Centre Number						Candidate Number				
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Other Names										
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MFP4

Unit Further Pure 4

Wednesday 20 May 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The points U, V and W have position vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ respectively relative to an origin O, where

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix} \text{ and } \mathbf{w} = \begin{bmatrix} a \\ 7 \\ -2 \end{bmatrix}$$

(a) Find $(\mathbf{u} \times \mathbf{v}).\mathbf{w}$ in terms of a.

[2 marks]

(b) Given that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are linearly dependent:

(i) find the value of a;

[1 mark]

(ii) express $\mathbf{u}$ as a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

[3 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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2 The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are such that $\mathbf{c} \times \mathbf{a} = 2\mathbf{i}$ and $\mathbf{b} \times \mathbf{a} = 3\mathbf{j}$.

Simplify $(\mathbf{a} + 2\mathbf{b} - 6\mathbf{c}) \times (\mathbf{a} - \mathbf{b} + 3\mathbf{c})$, giving your answer in the form $\lambda\mathbf{i} + \mu\mathbf{j}$.

[5 marks]

QUESTION PART REFERENCE

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3 (a) Factorise completely the determinant

$$\begin{vmatrix} a & b-c & -bc \\ b & a-c & -ca \\ -c & a+b & ab \end{vmatrix}$$

[6 marks]

(b) Hence, or otherwise, find the values of a for which the equations

$$\begin{aligned} ax + \quad\quad y - \ \ 6z &= 0 \\ 3x + (a-2)y - 2az &= 0 \\ -2x + (a+3)y + 3az &= 0 \end{aligned}$$

do not have a unique solution.

[3 marks]

QUESTION PART REFERENCE

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4 (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{M} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

[6 marks]

(b) Given that $\mathbf{U} = \begin{bmatrix} 4 & b \\ a & -2 \end{bmatrix}$ and $\mathbf{U}^{-1}\mathbf{M}\mathbf{U} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, find the values of a and b.

[3 marks]

QUESTION PART REFERENCE

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5 A system of equations is given by

$$2x - 11y - 3z = 1$$
$$5x - 10y - 4z = 6$$
$$9x - 17y - 7z = 11$$

(a) Find the solution of this system of equations, showing all your working.

[5 marks]

(b) Interpret this solution geometrically.

[1 mark]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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6 The line L has equation $\left(\mathbf{r} - \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}\right) \times \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

The plane Π contains the line L and the point $A(4,\ 1,\ -2)$.

(a) Show that A does not lie on the line L.

[1 mark]

(b) Find an equation of the plane Π, giving your answer in the form $\mathbf{r}.\mathbf{n} = c$.

[5 marks]

(c) The point D has coordinates $(8,\ -2,\ 6)$. Find the coordinates of the image of D after reflection in the plane Π.

[5 marks]

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7 The matrix $\mathbf{A} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}$ represents a transformation that is a shear S followed by a transformation T.

(a) The shear S is such that the image of the point $(1,\ 1)$ is $(5,\ -3)$ and the line $y = -x$ is a line of invariant points. Find the matrix that represents S.

[4 marks]

(b) (i) Hence find the matrix that represents the transformation T.

[4 marks]

(ii) Give a full description of the transformation T.

[2 marks]

QUESTION PART REFERENCE

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8 The linear transformation T is represented by the matrix $\mathbf{M} = \begin{bmatrix} 1 & 2 & k \\ 0 & 3 & 4 \\ -1 & 1 & -1 \end{bmatrix}$.

(a) In the case when $\mathbf{M}$ is a non-singular matrix:

(i) find the possible values of k;

[3 marks]

(ii) find $\mathbf{M}^{-1}$ in terms of k.

[5 marks]

(b) In the case when $k = 1$, the matrix $\mathbf{M}^{-1}$ is applied to a solid shape of volume $6\,\text{cm}^3$. Find the volume of the image.

[3 marks]

(c) In the case when $k = 5$, verify that the image of every point under T lies in the plane $x - y + z = 0$.

[3 marks]

(d) Find the value of k for which T has a line of invariant points and obtain the Cartesian equations of this line.

[5 marks]

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