A-LEVEL

Mathematics

MFP2 – Further Pure2

Report on the Examination

6360

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General

The paper seemed to provide a challenge for able students whilst at the same time allowing weaker students to demonstrate their basic understanding of particular topics such as properties of roots of cubic equations, loci of complex numbers, and hyperbolic functions. Proofs were very rarely set out in a logical manner with a concluding statement and the presentation of solutions from a significant number of students fell short of the standard expected for this level of examination. This careless approach was seen by many who omitted $\mathrm{d}x$ etc from their integrals and who confused the limits when the method of integration by substitution was used. In fact, many weaker students seemed unaware of the calculus techniques from core mathematics, in particular the quotient rule, integration of trigonometric functions and the important steps in doing integration by substitution.

Question 1

This was intended as an easy starter but it was not answered well by a large number of students.

(a) Almost everyone managed to find the correct value of r, although a few ignored the instruction that *r* was positive. An alarming number of students failed to give the correct value of θ in the given interval and a very common incorrect answer was $\theta = \frac{\pi}{2}$.

(b) Students were expected to simplify $9^{\frac{1}{4}}$ to $\sqrt{3}$ and although the general idea of dividing the argument by 4 was well known, only the best students seemed capable of finding the correct four roots of the equation $-z^4 + 9\mathrm{i} = 0$ in the requested form.

Question 2

(a) Some students drew their half line from $-2\mathrm{i}$, but the majority draw a half line in the second quadrant of the Argand diagram from $2\mathrm{i}$. Some were rather casual in their sketch and the angle between the half line and the imaginary axis was too close to $\frac{\pi}{4}$ to be worthy of full marks.

(b)(i) Some students drew a circle passing through $2\mathrm{i}$ rather than having a circle which touched $\mathrm{Im}(-z) = 2$. Acceptable sketches usually consisted of freehand circles which intersected the positive imaginary axis twice.

(ii) Those with a clear diagram could almost write down the correct value of *b*. In order to find the value of *a*, some used the equation $\frac{b-2}{a} = -\sqrt{3}$ whereas the majority wrote down a correct trigonometric expression for the distance of the centre from the imaginary axis and deduced that *a* was equal to $-\sqrt{3}$.

Question 3

(a) Almost every student was able to simplify the expression correctly.

(b) Most students had been taught well how to set out a proof by induction; others simply went through the motions and their poor algebraic skills caused problems.
Many who realised that the two different sides of the printed equation gave the value 2 when $n = 1$ failed to make a concluding statement such as "the formula is true when $n = 1$" . Many of those who made a good attempt at the algebra on the right hand side of the equation failed to consider the left hand side of the formula. Those students who use expressions such as "$\mathrm{P}(k)$ is true" are well advised to define what they mean by the proposition $\mathrm{P}(k)$. There was some confusion in that students also used $\mathrm{P}(n)$ to refer to both the value of the summation. Several students who set out their proof quite well spoiled their solution by writing "therefore result is true for all $n \geq 1$ ".

Question 4

Most students seemed well drilled in the topic of roots of the cubic equation and for the majority of students this was their highest scoring question.

(a)(i) There were a few sign errors in writing down the sum of the roots but on the whole the vast majority scored full marks in this part.

(ii) The identity for the sum of the squares of the roots was well known. Some students did not show the evaluation of the various terms and those who simply wrote $(-2)^2 - 2\times 3 = -2$ did not earn full marks since the answer was given.

(b) This part of the question was a little more challenging and required students to derive appropriate identities rather than simply memorising results. A common error in part **(b)(i)** was to write the expanded expression as $\alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\lambda + \gamma\alpha)$ and no credit was given for substituting values unless the initial expression was correct.

Part **(b)(ii)** proved even more challenging and most students only picked up a single mark for recognising that $\alpha\beta\gamma = 4$. However it was interesting to see many correct approaches to this part of the question from those who had the necessary algebraic skills to derive an appropriate identity.

(c) Those who had the correct results in part **(b)** usually had no problem in obtaining the correct cubic equation, although some lost a precious mark by failing to include "$= 0$".

Question 5

(a) Far too many treated this fairly standard proof too casually. Students were expected to present a logical proof of the identity, usually starting with $4\sinh^3\theta + 3\sinh\theta$ being expressed correctly in terms of exponentials and concluding with "$= \sinh 3\theta$". The minimum expected was a series of trailing equals signs from line to line but the best students concluded with a statement of the identity they had proved. Some decided to expand $\sinh(2\theta + \theta)$ but made no reference to the definition in terms of exponentials and scored no marks. Those who failed to write down the left hand side of the identity could not expect to earn full marks.

(b) Most students quickly linked the expression in x with the identity in part **(a)** and correctly wrote down that $4\sinh 3\theta = 3$. Some then made heavy weather of solving this equation by converting to exponentials rather than simply using the expression for $\sinh^{-1}\frac{3}{4}$ from the formula book or from memory.

(c) This final part of the question resulted in a very pleasing result for those students who were competent in handling logarithms. Quite a few could not simplify $e^{-\theta}$ as a power of 2 when $\theta = \frac{1}{3}\ln 2$.

Question 6

(a) (i) The result was fairly familiar to most students but many struggled to prove that $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$. Some simply assumed the result and others lost a mark for omitting brackets, with many writing $\cos - n\theta$, for example, instead of $\cos(-n\theta)$.

(ii) The even more familiar result in this part allowed most students to write down immediately that $z^n + z^{-n} = 2\cos n\theta$.

(b) (i) Some ignored the instruction to expand the expression in terms of *z* and poor algebraic skills prevented many from obtaining the correct result. Those who simply squared $(z^2 - z^{-2})$ were usually more successful than those who tried to multiply out a pair of quadratic expressions.

(ii) Some students ignored the instruction "Hence" and tried to obtain the result using double angle formulae but no marks were earned for this approach. Quite a few simply assumed that the left hand side was equivalent to the expression in part **(b)(i)** and so incorrectly assumed that the right hand side was equal to $2\cos 4\theta - 2$.

(c) It was alarming to see so many poor attempts at integration by substitution; weaker students made no attempt to differentiate and omitted dx completely; many students wrote down $\frac{dx}{d\theta} = 2\cos\theta$ but then replaced d*x* by $\frac{d\theta}{2\cos\theta}$ in the integral and consequently saw no connection with the earlier result. Many others had a correct unsimplified version of the integral in terms of θ but assumed that it must simplify to $8\sin^2\theta\cos^2\theta$ in order to use their result from part **(b)(ii).** Only the best students were able to obtain the correct value of the integral and some of these were penalised for using incorrect limits or omitting $d\theta$ throughout their working.

Question 7

(a) The differentiation caused more difficulties than had been anticipated. Many students did not use the quotient rule or product rule on the expression $\frac{1+x}{1-x}$; others could not apply the result from the formulae book for the derivative of $\tan^{-1}u$ in their chain rule attempt. Some used implicit differentiation efficiently to prove the given result whereas others expanded the expression to obtain equations such as $\tan y - x\tan y = 1 + x$ before differentiating implicitly. Many were successful in using this approach even though the working sometimes spanned two pages.

(b) It was necessary to realise that both $\tan^{-1}x$ and $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ had the same derivative in order to make any progress. Once students made this breakthrough it was obvious that the expression $\tan^{-1}\left(\frac{1+x}{1-x}\right) - \tan^{-1}x$ was constant when $x<1$, and by substituting $x = 0$ the printed result immediately followed. However, only the very best students scored full marks in this part.

Question 8

(a) All students should have been able to score full marks in deriving the expression for the arc length but, once again, carelessness in omitting the limits or missing out dx on the final integral deprived quite a few of a precious mark. Those who failed to write $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ as $\frac{1}{x-1}$ at some stage did not convince examiners that they had proved the given result.

(b) Most students wrote $\cosh^{-1}3 = \ln(3+\sqrt{8})$ but it was necessary to show clearly that $\left(3+\sqrt{8}\right) = \left(1+\sqrt{2}\right)^2$ in order to earn full marks for the proof.

(c) Once more there were some very poor attempts at the integration by substitution. A number of students did not feel comfortable differentiating $\cosh^2\theta$ and so tried to use double angle formulae to obtain an easier function to differentiate. Some were successful in taking this approach and others floundered. Once the resulting integrand was of the form $2\cosh^2\theta$ most students were able to make good progress towards the printed answer. The limits were tricky but many persevered by using the result from part **(b)(i)** and obtained an answer of the given form. There was some fudging at this stage and examiners had to be convinced that all working was correct in order to award full marks.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.

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UMS conversion calculator www.aqa.org.uk/umsconversion