the specified degree of accuracy.

Question 2

This question, which tested the relationship between polar and Cartesian coordinates, was very well answered with a high proportion of students scoring full marks. Less able students generally equated r at some stage to 10, the radius of the circle, and rarely scored more than 1 mark for the whole question.

Question 3

This question, which tested the solution of a second order differential equation with the form of the particular integral given, was a good source of marks for many students; especially before the boundary conditions in part (c) became part of the test. The majority of students found the correct values for the constants *a*, *b* and *c* in part (a) but sign errors were sometimes seen; the most common being to equate $-4c$ to 8 instead of -8 from consideration of the coefficients of e^{-3x}. In part (b), most students showed that they knew how to find the complementary function and then the general solution. Part (c) caused students more problems than the previous two parts. Although most students scored the mark for correctly dealing with the given boundary condition at $x=0$, a significant minority did not realise that the coefficient of e^x, in the expression for $\frac{dy}{dx}$, had to be 0 for the boundary condition $\frac{dy}{dx} \to -1$ as $x \to \infty$ to be satisfied. As mentioned in previous reports, the examiners expect students to give special attention to certain limits that are within the specification. In this question, the limit of xe^{-3x} as $x \to \infty$ needed to be treated in isolation, and this was not considered by a significant number of students.

Question 4

This question asked students to evaluate an improper integral, showing the limiting process used. The majority of students correctly equated the given integral to $\lim_{a \to \infty} \int_0^a \left(\frac{2x}{x^2+4} - \frac{4}{2x+3} \right) dx$ and most students integrated correctly to obtain the difference of two logarithms although an inverse trigonometric function was seen occasionally. A significant minority of students then dealt with the upper limit incorrectly by considering each logarithmic term separately and stating that each was ∞ so the limit of their difference was 0. More able students combined the two terms into a single log term, substituted the limits and reached $\lim_{a \to \infty} \left[\ln\left(\frac{a^2+4}{(2a+3)^2} \right) \right] - \ln\left(\frac{4}{9} \right)$. Further work, which was required on the first term before the limit taken, was not always shown. Those students who squared the denominator and then divided each term in the relevant fraction by a^2 to obtain $\lim_{a \to \infty} \left[\ln\left(\frac{1+\frac{4}{a^2}}{4+\frac{12}{a}+\frac{9}{a^2}} \right) \right] - \ln\left(\frac{4}{9} \right)$ usually obtained the correct answer convincingly.

Question 5

This question, which tested use of an integrating factor to solve a first-order differential equation, was a good source of marks for many students. The first two parts of the question were usually answered correctly and almost all students correctly reached the stage $y\ln x = \int 9x^2 \ln x \, \mathrm{d}x$ in part (b)(ii). A large majority of students used integration by parts correctly but a small number forgot the '9' at this stage and so lost the two remaining accuracy marks. Very few failed to include the '$+c$' and almost all of those students who had scored the method mark for their integration also scored the method mark for correctly using the given boundary condition. Surprisingly there was a minority of students who failed to score the final mark due to a miscopy of an earlier correct expression when substituting their value for c, as illustrated by $y \ln x = 3x^2 \ln x - x^3 + 2\mathrm{e}^3$.

Question 6

This question, which tested differentiation and subsequent application of Maclaurin's theorem, proved to be a challenge to a significant number of students in part (b). Most students answered part (a) correctly with the majority using the chain rule and only a minority finding and differentiating y^2 to reach $2y\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x$. In part (b) the majority of students attempted to find the third derivative by differentiating $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(4+\sin x)^{-1/2}(\cos x)$ twice. There were different forms presented with many mixtures of terms in x and y but a significant proportion of students gave incorrect expressions for the third derivative. Some students obtained the correct expression for $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ but then did not attempt to differentiate the term $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{1}{2}(4+\sin x)^{-1/2}(-\sin x)$ when finding the value of $\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}$ when x=0, presumably believing it would be zero since $\frac{1}{2}(4+\sin x)^{-1/2}(-\sin x)$ itself is 0 when x=0. A significant proportion of those students who used the implicit differentiation method made the same type of error as has been indicated in recent examiners' reports, namely, differentiating $y\frac{\mathrm{d}y}{\mathrm{d}x}$ correctly as $y\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ but then writing this as $y\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ or, more costly, just writing the incorrect form. In part (c) many students scored the method mark but the final accuracy mark depended on correct earlier work and so was frequently not awarded.

Question 7

This question on transforming and solving a second-order differential equation caused the usual problems but there were also some excellent solutions from some students. Part (a) did not have the structure that has been given in questions in some earlier series on this topic but it was pleasing to see a majority of students correctly differentiating $u\sin x$ twice although, as expected, less able students generally could not overcome this hurdle. Those who substituted the differentials to obtain a correct equation in u and x sometimes made a sign error in their simplification of the equation and then invariably made a second error, possibly deliberately, to 'obtain' the printed equation. Students had to convince examiners that a correct relevant trigonometric identity had been used in any rearrangement to reach the printed form. In part (b), most students realised that $\frac{d^2u}{dx^2}+u=\sin 2x$ had to be solved but not all realised that the solution would be of the form $u=g(x)$. Perhaps surprisingly a significant minority formed an incorrect auxiliary equation which led to real roots. In finding the particular integral, relatively few students realised that its form was simply $\alpha\sin 2x$ and the majority started with the form $u=\alpha\sin 2x+b\cos 2x$. This was perfectly acceptable provided the student produced sufficient work to show that the coefficient of $\cos 2x$ was zero. Those students who found the correct general solution for u normally went on to find the correct expression for y.

Question 8

This question on polar coordinates produced some excellent solutions, particularly in the final part of the question. Most students scored at least two of the three marks in part (a), but a significant proportion gave a θ-coordinate that was outside the stated interval. There were many students who gave the correct polar coordinates of A in part (b)(i). In part (b)(ii) the most common correct methods for finding the length of AQ were either to use the cosine rule in triangle AOQ or use of the Cartesian coordinates of the points A and Q. A significant number of students failed to gain credit because they just assumed that angle AQO was a right angle and then used Pythagoras to find AQ, some then claiming in the next part that since Pythagoras was true the angle was 90º and so the line was a tangent. Those students who made better attempts at explaining why AQ was a tangent sometimes failed to score both marks because they either did not show sufficient detail or did not explicitly identify the relevant angle within their explanations. In part (c) many students showed that they knew the method for finding the area of the minor region OPQ of the curve. A significant number of students realised that the minor sector OPQ of the circle was required but many good attempts at combining these results were spoilt by an incorrect sign at some stage, normally in one of the surd forms for a trigonometric term involving $\frac{7}{6}\pi$ or $\frac{11}{6}\pi$.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.

Converting Marks into UMS marks

Convert raw marks into Uniform Mark Scale (UMS) marks by using the link below.

UMS conversion calculator www.aqa.org.uk/umsconversion