
A-LEVEL

Mathematics

Further Pure 2 – MFP2
Mark scheme

6360
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Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

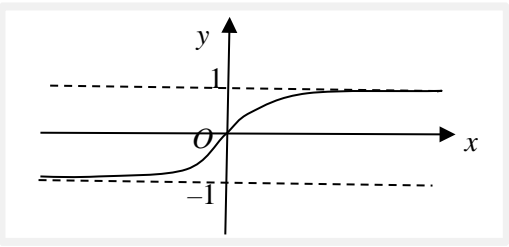
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

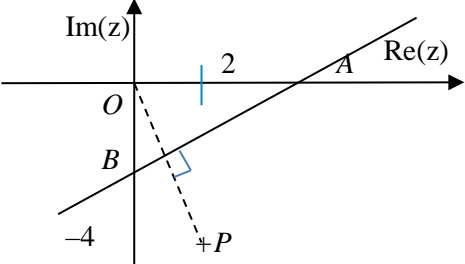
Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	<p>(a) $r+1 = A(r+2) + B$ or</p> $1 = \frac{A(r+2)}{r+1} + \frac{B}{r+1}$ <p>either $A=1$ or $B=-1$</p> $\frac{1}{(r+2)r!} = \frac{1}{(r+1)!} - \frac{1}{(r+2)!}$ <p>(b) $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \dots$</p> $\frac{1}{(n+1)!} - \frac{1}{(n+2)!}$ <p>Sum = $\frac{1}{2} - \frac{1}{(n+2)!}$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>3</p> <p>2</p> <p>5</p>	<p>OE with factorials removed</p> <p>correctly obtained</p> <p>allow if seen in part (b)</p> <p>use of their result from part (a) at least twice</p> <p>must simplify 2! and must have scored at least M1 A1 in part (a)</p>
	Total		5	
	<p>(a) Alternative Method Substituting two values of r to obtain two correct equations in A and B with factorials evaluated correctly</p> $r=0 \Rightarrow \frac{1}{2} = A + \frac{B}{2} \quad ; \quad r=1 \Rightarrow \frac{1}{3} = \frac{A}{2} + \frac{B}{6}$ <p>earns M1 then A1, A1 as in main scheme</p> <p>NMS $\frac{1}{(r+1)!} - \frac{1}{(r+2)!}$ earns 3 marks.</p> <p>However, using an incorrect expression resulting from poor algebra such as $1 = A(r+2)! + B(r+1)!$ with candidate often fluking $A=1, B=-1$ scores M0 ie zero marks which you should denote as FIW</p> <p>These candidates can then score a maximum of M1 in part (b).</p> <p>(b) ISW for incorrect simplification after correct answer seen</p>			

Q2	Solution	Mark	Total	Comment
(a)	 <p>Graph roughly correct through O</p> <p>Correct behaviour as $x \rightarrow \pm\infty$ & grad at O</p> <p>Asymptotes have equations $y = 1$ & $y = -1$</p>	<p>M1</p> <p>A1</p> <p>B1</p>	<p>3</p>	<p>condone infinite gradient at O for M1</p> <p>must state equations</p> <p>both correct ACF or correct squares of these expressions seen</p> <p>attempt to combine their squared terms with correct single denominator</p> <p>AG valid proof convincingly shown to equal 1 including LHS seen</p> <p>correct use of identity from part (b)</p> <p>forming quadratic in $\tanh x$</p> <p>obtained from correct quadratic</p> <p>FT a value of k provided $k < 1$</p> <p>both solutions correct and no others</p> <p>any equivalent form involving \ln</p>
(b)	$\operatorname{sech} x = \frac{2}{e^x + e^{-x}} ; \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $(\operatorname{sech}^2 x + \tanh^2 x) = \frac{2^2 + (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$ $\operatorname{sech}^2 x + \tanh^2 x = \frac{e^{2x} + 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}} = 1$	<p>B1</p> <p>M1</p> <p>A1</p>	<p>3</p>	
(c)	$6(1 - \tanh^2 x) = 4 + \tanh x$ $6 \tanh^2 x + \tanh x - 2 = 0$ $\tanh x = \frac{1}{2}, \quad \tanh x = -\frac{2}{3}$ $\tanh x = k \Rightarrow x = \frac{1}{2} \ln \left(\frac{1+k}{1-k} \right)$ $x = \frac{1}{2} \ln 3, \quad x = \frac{1}{2} \ln \frac{1}{5}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1F</p> <p>A1</p>	<p>5</p>	
Total			11	
(a)	<p>Actual asymptotes need not be shown, but if asymptotes are drawn then curve should not cross them for A1. Gradient should not be infinite at O for A1.</p>			
(b)	<p>Condone trailing equal signs up to final line provided “$\operatorname{sech}^2 x + \tanh^2 x =$” is seen on previous line for A1</p> <p>Denominator may be $e^{4x} + 4e^{2x} + 6 + e^{4x} + 4e^{-2x} + e^{-4x}$ etc for M1 and A1</p> <p>Accept $\operatorname{sech}^2 x + \tanh^2 x = \frac{(e^x + e^{-x})^2}{(e^x + e^{-x})^2} = 1$ for A1</p> <p>Alternative : $\left(\frac{1}{\cosh^2 x} + \frac{\sinh^2 x}{\cosh^2 x} \right) = \frac{1 + \left(\frac{1}{2}(e^x - e^{-x}) \right)^2}{\left(\frac{1}{2}(e^x + e^{-x}) \right)^2}$ scores B1 M1</p> <p>and then A1 for $\operatorname{sech}^2 x + \tanh^2 x = \frac{\frac{1}{4}e^{2x} + \frac{1}{4}e^{-2x} + \frac{1}{2}}{\frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}} = 1$, (all like terms combined in any order).</p>			

Q3	Solution	Mark	Total	Comment
(a)	$\frac{dx}{dt} = 1 - \frac{1}{t^2}$	B1		OE eg $\frac{t(2t) - (t^2 + 1)}{t^2}$ ACF
	$\frac{dy}{dt} = \frac{2}{t}$	B1		
	$\left(\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right) = 1 - \frac{2}{t^2} + \frac{1}{t^4} + \frac{4}{t^2}$	M1		squaring and adding their expressions and attempting to multiply out
	$1 + \frac{2}{t^2} + \frac{1}{t^4} = \left(1 + \frac{1}{t^2} \right)^2$	A1	4	AG be convinced
	(b)	$2\pi \int_1^2 (2\ln t) \left(1 + \frac{1}{t^2} \right) dt$	B1	
		M1		integration by parts - clear attempt to integrate $1 + \frac{1}{t^2}$ and differentiate $2\ln t$
$(2\pi) \left\{ (2\ln t) \left(t - \frac{1}{t} \right) - \int \frac{2}{t} \left(t - \frac{1}{t} \right) (dt) \right\}$		A1		correct (may omit limits, 2π and dt)
$2\pi \left[(2\ln t) \left(t - \frac{1}{t} \right) - \left(2t + \frac{2}{t} \right) \right]$ $= 2\pi(3\ln 2 - 5 + 4)$ $= \pi(6\ln 2 - 2)$		A1	5	correct including 2π (no limits required)
	Total		9	
(b)	May have two separate integrals and attempt both using integration by parts for M1 Must see $(2\pi) \left\{ 2t \ln t - \int 2(dt) - \left(2t^{-1} \ln t - \int 2t^{-2}(dt) \right) \right\}$ (may omit limits, 2π and dt) for first A1 and $2\pi \left[(2t \ln t - 2t) - (2t^{-1} \ln t + 2t^{-1}) \right]$ for second A1 Condone poor use of brackets if later recovered.			

Q4	Solution	Mark	Total	Comment
<p>(a)</p> <p>(b)</p>	$f(k+1) = 2^{4k+7} + 3^{3k+4}$ <p>convincingly showing $2^{4k+7} = 16 \times 2^{4k+3}$</p> $f(k+1) - 16f(k)$ $= (81 - 16 \times 3) \times 3^{3k}$ $= 33 \times 3^{3k}$	<p>M1</p> <p>E1</p> <p>A1</p>	<p>3</p>	<p>must see $16 = 2^4$ OE</p>
	<p><i>Assume</i> $f(k)$ is a multiple of 11 (*)</p> $f(k+1) = 16f(k) + 33 \times 3^{3k}$ $= 11M + 11N = 11(M + N)$ <p>Therefore $f(k+1)$ is a multiple of 11</p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>4</p>	<p>$f(1) = 209 = 11 \times 19$ or $209 \div 11 = 19$ etc therefore true when $n=1$</p> <p>attempt at $f(k+1) = \dots$ using their result from part (a) where M and N are integers</p>
	<p>Since $f(1)$ is multiple of 11 then $f(2), f(3), \dots$ are multiples of 11 by induction (or is a multiple of 11 for all integers $n \geq 1$)</p>	<p>E1</p>	<p>4</p>	<p>must earn previous 3 marks and have (*) before E1 can be awarded</p>
	Total			7
<p>(a)</p> <p>(b)</p>	<p>It is possible to score M1 E0 A1</p> <p>Withhold E1 for conclusion such as “a multiple of 11 for all $n \geq 1$” or poor notation, etc</p>			

Q5	Solution	Mark	Total	Comment
(a)	 <p data-bbox="236 548 746 649"> Straight line Through midpoint of OP, P correct Perpendicular to OP, P correct </p>	<p data-bbox="794 548 839 645"> M1 A1 A1 </p>	<p data-bbox="922 616 941 645">3</p>	<p data-bbox="997 286 1508 353">Ignore the line OP drawn in full or circles drawn as part of construction for locus L.</p> <p data-bbox="997 582 1220 616">P represents $2 - 4i$</p>
(b)(i)	$(x-2)^2 + (y+4)^2 = x^2 + y^2$ $2y - x + 5 = 0$ <p data-bbox="395 833 667 866">$A(5,0)$ & $B(0,-2.5)$</p> $C\left(\frac{5}{2}, -\frac{5}{4}\right) \Rightarrow \text{complex num} = \frac{5}{2} - \frac{5}{4}i$	<p data-bbox="794 721 839 750">M1</p> <p data-bbox="794 795 839 824">A1</p> <p data-bbox="794 840 839 869">A1</p> <p data-bbox="794 907 839 936">A1</p>	<p data-bbox="922 907 941 936">4</p>	<p data-bbox="997 833 1348 866">may have $5 + 0i$ and $0 - 2.5i$</p>
(ii)	<p data-bbox="236 996 654 1064"><i>either</i> $\alpha = \frac{5}{2} - \frac{5}{4}i$ <i>or</i> $k = \frac{5\sqrt{5}}{4}$</p> $\left z - \frac{5}{2} + \frac{5}{4}i\right = \frac{5\sqrt{5}}{4}$	<p data-bbox="794 1019 839 1048">M1</p> <p data-bbox="794 1153 839 1182">A1</p>	<p data-bbox="922 1153 941 1182">2</p>	<p data-bbox="997 1019 1452 1086">allow statement with correct value for centre or radius of circle</p> <p data-bbox="997 1153 1316 1187">must have exact surd form</p>
Total			9	
(a)	Withhold the final A1 (if 3 marks earned) if the straight line does not go beyond the $\text{Re}(z)$ axis and negative $\text{Im}(z)$ axis.			
	The two A1 marks can be awarded independently.			
(b)(i)	Alternative 1: $\text{grad } OP = -2 \Rightarrow \text{grad } L = 0.5$ M1 ; $y + 2 = \frac{1}{2}(x - 1)$ OE A1 then A1, A1 as per scheme			
	Alternative 2: substituting $z = x$ (or a) then $z = iy$ (or ib) into given locus equation			
	Both $(x - 2)^2 + 4^2 = x^2$ and $2^2 + (y + 4)^2 = y^2$ M1 ; $4 - 4x + 16 = 0$ and $4 + 8y + 16 = 0$ OE for A1 then A1, A1 as per scheme.			

Q6	Solution	Mark	Total	Comment
<p>(a)</p> <p>(b)</p>	$\sqrt{5+4x-x^2} + \frac{(x-2)\frac{1}{2}(4-2x)}{\sqrt{5+4x-x^2}}$ $(+)\frac{9 \times \frac{1}{3}}{\sqrt{1-\left(\frac{x-2}{3}\right)^2}}$ $\frac{5+4x-x^2}{\sqrt{5+4x-x^2}}$ $\left(\frac{dy}{dx}\right) = 2\sqrt{5+4x-x^2}$ $\frac{1}{k} \left\{ (x-2)\sqrt{5+4x-x^2} + 9\sin^{-1}\left(\frac{x-2}{3}\right) \right\}$ $\frac{1}{\text{"their" } k} \left[\frac{3}{2}\sqrt{\frac{27}{4}} + 9\sin^{-1}\frac{1}{2} \right]$ $= \frac{9}{8}\sqrt{3} + \frac{3}{4}\pi$	<p>M1</p> <p>A1</p> <p>B1</p> <p>A1</p> <p>A1cso</p> <p>M1</p> <p>m1</p> <p>A1 cso</p>	<p>5</p> <p>3</p>	<p>product rule (condone one error)</p> <p>correct unsimplified</p> <p>or $\frac{9}{\sqrt{3^2-(x-2)^2}}$ correct unsimplified</p> <p>last two terms above combined correctly</p> <p>$k = 2$</p> <p>ft “their” k</p> <p>correct sub of limits (simplified at least this far)</p> <p>must have earned 5 marks in part(a) to be awarded this mark</p>
	Total	8		
(a)	Second A1 ; may combine all three terms correctly and obtain $\frac{10+8x-2x^2}{\sqrt{5+4x-x^2}}$			

Q7	Solution	Mark	Total	Comment
(a)	$\alpha\beta + \beta\gamma + \gamma\alpha = 0$	B1	2	May use γ instead of β throughout (b)(i) Clear attempt to eliminate either α or β from “their” equations correct all 3 roots clearly stated or substituting correct root into equation correctly substituting “their” $\alpha^2 = -2i$ and “their” $\alpha^3 = -2 - 2i$ may use any letter instead of y sub their z into cubic equation removing denominators correctly correct and $(y-1)^3$ expanded correctly sum of new roots = 3 M1 for either of the other two formulae correct in terms of $\alpha\beta\gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha + \beta + \gamma$ may use any letter instead of y
	$\alpha\beta\gamma = -\frac{4}{27}$	B1		
(b)(i)	$\alpha\beta + \alpha\beta + \beta^2 = 0$; $\alpha\beta^2 = -\frac{4}{27}$	B1	5	
	$\alpha^3 = -\frac{1}{27}$ or $\beta^3 = \frac{8}{27}$	M1 A1		
	either $\alpha = -\frac{1}{3}$ or $\beta = \frac{2}{3}$	A1		
	$\alpha = -\frac{1}{3}$, $\beta = \frac{2}{3}$, $\gamma = \frac{2}{3}$	A1		
(ii)	$\left(\sum \alpha = 1 = -\frac{k}{27} \Rightarrow\right) k = -27$	B1	1	
(c)(i)	$\alpha^2 = -2i$	B1	2	
	$\alpha^3 = -2 - 2i$	B1		
(ii)	$27(-2 - 2i) - 2ik + 4 = 0$	M1	2	
	$k = -27 + 25i$	A1		
(d)	$y = \frac{1}{z} + 1 \Rightarrow z = \frac{1}{y-1}$	B1	5	
	$\frac{27}{(y-1)^3} - \frac{12}{(y-1)^2} + 4 = 0$	M1		
	$27 - 12(y-1) + 4(y-1)^3 = 0$	A1		
	$27 - 12y + 12 + 4(y^3 - 3y^2 + 3y - 1) = 0$	A1		
	$4y^3 - 12y^2 + 35 = 0$	A1		
	Alternative: $\sum \alpha' = 3 + \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} = 3$	(B1)		
	$\sum \alpha' \beta' = 3 + \frac{2(\alpha\beta + \beta\gamma + \gamma\alpha) + \alpha + \beta + \gamma}{\alpha\beta\gamma}$	(M1)		
	$= 0$	(A1)		
	$\prod = 1 + \frac{\alpha\beta + \beta\gamma + \gamma\alpha + 1 + \alpha + \beta + \gamma}{\alpha\beta\gamma}$	(A1)		
	$= \frac{-35}{4}$	(A1)		
	$4y^3 - 12y^2 + 35 = 0$	(A1)	(5)	
Total			17	

Q8	Solution	Mark	Total	Comment
(a)(i)	$(\omega^5 =) \cos 2\pi + i \sin 2\pi = 1$ So ω is a root of $z^5 = 1$	B1	1	must have conclusion plus verification that $\omega^5 = 1$
(ii)	$\omega^2, \omega^3, \omega^4.$	B1	1	OE powers mod 5 (must not include 1)
(b)(i)	$1 + \omega + \omega^2 + \omega^3 + \omega^4 = \frac{1 - \omega^5}{1 - \omega} = 0$	B1	1	or clear statement that sum of roots (of $z^5 - 1 = 0$) is zero
(ii)	$\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1$ $= \omega^2 + 2 + \frac{1}{\omega^2} + \omega + \frac{1}{\omega} - 1$ $= \frac{1 + \omega + \omega^2 + \omega^3 + \omega^4}{\omega^2} = 0$	M1 A1	2	correct expansion AG correctly shown to = 0 do not allow simply multiplying by ω^2
(c)	$\frac{1}{\omega} = \cos \frac{2\pi}{5} - i \sin \frac{2\pi}{5}$ $\Rightarrow \omega + \frac{1}{\omega} = 2 \cos \frac{2\pi}{5}$ Solving quadratic $\left(\omega + \frac{1}{\omega} = \frac{-1 \pm \sqrt{5}}{2}\right)$ Rejecting negative root since $\cos \frac{2\pi}{5} > 0$ Hence $\cos \frac{2\pi}{5} = \frac{\sqrt{5} - 1}{4}$	M1 A1 M1 A1	4	SC1 if result merely stated must see both values must see this line for final A1 It is possible to score SC1 M1 A1
Total			9	
(b)(ii)	May replace $\frac{1}{\omega^2}$ by ω^3 and $\frac{1}{\omega}$ by ω^4 and/or 1 by ω^5 in valid proof. Alternative: $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0 \Rightarrow \frac{1}{\omega^2} + \frac{1}{\omega} + 1 + \omega + \omega^2 = 0$ M1 $\left(\omega + \frac{1}{\omega}\right)^2 - 2 + \left(\omega + \frac{1}{\omega}\right) + 1 = 0 \Rightarrow \left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1 = 0$ A1			