A-LEVEL

Mathematics

MFP4 – Further Pure 4
Report on the Examination

6360
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General

All questions produced a wide range of responses from students. This was even true of the earlier more straightforward questions, in particular questions 1 and 5. Questions 4, 6 and 8 proved to be the most demanding of all. The best scripts were those where students had provided full working and clear explanation. Students must realise that method marks will not be awarded if full working out is not shown. Furthermore, marks were sometimes lost for not rounding answers to the appropriate degree as specified in the question.

Question 1

This question was meant to be an easy starter. Almost every student identified the correct axis for part (a). However it was surprising to see so many students give the answer to part (b) as 53°, particularly when the relevant matrix is provided in the formula book. Full marks could only be awarded for students who had specified correct values for both $\cos\theta$ and $\sin\theta$ before using inverse trigonometry to find the correct angle.

Question 2

This topic area still proves to be challenging for a significant number of students. The key test here is to use row and column operations to be able to find and extract factors. Some students do not help themselves by failing to state the row and column operations used. It is important for students to realise that marks will only be awarded for correct working at all stages, so making errors when extracting the correct factor will be heavily penalised. A small number of students try to avoid extracting factors by expanding directly or after creating two zeros; this approach is rarely successful as errors occur with signs. Many score zero marks because of such errors.

The second part proved more successful but whilst most students knew of and were able to apply the rule $\det \mathbf{AB} = \det \mathbf{A} \times \det \mathbf{B}$ it would appear that the rule $\det \mathbf{B}^{-1} = \dfrac{\mathbf{1}}{\det \mathbf{B}}$ is less well known. A number of students were not able to simplify the final algebraic fraction correctly.

Question 3

There was a good response to this question. Expansion of determinants in part a) appears to be well understood. Students need to realise that it is important to demonstrate they know what non-singular means by commenting on their final answer and mentioning that it is non-zero. Many students gave excellent answers by referring to there being no k's left and stating that the final answer could never equal zero.

Many students were able to use the appropriate algorithm to find a valid inverse matrix. It would help if students became clearer about the terminology used; the matrix of minors, matrix of cofactors and the adjoint matrix were often confused. A few students thought that the cofactor matrix was the inverse matrix and lost the final two marks.

In part c) there was a significant number of students who did not know how to use the inverse matrix to solve a system of equations. On this occasion any other attempted method scored zero marks as this was a specific request to test the understanding of solving equations using the inverse matrix. Algebraic skills were good and final answers were simplified fully in most cases.

Question 4

A significant number of students were unable to make any progress with this question and it does suggest that understanding of algebraic properties of the vector product are not well understood. The key was to rearrange and factorise the expression to create a zero vector and then consider the implications of the result. Although a number of students scored 0 or 1 mark it was encouraging to see many scores of 2 and 3 marks. Improving students ability to write out a proof should be a key development area.

Question 5

Students were very successful on this question and it was clear that students were helped by the structure of the question, particularly with the "Show that.." in part (b). Part (a) was excellent with almost every student scoring both marks. For part (b) there was an even split between the choice of method: triple scalar product or vector product followed by scalar product. The latter proved to be the more successful method. Errors were more often seen with direct evaluation of the triple scalar product due to omission of minus signs when expanding the determinant or expanding brackets incorrectly. Part (c) was answered well by most students. Marks were lost by students who referred to the points "being linearly independent" and on occasions consistency or inconsistency were randomly mentioned. Part (d) proved to be discriminating with a significant number of students failing to appreciate that a volume of 60 cubic units could result from a determinant of ± 60. It was disappointing to see that some students who did realise this then failed to solve the resulting equations correctly.

Question 6

Almost every student realised that part (a) involved evaluating the determinant of the matrix given and setting it equal to 1. However not every student justified the reason for this and lost the final accuracy mark. However, the vast majority of students do fully appreciate that a shear transformation leaves areas unchanged. Part (b) was attempted quite well, although a number of students were unable to formulate the equation using the fixed point, clearly not understanding what a fixed point meant. Again it was disappointing to see that some students who had set up the equation correctly then failed to solve it fully. A common error was to write $2a + 2b = 2$ hence $a + b = 2$. Part (c) proved more successful than in the past with many students able to at least score the first M1 mark. The proof was not always fully completed correctly. Those who chose to consider $\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ mx+k \end{pmatrix}$ and then used $y' = mx' + k$ were usually able to score both M1 marks and obtain a condition on the value of m, although often failing to fully explain what then happens to k.

Question 7

Almost all students attempted the vector product in part (a). Most students then correctly indicated how the vector $\begin{pmatrix}1\\5\\2\end{pmatrix}$ was related to their answer, whilst a small number lost a mark by making the valid connection. Many then used a valid point to find the value of d correctly. Common errors were to try to use (-1, -2, 1) or one of the direction vectors. Surprisingly part (b) caused more problems than expected with some unable to find a value for c. Part (c)(i) was exceptionally well done. A serious error was to not use the two normal vectors and such attempts scored no marks. When more minor errors occurred they related to using sine instead of cosine with the scalar product, evaluating the scalar product or modulus of one vector incorrectly and subtracting their valid answer from 90° at the end. Part (c)(ii) was well attempted. Both methods indicated in the mark scheme were seen with equal frequency. The method of finding a perpendicular vector to both normals and attempting to find a common point was generally more successful. The other method often led to algebraic slips. There was evidence that students were confident with the different forms for straight lines with almost all students able to put their values in the appropriate forms.

Question 8

This proved to be the most challenging question on the paper with an even split of very high or very low marks. All students knew how to set up the characteristic equation, but a very significant number could not find the solutions to the relevant quadratic. It was disappointing to see so many poor attempts at using the quadratic formula. Those who attempted factorisation often had incorrect signs. Students who failed to solve the quadratic were then very limited in the subsequent marks they could get, although some recovery would have been possible in latter parts. For those students who did solve the quadratic correctly to obtain two distinct solutions sometimes then failed to generate the correct equations in part (b) through errors with minus signs. Constructing the **U** and **D** matrices is well understood. Part (d) confounded most students which was surprising given that it is a standard piece of book work. Some students were able to score M1 by focussing on a specific value of n. Many students failed to explain how the $\mathbf{U}^{-1}\mathbf{U}$ pairs can be simplified, often referring to “cancelling out” or writing **U**/**U.** It was critical to refer explicitly to the identity matrix. Some students tried to show the result using the matrix given in this question with very little success and using very spurious algebra. Part (e) was answered by the best students who had a good understanding of limiting values. When errors occurred here they were the result of an incorrect $\mathbf{U}^{-1}$ or careless slips in multiplication. On occasions the order of matrices was switched showing that some students did not understand key properties of matrix algebra.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.

Converting Marks into UMS marks

Convert raw marks into Uniform Mark Scale (UMS) marks by using the link below.

UMS conversion calculator www.aqa.org.uk/umsconversion