

A-LEVEL

Mathematics

Pure Core 1 – MPC1
Mark scheme

6360
June 2014

Version/Stage: Final V1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

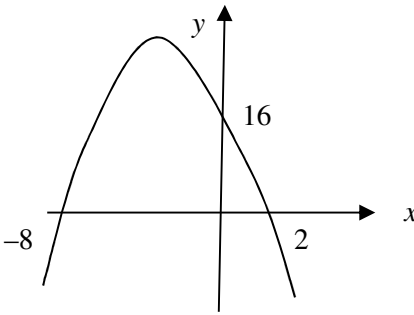
Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment	
1	(a)(i) Grad $AB = \frac{-5-2}{3--1}$ OE $= -\frac{7}{4}$	M1	2	correct unsimplified eg $\frac{2--5}{-1-3}$	
		A1			
	(ii) $y--5 = \text{'their grad' } (x-3)$ $y-2 = \text{'their grad' } (x--1)$ $y-2 = -\frac{7}{4}(x+1)$ $y+5 = -\frac{7}{4}(x-3)$ $y = -\frac{7}{4}x + \frac{1}{4}$ $7x+4y=1$	M1		either pair of coordinates used correctly and attempt to find c if using $y=mx+c$	
		A1		OE, any form of correct equation with -- simplified to +	
		A1	3	integer coefficients & in this form	
	(b)(i)	(M) $(1, -1.5)$	B1	1	condone $x=1, y = -\frac{3}{2}$
	(ii) Perp grad = $\frac{4}{7}$ $y--\frac{3}{2} = \text{'their' } \frac{4}{7}(x-1)$ $y+\frac{3}{2} = \frac{4}{7}(x-1)$	B1 ✓			perp grad = $-1/\text{'their' grad } AB$
		M1			ft 'their M ' but must have attempted perpendicular gradient
		A1	3	any correct form with -- simplified to + eg $8x-14y=29$; $y = \frac{4}{7}x+c, c = -\frac{29}{14}$	
	(c) $(AC^2) = (k--1)^2 + (2k+3-2)^2$ $k^2+2k+1+4k^2+4k+1=13$ $5k^2+6k-11=0$ $(5k+11)(k-1)=0$ $\Rightarrow k=1, k = -\frac{11}{5}$	M1			$(k+1)^2 + (2k+1)^2$
A1					
A1				correct factors or correct use of formula as far as $\frac{-6 \pm \sqrt{256}}{10}$	
A1		4			
Total			13		

(a) (i) NMS grad $AB = -\frac{7}{4}$ earns 2 marks.
(ii) must simplify $y--5$ to $y+5$ or $x--1$ to $x+1$ for first **A1**
 Condone $8y+14x=2$ etc for final **A1**, but not $7x+4y-1=0$ etc
(b)(ii) If their gradient of AB is m , then use of $-m$ or $1/m$ can earn **M1**. For **A1**, $1/(\frac{7}{4})$, $\frac{14.5}{7}$ etc must be simplified.

Q	Solution	Mark	Total	Comment
2	$\frac{15+7\sqrt{3}}{9+5\sqrt{3}} \times \frac{9-5\sqrt{3}}{9-5\sqrt{3}}$ <p>(Numerator =) $135 - 75\sqrt{3} + 63\sqrt{3} - 105$</p> <p>(Denominator = $81 - 45\sqrt{3} + 45\sqrt{3} - 75$) = 6</p> $\left(\frac{30-12\sqrt{3}}{6} = \right) 5 - 2\sqrt{3}$ <p>Alternative</p> $(9+5\sqrt{3})(m+n\sqrt{3})$ $= 9m+15n+5m\sqrt{3}+9n\sqrt{3}$ $9m+15n=15, \quad 5m+9n=7$ $m=5, \quad n=-2$ $5-2\sqrt{3}$	<p>M1</p> <p>A1</p> <p>B1</p> <p>A1cso</p> <p>(M1)</p> <p>(A1)</p> <p>(A1)</p> <p>(A1)</p>	<p>4</p>	<p>writing correct quotient and multiplying by correct conjugate of denominator</p> <p>$30 - 12\sqrt{3}$</p> <p>must be seen as denominator</p> <p>units (cm) need not be given</p> <p>must be correct both equations correct either correct</p>
Total			4	
<p>No marks if candidate uses $\frac{9+5\sqrt{3}}{15+7\sqrt{3}}$</p> <p>Condone multiplication by $9-5\sqrt{3}$ instead of $\frac{9-5\sqrt{3}}{9-5\sqrt{3}}$ for M1 only if subsequent working shows multiplication by both numerator and denominator – otherwise M0.</p> <p>May use alternative conjugate $\frac{15+7\sqrt{3}}{9+5\sqrt{3}} \times \frac{5\sqrt{3}-9}{5\sqrt{3}-9}$ M1 numerator = $12\sqrt{3}-30$ A1 denominator = -6 B1</p> <p>Ignore any incorrect units</p>				


Q	Solution	Mark	Total	Comment
3 (a)(i)	$\left(\frac{dy}{dx} =\right) 10x^4 + 20x^3$	M1 A1	2	one term correct all correct (no + c etc)
(ii)	$\left(\frac{d^2y}{dx^2} =\right) 40x^3 + 60x^2$	B1 ✓	1	ft their $\frac{dy}{dx}$
(b)(i)	$\left(\frac{dy}{dx} =\right) 10 - 20 = -10$	B1 ✓		correctly sub $x = -1$ into their $\frac{dy}{dx}$ and evaluated correctly
	$\frac{dy}{dx} < 0$ (therefore y is) decreasing	E1 ✓	2	Must state “decreasing” and $\frac{dy}{dx} < 0$ ft ‘therefore y is increasing’ and reason if their value of $\frac{dy}{dx} > 0$
(ii)	(When $x = -1$) $y = 2$	B1		ft ‘ their’ value of $\frac{dy}{dx}$ when $x = -1$ and ‘ their’ y-coordinate
	$y - 'their' 2 = 'their grad'(x - -1)$ but must be tangent and not normal	M1		
	$y - 2 = -10(x + 1)$ or $y = -10x - 8$ etc	A1	3	any correct tangent eqn from correct $\frac{dy}{dx}$
(c)	$\left(\frac{dy}{dx} =\right) 10(-2)^4 + 20(-2)^3$	M1		correctly sub $x = -2$ into their $\frac{dy}{dx}$
	$= 160 - 160 = 0 \Rightarrow$ stationary point (when $x = -2$)	A1		correctly shown that $\frac{dy}{dx} = 0$ plus correct statement
	$\left(\frac{d^2y}{dx^2} =\right) 40(-2)^3 + 60(-2)^2$	M1		correctly sub $x = -2$ into their $\frac{d^2y}{dx^2}$ or other suitable test for max/min
	$= -320 + 240 = -80 < 0$ (Therefore) maximum (point at Q)	A1	4	either $\frac{d^2y}{dx^2} = -320 + 240 < 0$ or $\frac{d^2y}{dx^2} = -80 < 0$ plus conclusion
	Total		12	
(b) (i)	Accept “gradient is negative so decreasing” for E1 Do not accept “because it is negative” or “ $\frac{dy}{dx} = -10$ ” as reasons for E1			
(ii)	May earn M1 for attempt to find c using $y = mx + c$ if clearly finding tangent and not normal. Must simplify $x - -1$ to $x + 1$ for A1			
(c)	May write “their” $10x^4 + 20x^3 = 0$ and attempt to find x for first M1 leading to “ $x = -2$...stationary pt” for A1			

Q	Solution	Mark	Total	Comment
4	(a)(i) $k - (x + 3)^2$	M1		<i>or</i> $x^2 + 6x - 16 = (x + 3)^2 - 25$ <i>or</i> $q = 3$ stated
	$25 - (x + 3)^2$	A1	2	
	(ii) (Max value =) 25	B1 ✓	1	ft their p
	(b)(i) $(8 + x)(2 - x)$	B1	1	
	(ii)		M1	\cap shape
	crosses x -axis at -8 and 2	A1	3	curve roughly symmetrical with max to left of y -axis, curve in all 4 quadrants and y -intercept 16 stated or marked on y -axis
Total			7	
(a)(i)	Example $16 - (x + 3)^2 - 9$ earns M1			
(ii)	$(-3, 25)$ scores B0 since maximum value not identified Allow maximum given as “ $y = 25$ ”			
(b)(i)	Condone $-(x - 2)(x + 8)$, $(x - 2)(-x - 8)$ etc			
(ii)	Withhold B1 if more than 2 intercepts			

Q	Solution	Mark	Total	Comment
5	(a)	$(-3)^3 + c(-3)^2 + d(-3) + 3$	M1	p(-3) attempted
		$-27 + 9c - 3d + 3 = 0$		AG { must see this line or equivalent, and must have = 0 on right or left before final result be convinced
		$\Rightarrow 3c - d = 8$	A1	
(b)	$2^3 + c \times 2^2 + d \times 2 + 3 = 65$	M1		p(2) attempted & ... = 65
	$8 + 4c + 2d + 3 = 65$	A1	2	correct equation in any form simplifying powers of 2 eg $4c + 2d = 54$
(c)	$5c = 35$			correct elimination of c or d using both $3c - d = 8$ and their equation from (b)
	or $10d = 130$ OE	M1		
	$c = 7$ $d = 13$	A1 A1	3	
		Total	7	
(a)	May use long division by $x + 3$ but must reach remainder term for M1 Condone missing brackets in p(-3) expression if recovered later as $-27 + 9c + \dots$ to earn A1			
(b)	Treat parts (b) and (c) holistically May use long division by $x - 2$ as far as remainder and equate their remainder to 65 for M1			
(c)	Example $4c + 2(3c - 8) = 54$ earns M1 for eliminating d if equation in part (b) is correct			

Q	Solution	Mark	Total	Comment
6	(a)(i) $x^3 - x^2 - 5x + 7 = x + 7$ $\Rightarrow x^3 - x^2 - 5x = x$ $(x \neq 0) \Rightarrow x^2 - x - 6 = 0$	M1	2	must see this line OE eg $x^3 - x^2 - 6x = 0$ AG
		A1		
	(ii) $(x-3)(x+2)$ $x = 3, x = -2$ A(-2,5) and C(3,10)	M1	3	correct
		A1		both x values correct
		A1		both pairs of coordinates correct
	(b) $\frac{x^4}{4} - \frac{x^3}{3} - \frac{5x^2}{2} + 7x$ (+c)	M1	3	2 terms correct
		A1		another term correct
		A1		all correct
	(c) $F(-2) = \left[\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - \frac{5(-2)^2}{2} + 7(-2) \right]$ $F(0) - F(-2) =$ $0 - \left(\frac{16}{4} + \frac{8}{3} - \frac{20}{2} - 14 \right) = \frac{52}{3}$ Area of trapezium = $\left(\frac{1}{2}(5+7) \times 2 \right) = 12$ Area of $R = \frac{52}{3} - 12 = \frac{16}{3}$	M1	4	F('their' -2) correctly substituting into their answer to (b), but must have scored M1 in part (b)
		A1		correct value using limits correctly
		B1		or rectangle plus triangle
		A1		$5\frac{1}{3}$ or $5.\dot{3}$
Total			12	
(a)(ii)	NMS either (-2,5) or (3,10) scores SC1 and both correct scores SC3 Allow "when $x = 3, y = 10$ and when $x = -2, y = 5$ " instead of coordinates for final A1			
(c)	Condone missing brackets around "their" -2 for M1 and if recovered and correct on next line for A1 Area of trapezium found by integration $\int_{-2}^0 (x+7) dx = \left[\frac{x^2}{2} + 7x \right]_{-2}^0 = 12$ earns B1 Accept rounded answer of 5.3 etc after correct exact answer seen.			

Q	Solution	Mark	Total	Comment
7				
(a)	$(x-5)^2 + (y-6)^2$ $(x-5)^2 + (y+6)^2 = 20$	M1 A1 A1	 3	one term correct LHS correct with perhaps extra constant terms equation completely correct
(b) (i)	$C(5, -6)$	B1 ✓	1	correct or ft their (a)
(ii)	(radius =) $\sqrt{20}$ $= 2\sqrt{5}$	M1 A1	 2	correct or ft 'their' \sqrt{k} provided RHS > 0 must see $\sqrt{20}$ first
(c)	Grad AC = $\frac{-6-6}{5-3} (= -2)$ Grad of tangent = $\frac{1}{2}$ Equation of tangent is $(y-6) = \frac{1}{2}(x-3)$ $y+6 = \frac{1}{2}(x-3)$ $x-2y=7$	M1 B1 ✓ M1 A1 A1 cso	 5	correct unsimplified, ft their coords of C ft their $-1/\text{grad AC}$ clear attempt at tangent not normal through (3, -2) correct equation in any form but $y-6$ must be simplified to $y+6$
(d)	$AB^2 + (\text{their } r)^2 = 6^2$ $d^2 + 20 = 36 \text{ or } (AB^2) = 36 - 20$ $AB^2 = 16$ Hence $AB = 4$	M1 A1 A1cso	 3	Pythagoras used with 6 as hypotenuse values correct with $(2\sqrt{5})^2 = 20$ PI notation all correct
	Total		14	
(a)	$(x-5)^2 + (y-6)^2 = (\sqrt{20})^2$ scores full marks If final equation is correct then award 3 marks, treating earlier lines with extra terms etc as rough working. If final equation has sign errors then check to see if M1 is earned. Example $(x-5)^2 + (y+6)^2 - 25 + 36 + 41 = 0$ earns M1 A1 but if this is part of preliminary working and final equation is offered as $(x-5)^2 + (y+6)^2 = 20$ then award M1 A1 A1 . Example $(x-5)^2 + (y-6)^2 = 20$ earns M1 A0 ; Example $(x+5)^2 + (y-6)^2 = 20$ earns M0			
(b)(ii)	Candidates may still earn A1 here provided RHS of circle equation is 20. Example $(x+5)^2 + (y-6)^2 = 20$ earns M0 in (a) but can then earn M1 A1 for radius = $\sqrt{20} = 2\sqrt{5}$ NMS or no $\sqrt{20}$ seen; “radius = $2\sqrt{5}$ ” scores SC1 since question says “show that”			
(c)	May earn second M1 for attempt to find c using $y=mx+c$ if clearly finding tangent and not normal. If their gradient of AC is m , then use of $-m$ or $1/m$ with correct coordinates can earn second M1			
(d)	Example $AB = 36 - (2\sqrt{5})^2 = 16 = 4$ scores M1 A1 A0 for poor notation NMS $AB = 4$ scores SC1 since no evidence that exact value of radius has been used.			

Q	Solution	Mark	Total	Comment
8	<p>(a) $3 - 6x - 15x - 10 > 0$</p> $-21x > 7$ $\Rightarrow x < -\frac{1}{3}$	M1		Correctly multiplied out with > 0
		A1cso	2	all working correct
	<p>(b) $6x^2 - x - 12 \leq 0$</p> $(3x + 4)(2x - 3)$ <p>CVs are $-\frac{4}{3}, \frac{3}{2}$</p> $\begin{array}{c} + \quad \quad \quad - \quad \quad \quad + \\ \hline -\frac{4}{3} \quad \quad \quad \frac{3}{2} \end{array}$	M1		correct factors or correct use of formula as far as $\frac{1 \pm \sqrt{289}}{12}$
		A1		
		M1		use of sign diagram or graph with CVs clearly shown
		A1	4	or $\frac{3}{2} \geq x \geq -\frac{4}{3}$
	Total		6	
	TOTAL		75	
(a)	Allow final answer in form $-\frac{1}{3} > x$.			
(b)	<p>For second M1, if critical values are correct then sign diagram or sketch  must be correct with correct CVs marked.</p> <p>However, if CVs are not correct then second M1 can be earned for attempt at sketch or sign diagram but their CVs MUST be marked on the diagram or sketch.</p> <p>Final A1, inequality must have x and no other letter.</p> <p>Final answer of $x \leq \frac{3}{2}$ AND $x \geq -\frac{4}{3}$ (with or without working) scores 4 marks .</p> <p>(A) $-\frac{4}{3} < x < \frac{3}{2}$ (B) $x \leq \frac{3}{2}$ OR $x \geq -\frac{4}{3}$ (C) $x \leq \frac{3}{2}$, $x \geq -\frac{4}{3}$ (D) $-\frac{4}{3} \leq k \leq \frac{3}{2}$</p> <p>with or without working each score 3 marks (SC3)</p> <p>Example NMS $\frac{4}{3} \leq x \leq \frac{3}{2}$ scores M0 (since one CV is incorrect)</p> <p>Example NMS $x < \frac{3}{2}$, $x < -\frac{4}{3}$ scores M1 A1 M0 (since both CVs are correct)</p>			