A-LEVEL
MATHEMATICS

MFP2 – Further Pure 2
Report on the Examination

6360
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General

The questions seemed to provide a challenge for able candidates whilst at the same time allowing weaker candidates to demonstrate their basic understanding of particular topics such as hyperbolic functions, properties of roots of cubic equations, loci of complex numbers and calculus. Most candidates were unable to handle expressions with factorials and algebraic skills were not always sufficient to cope with the demands of various questions. Proofs were very rarely set out in a logical manner with a concluding statement. The presentation of solutions from a significant number of candidates fell short of the standard expected for this level of examination.

Question 1

(a) Many candidates were unable to simplify expressions involving factorials and incorrect expressions such as $A(r+2)!+B(r+1)!=1$ abounded. Many candidates then substituted $r=-2$ being convinced that $0!=0$ and $(-1)!=-1$, obtaining the correct values of A and B from totally incorrect work and therefore scored no marks.

(b) Although the difference method was widely understood and many wrote down the correct answer of $\frac{1}{2}-\frac{1}{(n+2)!}$, full marks were only awarded if the correct values of A and B had been obtained correctly in part **(a)**. Candidates were expected to simplify $2!$ for full marks.

Question 2

(a) Most candidates sketched the graph of $y=\tanh x$ correctly and many indicated the asymptotes by broken lines. It was not sufficient to mark the numbers ± 1 on the axes; the actual equations $y=1$ and $y=-1$ were required.

(b) Since this was a proof, candidates were expected to write $\text{sech}^2 x+\tanh^2 x=...$ before writing the correct expressions in terms of exponentials. Most candidates then realised the need to combine the expressions using a common denominator; the best candidates wrote a conclusion, namely $\text{sech}^2 x+\tanh^2 x=1$. A large number of proofs were incomplete and so did not earn full marks.

(c) This was one of the best answered sections of the whole paper and showed that candidates have been well drilled to solve equations of this type. Having found the correct values of $\tanh x$, candidates were expected to use the formulae book to write down answers in terms of natural logarithms. Some wasted time by using the definition of $\tanh x$ in terms of exponentials.

Question 3

(a) Almost every candidate was able to obtain the printed result. A large number of candidates used the quotient rule to differentiate $\frac{t^2+1}{t}$ but were usually successful in finding a correct form of the derivative; others wrote $2\ln t$ as $\ln t^2$ before differentiating, thus making the task more difficult than intended.

(b) One of the marks was allocated for a correct expression for the curved surface area. Most candidates used the formulae book effectively with hardly anyone omitting the 2π . Many candidates failed to earn this mark if they omitted the limits or $\mathrm{d}t$ from their expression. The resulting integral required the use of integration by parts. The most efficient way was to integrate $1+t^{-2}$ and to differentiate $\ln t$ but many chose to split their expression into two separate integrals and often made sign errors. Others made the substitution $u=\ln t$, but no marks were earned until they used integration by parts to find the integral of $u(\mathrm{e}^u+\mathrm{e}^{-u})$ or $u\cosh u$.

Question 4

(a) This part served as an easy introduction to part **(b)** with many candidates scoring full marks. It was necessary to show convincingly that the powers of 2 actually cancelled out and many made careless algebraic slips such as writing their final answer as 33×3^k .

(b) Many candidates struggled to convince examiners what was actually being proved by induction by inserting terms such as " hence result is true" when there was no mention of divisibility by 11 throughout their solution. Many who found that $\mathrm{f}(1)=209$ failed to show that this was a multiple of 11 and to make a statement to that effect. Those who understood what was required usually made a statement such as "Assume $\mathrm{f}(k)$ is a multiple of 11" and then proceeded to find an explicit expression for $\mathrm{f}(k+1)$ using their result from part **(a).** Many candidates seemed to think that they had to show that $\mathrm{f}(k+1)-16\mathrm{f}(k)$ was divisible by 11. Several candidates who set out their proof quite well spoiled their solution by writing "therefore $\mathrm{f}(n)$ is a multiple of 11 for all $n\geqslant 1$ ".

Question 5

(a) It was encouraging to see most candidates drawing the perpendicular bisector of OP , although quite a few drew other incorrect lines; a few thought the locus was a circle. In order to score full marks the line needed to be drawn beyond the positive real axis and the negative imaginary axis.

(b) (i) The problem here was that candidates simply assumed that the line cut the axes when $\mathrm{Re}(z)=5$ and when $\mathrm{Im}(z)=-2.5$ and then proceeded to find the mid-point of AB without realising that more would be required to earn 4 marks. Simple proofs were based on coordinate geometry using the equation of the straight line or substituting $z=x+\mathrm{i}y$ into the locus equation. Other correct proofs involved trigonometry or similar triangles.

(b) (ii) This final part of the question was omitted by many candidates, even though the centre of the circle was given in the earlier part of the question. Some made arithmetic errors when finding the value of k but any correct surd value was acceptable here.

Question 6

(a) The algebra proved difficult for most candidates with many fudging their working in order to find a value of k ; examiners were not deceived. The chain rule often contained errors when differentiating $\sqrt{5+4x-x^2}$ and $\sin^{-1}\left(\frac{x-2}{3}\right)$ with the factor $\frac{1}{3}$ usually being omitted from the final term. Even those candidates who differentiated the two terms correctly often lacked the necessary algebraic skills to combine at least two of their terms so as to obtain a result in the required form. Some used the substitution $u = x-2$ effectively and this simplified the algebra considerably.

(b) Most candidates scored the first two follow through marks by using their value of k from part **(a)** but full marks were only awarded to those who had legitimately obtained $k = 2$ in the earlier part.

Question 7

Most candidates seemed will drilled in the topic of roots of the cubic equation although careless sign errors were made in the various parts of this question.

(a)(i) The vast majority of candidates scored full marks in this part, though some wrote down the value of the product of the roots as $-4, 4$ or $\frac{4}{27}$.

(b)(i) The basic idea of using their results from earlier and putting two roots equal seemed to be well understood. Those who obtained the correct answers in **(a)** often scored full marks here also. However, some made careless algebraic errors and others were unable to finding the third root, even though they had been successful in finding one of the roots.
(ii) Most candidates who had found the three correct values of the roots were able to find the correct value of k , though quite a few made a sign error or forgot to multiply by 27 .

(c)(i) Some who earned very few marks on this question clearly managed to use the complex number facility on their calculators to obtain α^2 and α^3 without any working shown. Those who tried to square and cube by hand often made arithmetic errors but the majority of candidates did earn both marks for this part.

(ii) Many thought that the sum of the roots was $\alpha+\alpha^2+\alpha^3$ and did not find the correct value of k . Most candidates substituted α into the original cubic equation but many were unable to divide by 2i in order to obtain the correct value of k .

(d) The easiest method was to use a substitution but many lacked confidence as to how to proceed after obtaining an equation such as $\frac{27}{(y-1)^3}-\frac{12}{(y-1)^2}+4=0$.

The usual algebraic errors occurred when candidates tried to find the product of the roots and other necessary expressions in terms of symmetric functions of the original roots. A few candidates who had worked well using this approach left their final answer as $z^3 - 3z^2 + \frac{35}{4} = 0$ and so missed out on the easy final mark.

Question 8

There appeared to be some confusion in this question for candidates who had previously used ω to represent a complex cube root of unity and they insisted on using $1 + \omega + \omega^2 = 0$ throughout.

(a)(i) Although most candidates verified that $\omega^5 = 1$ or solved the equation $z^5 = 1$ by de Moivre's theorem, many candidates failed to write a concluding statement, namely that ω was a root of the equation $z^5 = 1$.

(ii) Although the request seemed fairly straightforward to "write down the other non-real roots in terms of ω", most candidates ignored the instruction and decided to present answers in the form $\cos\theta + \mathrm{i}\sin\theta$ instead and so lost out on an easy mark.

(b) (i) There were two approaches here: summing a geometric progression and using $\omega^5 - 1 = 0$ or stating that the sum of the roots of $z^5 - 1 = 0$ was zero since the coefficient of $z^4 = 0$. Each of these led nicely to the printed equation. Those who started with $\omega^5 - 1 = 0$ obtained the factorised form $(\omega - 1)(\omega^4 + \omega^3 + \omega^2 + \omega + 1) = 0$, but these candidates needed to state that since ω was not real, $\omega \neq 1$, before concluding that $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$.

(ii) A common mistake in this part was multiplying the expanded form of the left hand side, namely $\omega^2 + 2 + \frac{1}{\omega^2} + \omega + \frac{1}{\omega} - 1 = 0$ by ω^2 and then stating that $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$.
Those who wrote the expression as a quotient with ω^2 as a common denominator were usually successful in proving the result. Others started with the equation $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$ and divided by ω^2 to obtain the printed result.

(c) It was good to see not just the very able candidates doing well on this last part of the question. Some credit was given for simply writing down $\omega + \frac{1}{\omega} = 2\cos\frac{2\pi}{5}$ but better candidates proved the result before finding the two roots of the quadratic equation in part **(b)(ii) .** It was then necessary to explain why the negative root was being rejected in order to earn the final mark for obtaining the printed answer.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.

Converting Marks into UMS marks

Convert raw marks into Uniform Mark Scale (UMS) marks by using the link below.

UMS conversion calculator