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| For Examiner's Use | |
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General Certificate of Education
Advanced Level Examination
June 2013

# Mathematics MFP4

## Unit Further Pure 4

**Tuesday 18 June 2013 9.00 am to 10.30 am**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Time allowed
- 1 hour 30 minutes

### Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

### Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

**1** The points $A$, $B$, $C$ and $D$ have position vectors **a**, **b**, **c** and **d** respectively relative to the origin $O$, where

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$$

**(a)** Find $\overrightarrow{AB} \times \overrightarrow{AC}$. *(3 marks)*

**(b)** The points $A$, $B$ and $C$ lie in the plane $\Pi$. Find a Cartesian equation for $\Pi$. *(2 marks)*

**(c)** Find the volume of the parallelepiped defined by $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$. *(3 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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**2** The system of equations

$$\begin{aligned} 2x - \phantom{2}y - \phantom{3}z &= 3 \\ x + 2y - 3z &= 4 \\ 2x + \phantom{2}y + az &= b \end{aligned}$$

does not have a unique solution.

**(a)** Show that $a = -3$. *(3 marks)*

**(b)** Given further that the equations are inconsistent, find the possible values of $b$. *(2 marks)*

**(c)** State, with a reason, whether the vectors $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ -3 \\ -3 \end{bmatrix}$ are linearly dependent or linearly independent. *(1 mark)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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**3** The determinant $\Delta$ is given by

$$\Delta = \begin{vmatrix} x^2 - x & y^2 - y & z^2 - z \\ x & y & z \\ x^2 + y^2 + z^2 & x^2 + y^2 + z^2 & x^2 + y^2 + z^2 \end{vmatrix}$$

where $x$, $y$ and $z$ are distinct real numbers.

**(a)** Express $\Delta$ as a product of one quadratic factor and three linear factors. *(6 marks)*

**(b)** Deduce that $\Delta \neq 0$. *(2 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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**4** Two planes have equations

$$2x - 2y + z = 24$$

and

$$x + 3y + 4z = 8$$

They meet in a line $L$.

**(a)** Find Cartesian equations for the line $L$. *(5 marks)*

**(b)** The direction cosines of the line $L$ are given by $\cos\alpha$, $\cos\beta$ and $\cos\gamma$.

**(i)** Find the exact value of each of the direction cosines. *(2 marks)*

**(ii)** Show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$. *(3 marks)*

QUESTION PART REFERENCE

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**5** The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$$

**(a)** Show that $\lambda = 2$ is an eigenvalue for $\mathbf{M}$, and find the other two eigenvalues. *(5 marks)*

**(b)** Find an eigenvector that corresponds to $\lambda = 2$. *(3 marks)*

**(c)** The matrix $\mathbf{N}$ is given by

$$\mathbf{N} = \begin{bmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{bmatrix}$$

**(i)** Show that $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector for $\mathbf{N}$, and find the corresponding eigenvalue. *(2 marks)*

**(ii)** Hence state one eigenvector for the matrix $\mathbf{MN}$, and find the corresponding eigenvalue. *(3 marks)*

QUESTION PART REFERENCE

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**6** The plane transformation T is defined by

$$\mathrm{T}: \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

**(a)** A shape has an area of 3 square units. Find the area of the shape after being transformed by T. *(2 marks)*

**(b) (i)** Find the equations of all the invariant lines of T. *(5 marks)*

**(ii)** State the equation of the line of invariant points of T. *(1 mark)*

QUESTION PART REFERENCE

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**7** The $3 \times 3$ matrices $\mathbf{A}$ and $\mathbf{B}$ satisfy

$$\mathbf{AB} = \begin{bmatrix} k & 8 & 1 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix}, \text{ where } \mathbf{A} = \begin{bmatrix} k & 6 & 8 \\ 0 & 1 & 2 \\ -3 & 4 & 8 \end{bmatrix}$$

and $k$ is a constant.

**(a)** Show that $\mathbf{AB}$ is non-singular. *(1 mark)*

**(b)** Find $(\mathbf{AB})^{-1}$ in terms of $k$. *(5 marks)*

**(c)** Find $\mathbf{B}^{-1}$. *(4 marks)*

QUESTION PART REFERENCE

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**8** A line and a plane have equations

$$\frac{x-3}{p} = \frac{y-q}{3} = \frac{z-1}{-1}$$

and $$\mathbf{r} \,.\, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 10$$

respectively, where $p$ and $q$ are constants.

**(a)** Show that the line is **not** perpendicular to the plane. *(1 mark)*

**(b)** In the case where the line lies in the plane, find the values of $p$ and $q$. *(4 marks)*

**(c)** In the case where the angle, $\theta$, between the line and the plane satisfies $\sin\theta = \frac{1}{\sqrt{6}}$, and the line intersects the plane at $z = 2$:

**(i)** find the value of $p$; *(5 marks)*

**(ii)** find the value of $q$. *(2 marks)*

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**END OF QUESTIONS**