Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MFP3

Unit Further Pure 3

Monday 19 May 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 It is given that $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \frac{\ln(x + y)}{\ln y}$$

and $$y(6) = 3$$

Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.4$, to obtain an approximation to $y(6.4)$, giving your answer to three decimal places.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 1

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QUESTION PART REFERENCE

Answer space for question 1

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2 (a) Find the values of the constants a, b and c for which $a + b\sin 2x + c\cos 2x$ is a particular integral of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = 20 - 20\cos 2x$$

[4 marks]

(b) Hence find the solution of this differential equation, given that $y = 4$ when $x = 0$.

[4 marks]

QUESTION PART REFERENCE

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3 A curve has polar equation $r(4 - 3\cos\theta) = 4$. Find its Cartesian equation in the form $y^2 = \mathrm{f}(x)$.

[4 marks]

QUESTION PART REFERENCE

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4 Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{-x}$$

given that $y \to 0$ as $x \to \infty$ and that $\frac{dy}{dx} = -3$ when $x = 0$.

[10 marks]

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5 (a) Find $\int x \cos 8x \, dx$.

[3 marks]

(b) Find $\lim_{x \to 0} \left[\frac{1}{x} \sin 2x\right]$.

[2 marks]

(c) Explain why $\int_0^{\frac{\pi}{4}} \left(2 \cot 2x - \frac{1}{x} + x \cos 8x\right) dx$ is an improper integral.

[1 mark]

(d) Evaluate $\int_0^{\frac{\pi}{4}} \left(2 \cot 2x - \frac{1}{x} + x \cos 8x\right) dx$, showing the limiting process used. Give your answer as a single term.

[4 marks]

QUESTION PART REFERENCE

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6 (a) By using an integrating factor, find the general solution of the differential equation

$$\frac{\mathrm{d}u}{\mathrm{d}x} - \frac{2x}{x^2+4}u = 3(x^2+4)$$

giving your answer in the form $u = \mathrm{f}(x)$.

[6 marks]

(b) Show that the substitution $u = x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ transforms the differential equation

$$x^2(x^2+4)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 8x\frac{\mathrm{d}y}{\mathrm{d}x} = 3(x^2+4)^2$$

into

$$\frac{\mathrm{d}u}{\mathrm{d}x} - \frac{2x}{x^2+4}u = 3(x^2+4)$$

[4 marks]

(c) Hence, given that $x > 0$, find the general solution of the differential equation

$$x^2(x^2+4)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 8x\frac{\mathrm{d}y}{\mathrm{d}x} = 3(x^2+4)^2$$

[2 marks]

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7 (a) It is given that $y = \ln(\cos x + \sin x)$.

(i) Show that $\dfrac{d^2y}{dx^2} = -\dfrac{2}{1 + \sin 2x}$.

[4 marks]

(ii) Find $\dfrac{d^3y}{dx^3}$.

[1 mark]

(b) (i) Hence use Maclaurin's theorem to show that the first three non-zero terms in the expansion, in ascending powers of x, of $\ln(\cos x + \sin x)$ are $x - x^2 + \frac{2}{3}x^3$.

[3 marks]

(ii) Write down the first three non-zero terms in the expansion, in ascending powers of x, of $\ln(\cos x - \sin x)$.

[1 mark]

(c) **Hence** find the first three non-zero terms in the expansion, in ascending powers of x, of $\ln\left(\dfrac{\cos 2x}{e^{3x-1}}\right)$.

[4 marks]

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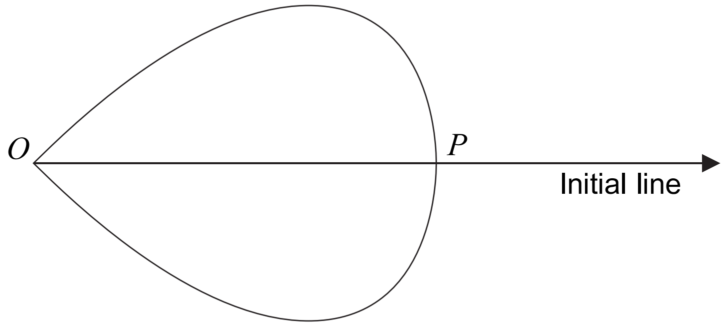
8 The diagram shows a sketch of a curve C, the pole O and the initial line. The curve C intersects the initial line at the point P.



The polar equation of C is $r = (1 - \tan^2\theta)\sec\theta$, $-\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$.

(a) Show that the area of the region bounded by the curve C is $\frac{8}{15}$.

[5 marks]

(b) The curve whose polar equation is

$$r = \frac{1}{2}\sec^3\theta, \quad -\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$$

intersects C at the points A and B.

(i) Find the polar coordinates of A and B.

[3 marks]

(ii) Given that angle $OAP =$ angle $OBP = \alpha$, show that $\tan\alpha = k\sqrt{3}$, where k is an integer.

[4 marks]

(iii) Using your value of k from part **(b)(ii)**, state whether the point A lies inside or lies outside the circle whose diameter is OP. Give a reason for your answer.

[1 mark]

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END OF QUESTIONS