| Centre Number | | | | | | Candidate Number | | | | |
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| For Examiner's Use | |
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| Examiner's Initials | |
| Question | Mark |
| 1 | |
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| TOTAL | |

AQA

General Certificate of Education
Advanced Level Examination
June 2015

# Mathematics MM04

## Unit Mechanics 4

**Wednesday 24 June 2015 9.00 am to 10.30 am**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Time allowed
- 1 hour 30 minutes

### Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

### Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

### Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

**1** Three forces, acting in the $x$–$y$ plane, act at the points with coordinates as listed below, where $a$ and $b$ are constants and **i** and **j** are unit vectors parallel to the $x$-axis and $y$-axis respectively.

| Force | Acting at point |
|---|---|
| $2a\mathbf{i} + \mathbf{j}$ | $(1, 3)$ |
| $8b\mathbf{i} - 2a\mathbf{j}$ | $(-1, 4)$ |
| $11\mathbf{i} + 4b\mathbf{j}$ | $(-5, -2)$ |

These three forces form a couple.

**(a)** Find the value of $a$ and the value of $b$.

**[3 marks]**

**(b)** Determine the magnitude of the couple.

**[4 marks]**

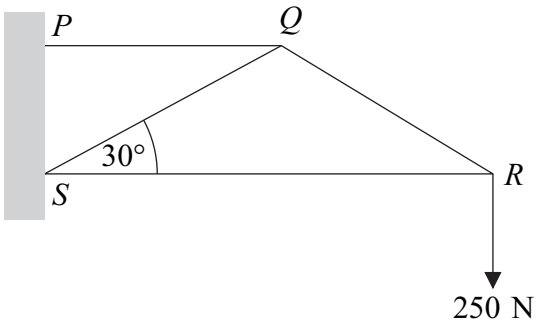
QUESTION PART REFERENCE

**Answer space for question 1**

QUESTION PART REFERENCE

**Answer space for question 1**

**2** A wall-mounted sign is modelled by a framework $PQRS$. The framework consists of four light smoothly jointed rods $PQ$, $QR$, $RS$ and $QS$. It is smoothly hinged to the wall at $P$ and $S$. The rods $QR$ and $QS$ have length $2\,\text{m}$ and angle $QSR$ is $30°$. The rods $PQ$ and $RS$ are horizontal. The rods are all in the same vertical plane which is perpendicular to the wall. A weight of $250\,\text{N}$ hangs from $R$. The framework, as shown in the diagram, is in equilibrium.




**(a)** Find the magnitude of the force in rod $QR$, stating whether this rod is in tension or compression.

**[3 marks]**

**(b)** Determine the magnitude of the force that the wall exerts on the framework at $S$.

**[5 marks]**

QUESTION PART REFERENCE

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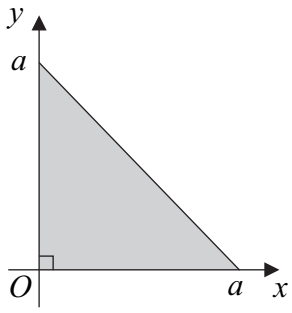
**3** A uniform triangular lamina is bounded by the line $y = a - x$ and the positive $x$ and $y$ axes, as shown in the diagram.



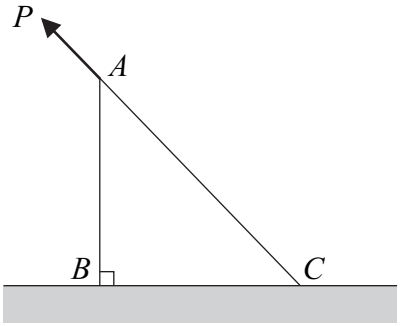

**(a) (i)** Explain why the centre of mass of the lamina lies on the line $y = x$.

**[1 mark]**

**(ii)** By using integration, find the coordinates of the centre of mass of the lamina.

**[5 marks]**

**(b)** A lamina is in the shape of an isosceles right-angled triangle, $ABC$, with its equal sides $AB$ and $BC$ of length $a$. The lamina has weight $W$. It rests in a vertical plane with the side $BC$ on a rough horizontal surface. The coefficient of friction between the lamina and the surface is $\mu$. A force of magnitude $P$ acts at $A$ in the direction parallel to $CA$, as shown in the diagram. The magnitude of the force is gradually increased.




**(i)** Show that when the lamina is on the point of toppling about $B$, $P = \dfrac{W\sqrt{2}}{3}$.

**[3 marks]**

**(ii)** The lamina slides before it topples if and only if $\mu < k$. Determine the value of $k$.

**[6 marks]**

| QUESTION PART REFERENCE | **Answer space for question 3** |
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**4** The forces $3\mathbf{i} - 2\mathbf{j}$, $4\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{j} - 4\mathbf{k}$ act at the points with coordinates $(-1, 1, 0)$, $(2, 0, 5)$ and $(-6, 2, 1)$ respectively.

**(a)** Find the total moment of these forces about the origin.

**[5 marks]**

**(b)** This system of forces is equivalent to a single force $\mathbf{F}$.

**(i)** Find $\mathbf{F}$.

**[1 mark]**

**(ii)** Determine a vector equation, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, for the line of action of $\mathbf{F}$.

**[6 marks]**

QUESTION PART REFERENCE

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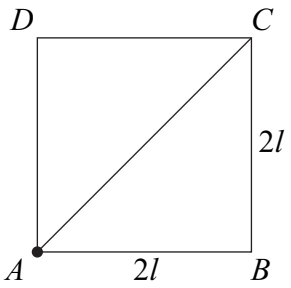
**5** A rigid square framework is formed from five uniform rods, $AB$, $BC$, $CD$, $AD$ and $AC$. Rods $AB$, $BC$, $CD$ and $AD$ are identical with length $2l$ and mass $m$. Rod $AC$ has mass $2m$.




The framework can rotate freely in a vertical plane about a horizontal axis through $A$, perpendicular to the plane of the square $ABCD$.

**(a)** Find the moment of inertia of the rod $AC$ about the axis through $A$.

**[2 marks]**

**(b)** Find the moment of inertia of the rod $BC$ about the axis through $A$.

**[2 marks]**

**(c)** Show that the moment of inertia of the whole framework about the axis through $A$ is $\frac{56}{3}ml^2$.

**[3 marks]**

**(d)** The framework is released from rest with $AB$ horizontal and $D$ vertically above $A$. Find the angular velocity of the framework, in terms of $g$ and $l$, when $B$ is vertically below $A$.

**[5 marks]**

**(e)** At the instant when $B$ is vertically below $A$, rod $AB$ collides with a stationary particle, $P$, of mass $3m$, where $AP = \frac{1}{3}l$ and $P$ is vertically below $A$. Throughout the subsequent motion, $P$ sticks to the rod $AB$. Find, in terms of $g$ and $l$, the angular velocity of the framework immediately after impact.

**[5 marks]**

QUESTION PART REFERENCE

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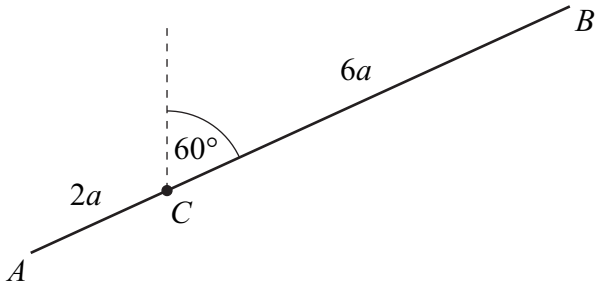
**6** A uniform rod $AB$ has length $8a$ and mass $m$. The point $C$ lies on the rod such that $AC = 2a$. The rod is free to rotate in a vertical plane about a smooth fixed horizontal axis through $C$ which is perpendicular to the rod.

**(a)** Prove by integration that the moment of inertia of the rod about this axis is $\frac{28}{3}ma^2$.

**[4 marks]**

**(b)** Initially the rod is held at rest in the position making an angle of $60°$ with the vertical line through $C$, as shown in the diagram.

**Initial position**




The rod is released from rest and begins to rotate. During the subsequent motion the rod makes an angle $\theta$ with the upward vertical.

**(i)** Find the angular acceleration of the rod in terms of $a$, $g$ and $\theta$.

**[3 marks]**

**(ii)** Find the angular velocity of the rod in terms of $a$, $g$ and $\theta$.

**[6 marks]**

**(iii)** The component of the reaction force parallel to the rod at $C$ is $X$. Find $X$ in terms of $m$, $g$ and $\theta$.

**[3 marks]**

QUESTION PART REFERENCE

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**END OF QUESTIONS**

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